

Visualization and optimization

- Jupyter
- Matplotlib
- `scipy.optimize.minimize`

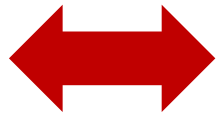


The Jupyter Notebook

The Jupyter Notebook is an open-source web application that allows you to create and share documents that contain live code, equations, visualizations and narrative text. Uses include: data cleaning and transformation, numerical simulation, statistical modeling, data visualization, machine learning, and much more.



IP[y]:
IPython



Jupyter Server
(e.g. running on
local machine)

Prime Number Theorem

$\pi(n)$ = the number of prime numbers $\leq n$. The Prime Number Theorem states that $\pi(n) \approx \frac{n}{\ln(n)}$. In the following we consider all primes $\leq 1,000,000$. First we compute a bitvector 'prime' such that prime[p] is true if and only p is a prime number.

```
In [1]: prime = [True] * 1000001
for p in range(2, 1000001):
    for f in range(2 * p, 1000001, p):
        prime[f] = False
```

We next compute select all the prime numbers in the range 2..100, i.e. idx where prime[idx] = True.

```
In [2]: primes = [p for p in range(2, 1000001) if prime[p]]
```

```
In [3]: primes[:10]
```

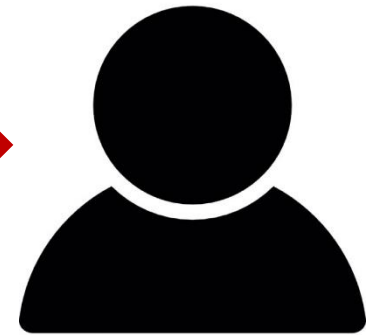
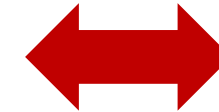
```
Out[3]: [2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
```

```
In [4]: import matplotlib.pyplot as plt
import math
```

```
In [5]: X = range(2, 1000001, 25000)
Y = [len([p for p in primes if p<=x]) for x in X] # slow
plt.plot(X, Y, '.g')
plt.plot(X, [x / math.log(x) for x,y in zip(X,Y)], 'r-')
plt.show()
```

x	Y (primes)	x / ln(x)
0	0	0
200000	15000	15000
400000	30000	30000
600000	45000	45000
800000	60000	60000
1000000	75000	75000

Web Browser



User

cells

python code

The screenshot shows a Jupyter Notebook interface with the following content:

- Cell 1 (Text):** A title "Prime Number Theorem" followed by a paragraph explaining the theorem: $\pi(n)$ is the number of prime numbers $\leq n$. The Prime Number Theorem states that $\pi(n) \approx \frac{n}{\ln(n)}$. In the following we consider all primes $\leq 1,000,000$. First we compute a bitvector 'prime' such that prime[p] is true if and only if p is a prime number.
- Cell 2 (Code):**

```
In [1]: prime = [True] * 1000001
for p in range(2, 1000001):
    for f in range(2 * p, 1000001, p):
        prime[f] = False
```
- Cell 3 (Text):** A comment: "We next compute select all the prime numbers in the range 2..100, i.e. idx where prime[idx] = True."
- Cell 4 (Code):**

```
In [2]: primes = [p for p in range(2, 1000001) if prime[p]]
```
- Cell 5 (Code):**

```
In [3]: primes[:10]
```
- Cell 6 (Output):** The output of Cell 5: `Out[3]: [2, 3, 5, 7, 11, 13, 17, 19, 23, 29]`
- Cell 7 (Code):**

```
In [4]: import matplotlib.pyplot as plt
import math
```
- Cell 8 (Code):**

```
In [5]: X = range(2, 1000001, 25000)
Y = [len([p for p in primes if p<=x]) for x in X] # slow
plt.plot(X, Y, '.g')
plt.plot(X, [x / math.log(x) for x,y in zip(X,Y)], 'r-')
plt.show()
```
- Cell 9 (Figure):** A plot showing the number of primes up to x (green dots) and the approximation $x / \ln(x)$ (red line). The x-axis ranges from 0 to 1,000,000, and the y-axis ranges from 0 to 80,000.

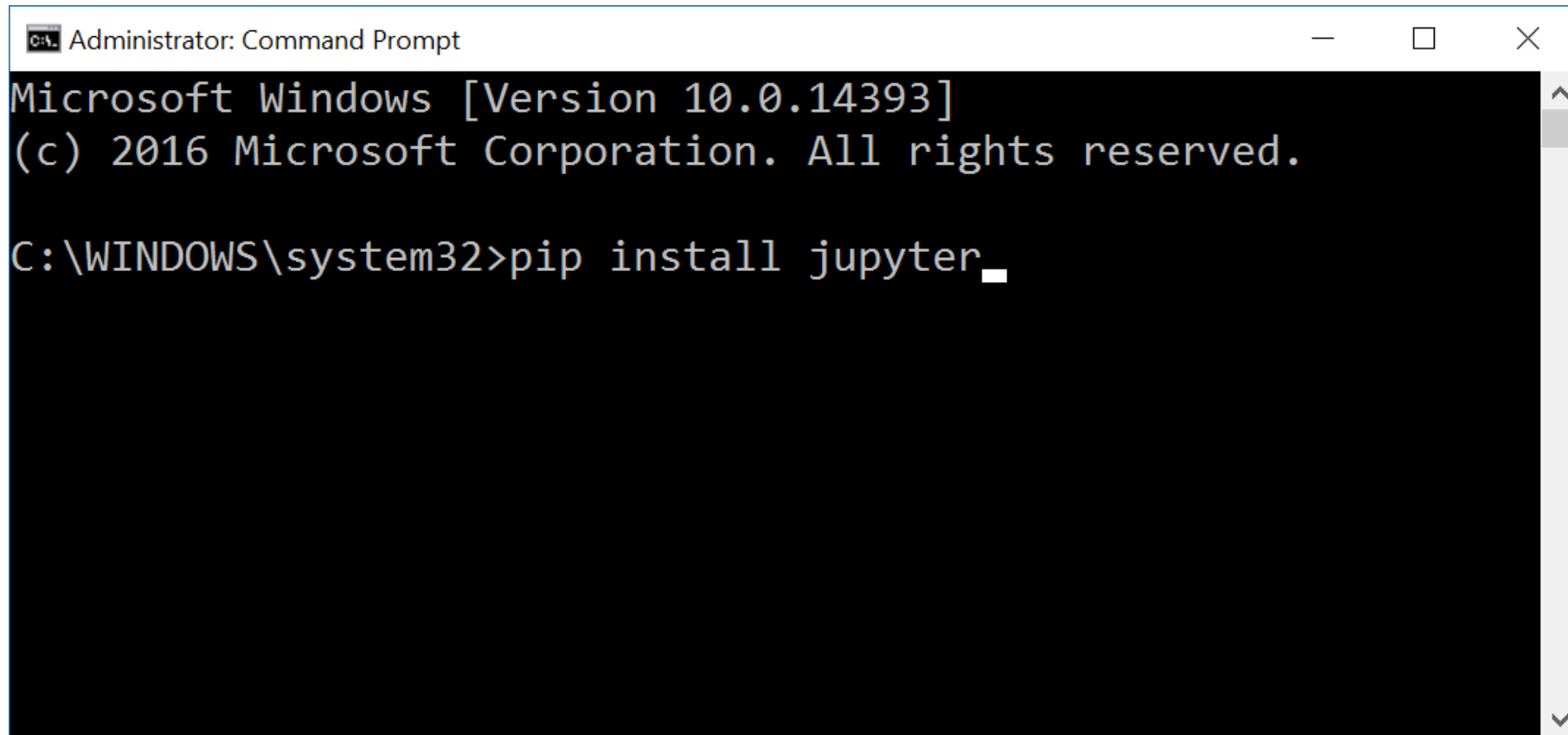
formatted text:
Markdown /
LaTeX / HTML /
...

python shell
output

matplotlib /
numpy / ...
output

Jupyter - installing

- Open a windows shell and run: `pip install jupyter`

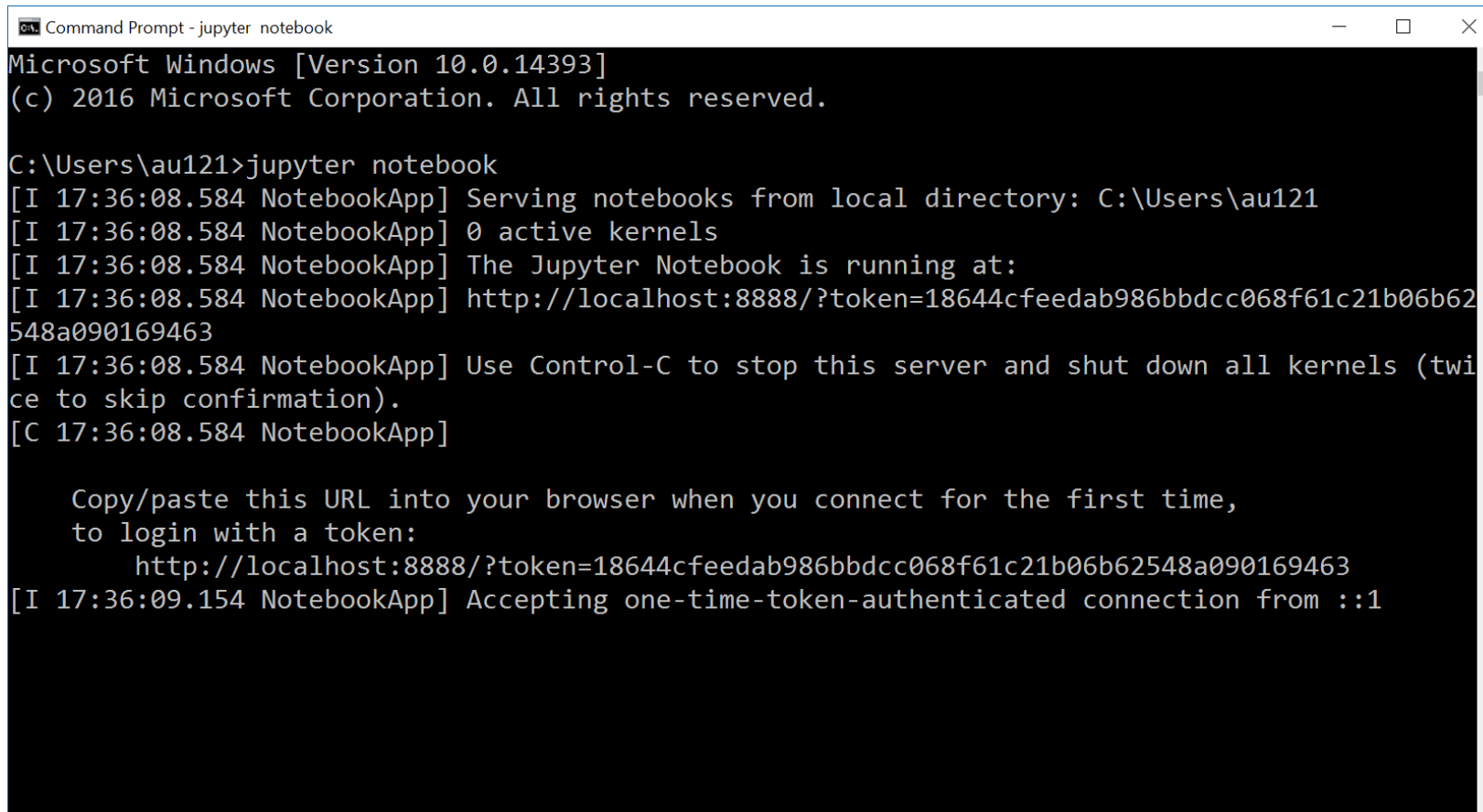


```
Administrator: Command Prompt
Microsoft Windows [Version 10.0.14393]
(c) 2016 Microsoft Corporation. All rights reserved.

C:\WINDOWS\system32>pip install jupyter
```

Jupyter – launching the jupyter server

- Open a windows shell and run: `jupyter notebook`



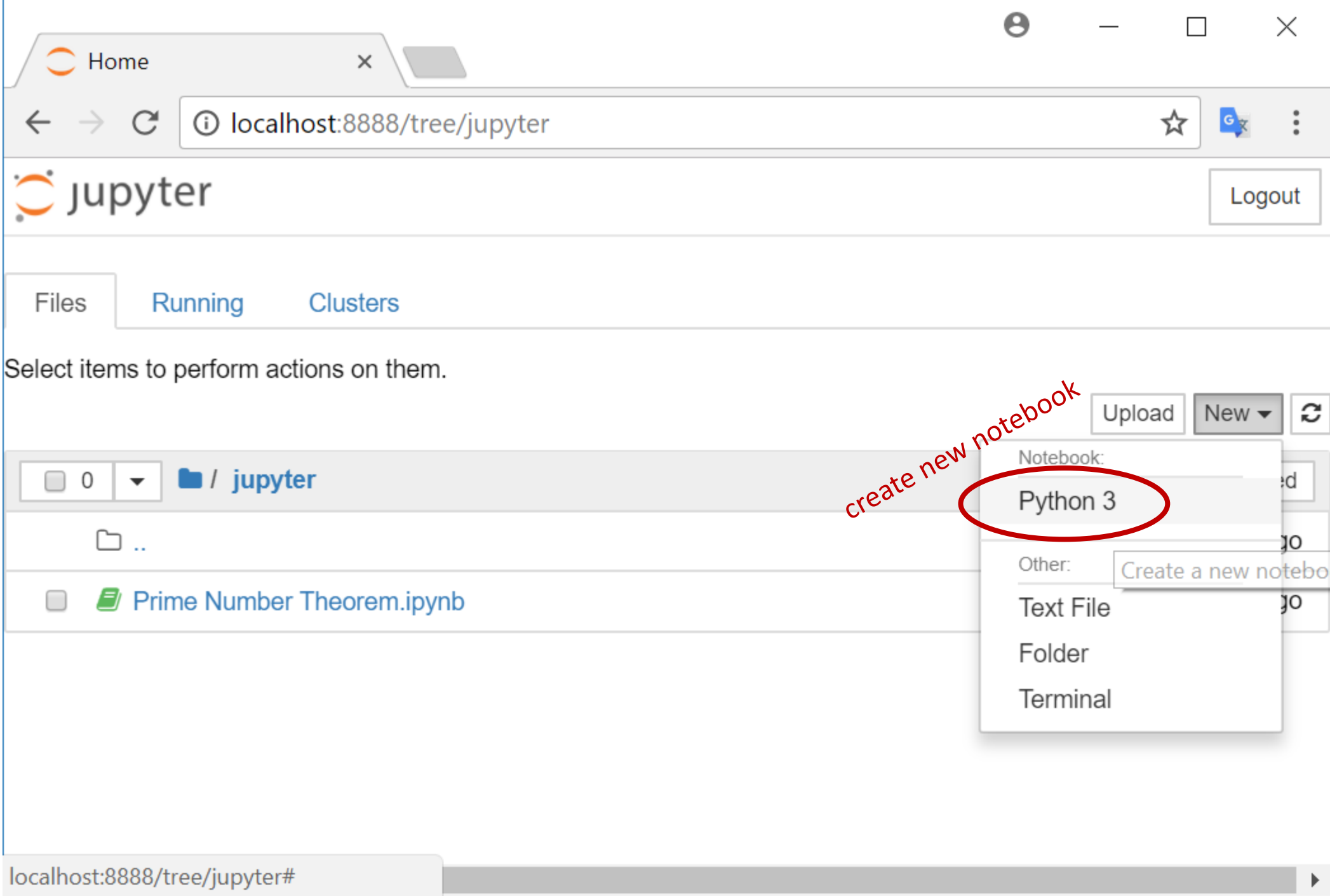
A screenshot of a Windows Command Prompt window titled "Command Prompt - jupyter notebook". The window shows the following text:

```
Microsoft Windows [Version 10.0.14393]
(c) 2016 Microsoft Corporation. All rights reserved.

C:\Users\au121>jupyter notebook
[I 17:36:08.584 NotebookApp] Serving notebooks from local directory: C:\Users\au121
[I 17:36:08.584 NotebookApp] 0 active kernels
[I 17:36:08.584 NotebookApp] The Jupyter Notebook is running at:
[I 17:36:08.584 NotebookApp] http://localhost:8888/?token=18644cfeedab986bbdcc068f61c21b06b62548a090169463
[I 17:36:08.584 NotebookApp] Use Control-C to stop this server and shut down all kernels (twice to skip confirmation).
[C 17:36:08.584 NotebookApp]

Copy/paste this URL into your browser when you connect for the first time,
to login with a token:
    http://localhost:8888/?token=18644cfeedab986bbdcc068f61c21b06b62548a090169463
[I 17:36:09.154 NotebookApp] Accepting one-time-token-authenticated connection from ::1
```

A red arrow points to the command prompt line `C:\Users\au121>jupyter notebook`.



Home

localhost:8888/tree/jupyter

jupyter

Logout

Files Running Clusters

Select items to perform actions on them.

Upload New

0 / jupyter

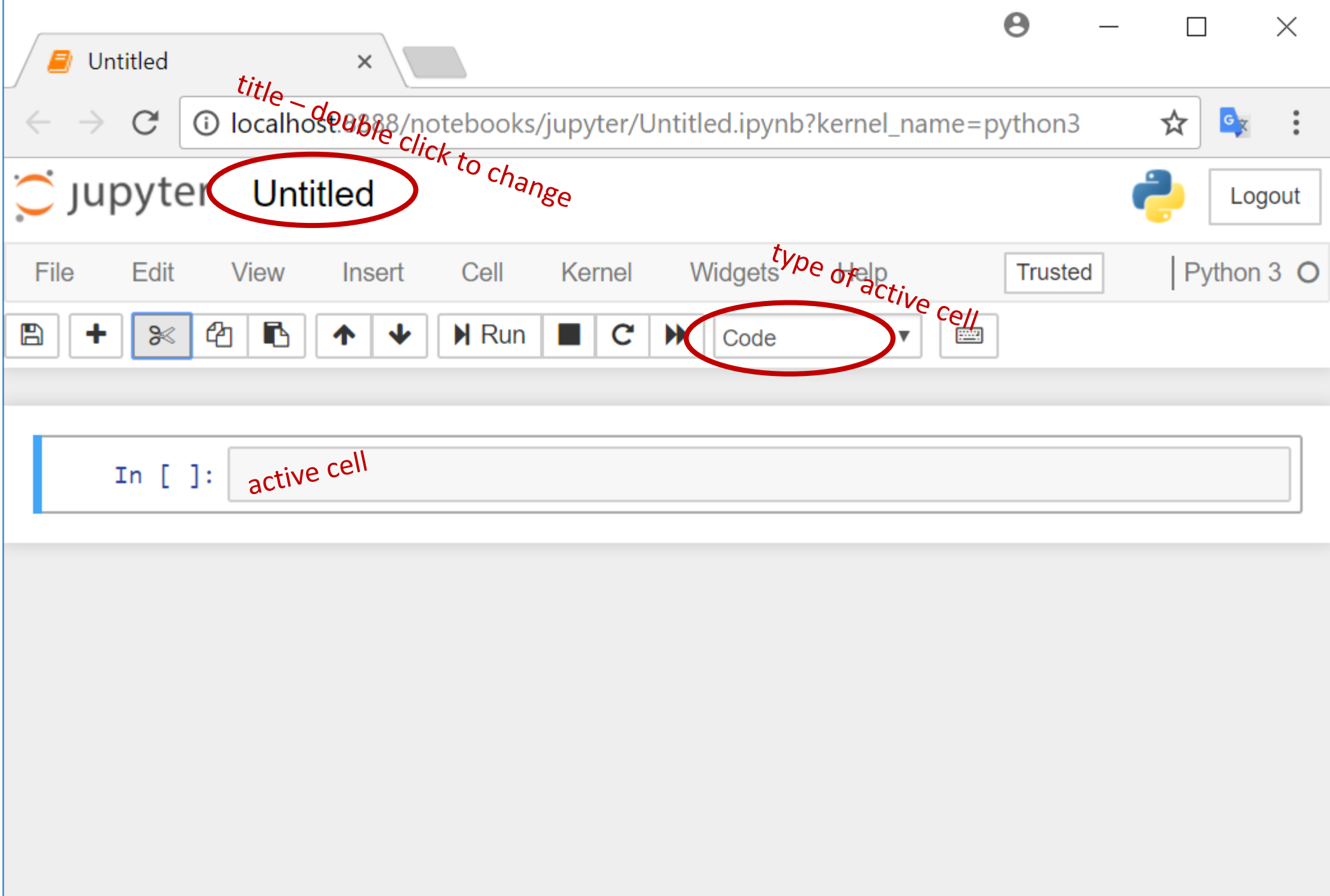
..

Prime Number Theorem.ipynb

create new notebook

- Notebook:
- Python 3
- Other:
- Create a new notebook
- Text File
- Folder
- Terminal

localhost:8888/tree/jupyter#



Markdown test

localhost:8888/notebooks/jupyter/Markdown%20test.ipynb

jupyter Markdown test

File Edit View Insert Cell Kernel Widgets Help Trusted Python 3

Run Markdown

Title of markdown cell

1. first item
2. second item

Here `<i>comes</i> some` math `$x^2 \cdot \left(\int_1^n x\,dx\right)$`

HTML formatting

LaTeX mathematics

Try:
Help > User Interface Tour
Help > Markdown

after pressing
Ctrl + Enter (evaluates)
Alt + Enter (evaluates + new cell)

Title of markdown cell

1. first item
2. second item

Here comes **some** math $x^2 \cdot \left(\int_1^n x dx\right)$

Jupyter

- Widespread tool used for data science applications
- Documentation, code for data analysis, and resulting visualizations are stored in one common format
- Easy to update visualizations
- Works with about 100 different programming languages (not only Python 3), many special features,
- Easy to share data analysis
- *Many online tutorials and examples are available*

matplotlib

Matplotlib is a Python 2D plotting library which produces publication quality figures in a variety of hardcopy formats and interactive environments across platforms. Matplotlib can be used in Python scripts, the Python and IPython shells, the Jupyter notebook, web application servers, and four graphical user interface toolkits.

Matplotlib tries to make easy things easy and hard things possible. You can generate plots, histograms, power spectra, bar charts, errorcharts, scatterplots, etc., with just a few lines of code. For simple plotting the pyplot module provides a MATLAB-like interface, particularly when combined with IPython. For the power user, you have full control of line styles, font properties, axes properties, etc, via an object oriented interface or via a set of functions familiar to MATLAB users.

Some simple matplotlib examples

Plot and Scatter

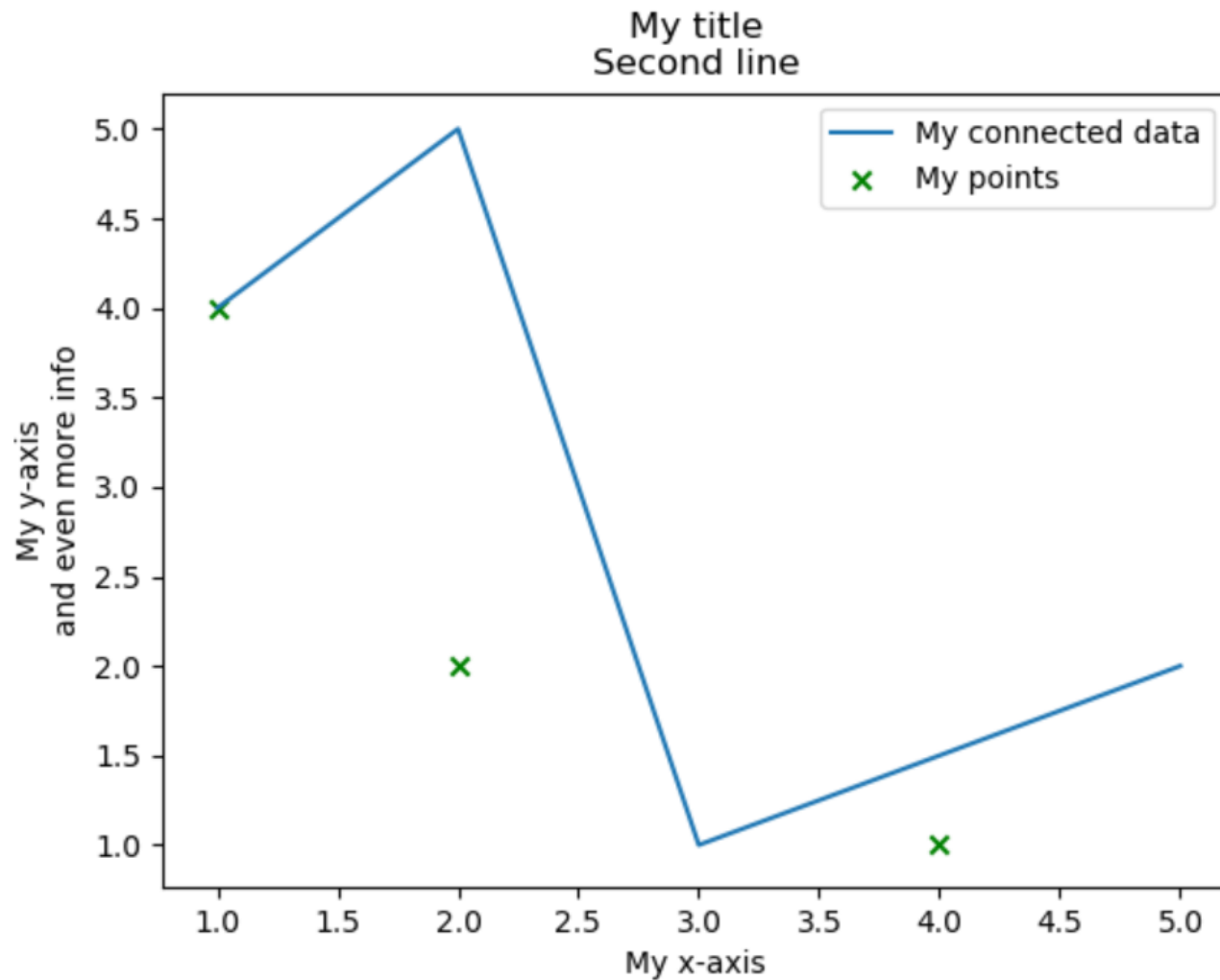
matplotlib-plot.py

```
import matplotlib.pyplot as plt

x1 = [1, 2, 3, 5]
y1 = [4, 5, 1, 2]

x2 = [1, 2, 4]
y2 = [4, 2, 1]

plt.plot(x1, y1, label='My connected data')
plt.scatter(x2, y2, label='My points',
            marker='x', color='green')
plt.xlabel('My x-axis')
plt.ylabel('My y-axis\nand even more info')
plt.title('My title\nSecond line')
plt.legend()
plt.show()
```



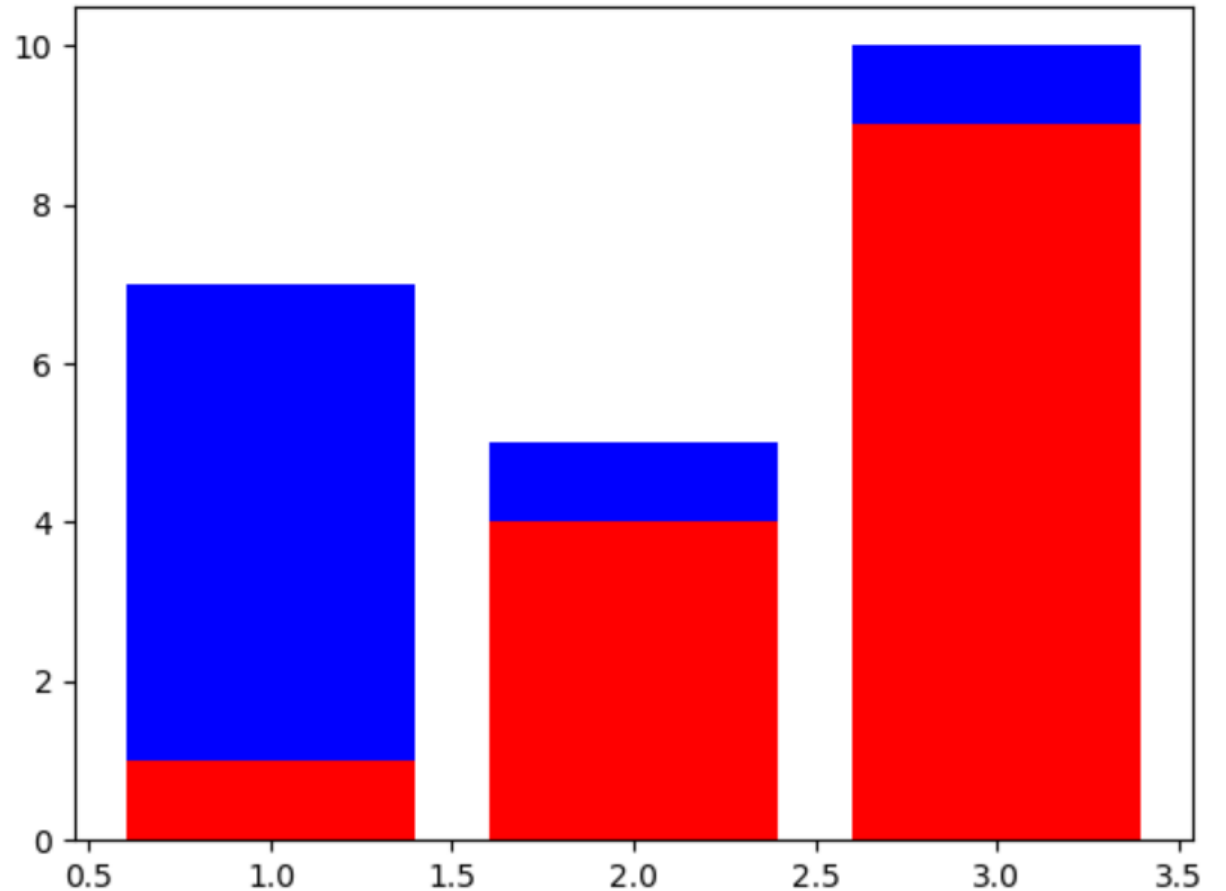
Bars

matplotlib-bars.py

```
import matplotlib.pyplot as plt
```

```
x = [1, 2, 3]  
y = [7, 5, 10]
```

```
plt.bar(x, y, color='blue')  
plt.bar(x, [v**2 for v in x], color='red')  
plt.show()
```



Histogram

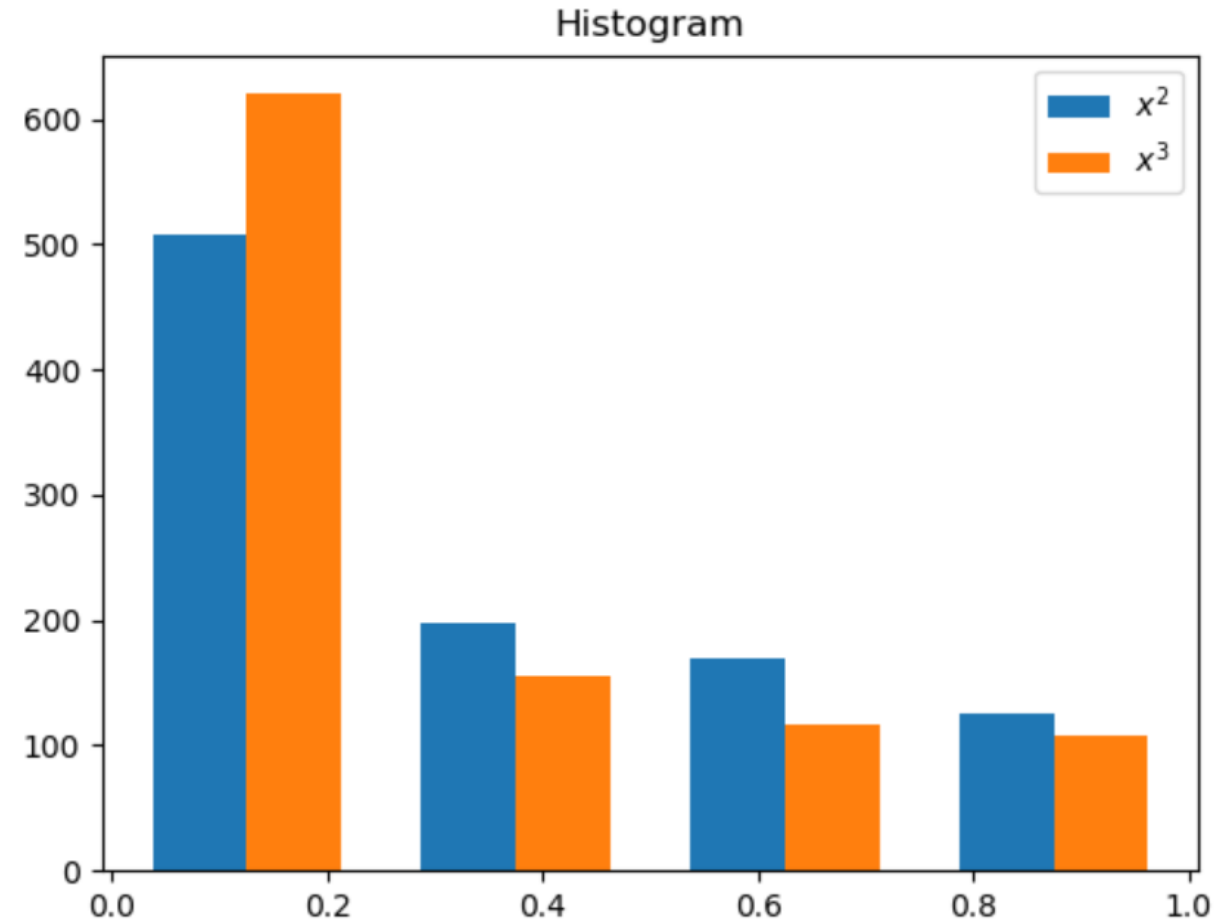
matplotlib-histogram.py

```
import matplotlib.pyplot as plt
from random import random

values1 = [random()**2 for _ in range(1000)]
values2 = [random()**3 for _ in range(1000)]

bins = [0.0, 0.25, 0.5, 0.75, 1.0]

plt.hist([values1, values2], bins, histtype='bar',
         rwidth=0.7, label=[' $x^2$ ', ' $x^3$ '])
plt.title('Histogram')
plt.legend()
plt.show()
```

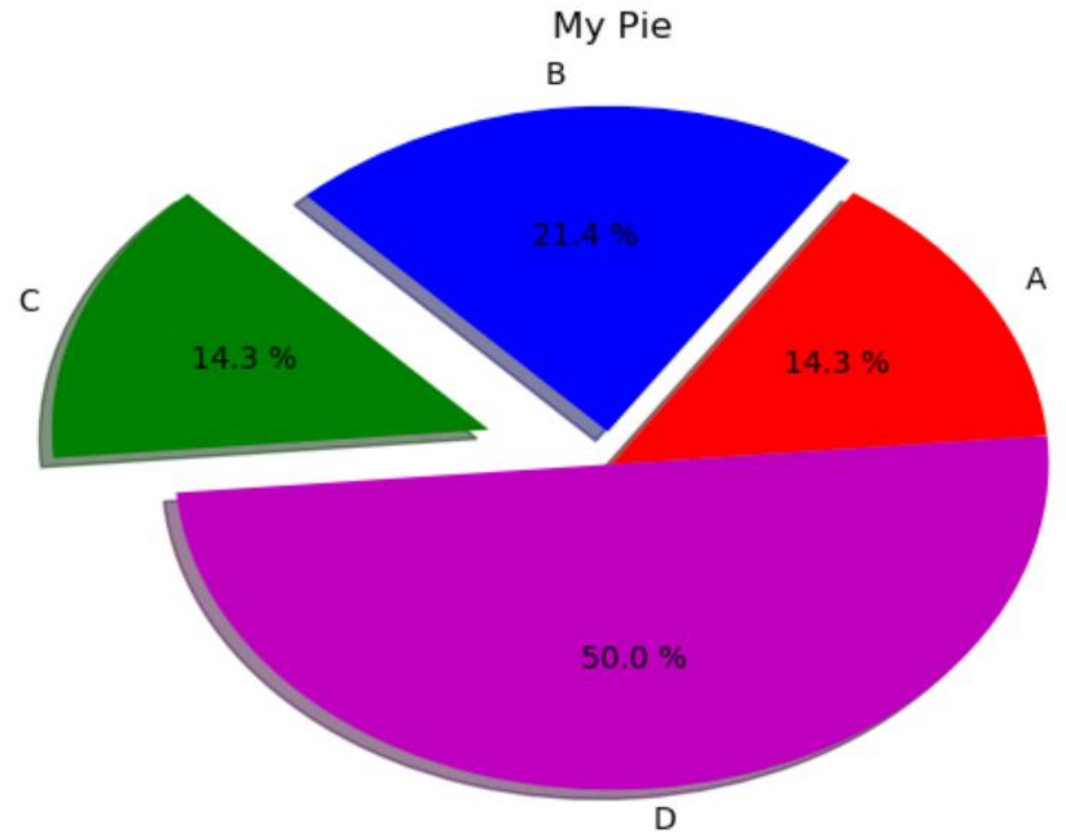


Pie

matplotlib-pie.py

```
import matplotlib.pyplot as plt

plt.title('My Pie')
plt.pie([2, 3, 2, 7],
        labels=['A', 'B', 'C', 'D'],
        colors=['r', 'b', 'g', 'm'],
        startangle=5,
        shadow=True,
        explode=(0, 0.1, 0.3, 0),
        autopct='%1.1f %%' # percent formatting
    )
plt.show()
```



Stackplot

```
matplotlib-stackplot.py
```

```
import matplotlib.pyplot as plt
from matplotlib import style

style.use('ggplot')

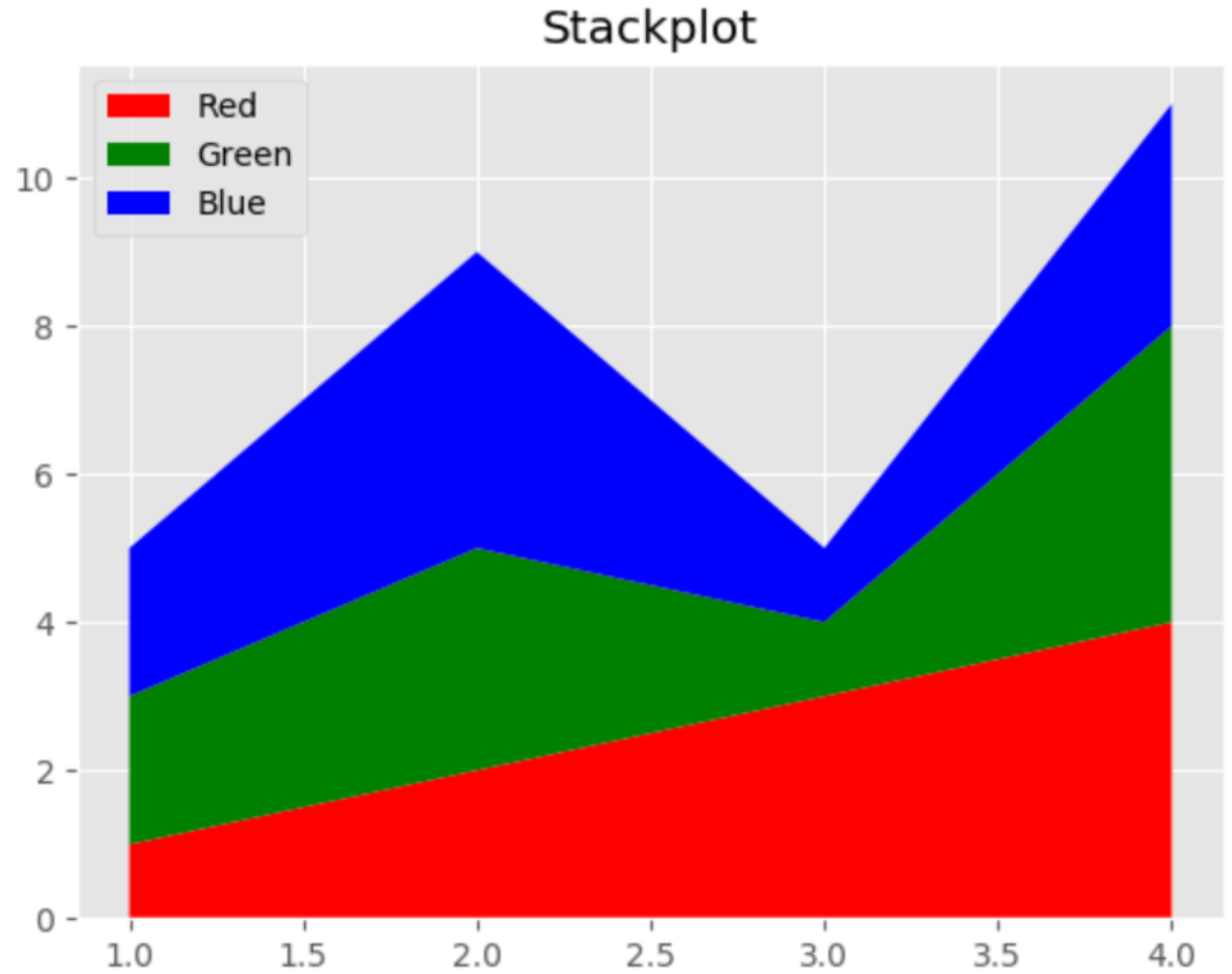
x = [1, 2, 3, 4]

y1 = [1, 2, 3, 4]
y2 = [2, 3, 1, 4]
y3 = [2, 4, 1, 3]

plt.title('Stackplot')

plt.stackplot(x, y1, y2, y3,
              colors=['r', 'g', 'b'],
              labels=["Red", "Green", "Blue"])

plt.grid(True)
plt.legend(loc=2)
plt.show()
```



matplotlib-subplots.py

```
import matplotlib.pyplot as plt
import math

x_min, x_max, n = 0, 2 * math.pi, 20
x = [x_min + (x_max - x_min) * i / n
      for i in range(n + 1)]
y = [math.sin(v) for v in x]

plt.subplot(2, 3, 1)
plt.plot(x, y, 'r-')
plt.title('Plot A')

plt.subplot(2, 3, 2)
plt.plot(x, y, 'g.')
plt.title('Plot B')

plt.subplot(2, 3, 3)
plt.plot(x, y, 'b--')
plt.title('Plot C')

plt.subplot(2, 3, 4)
plt.plot(x, y, 'mx:')
plt.title('Plot D')

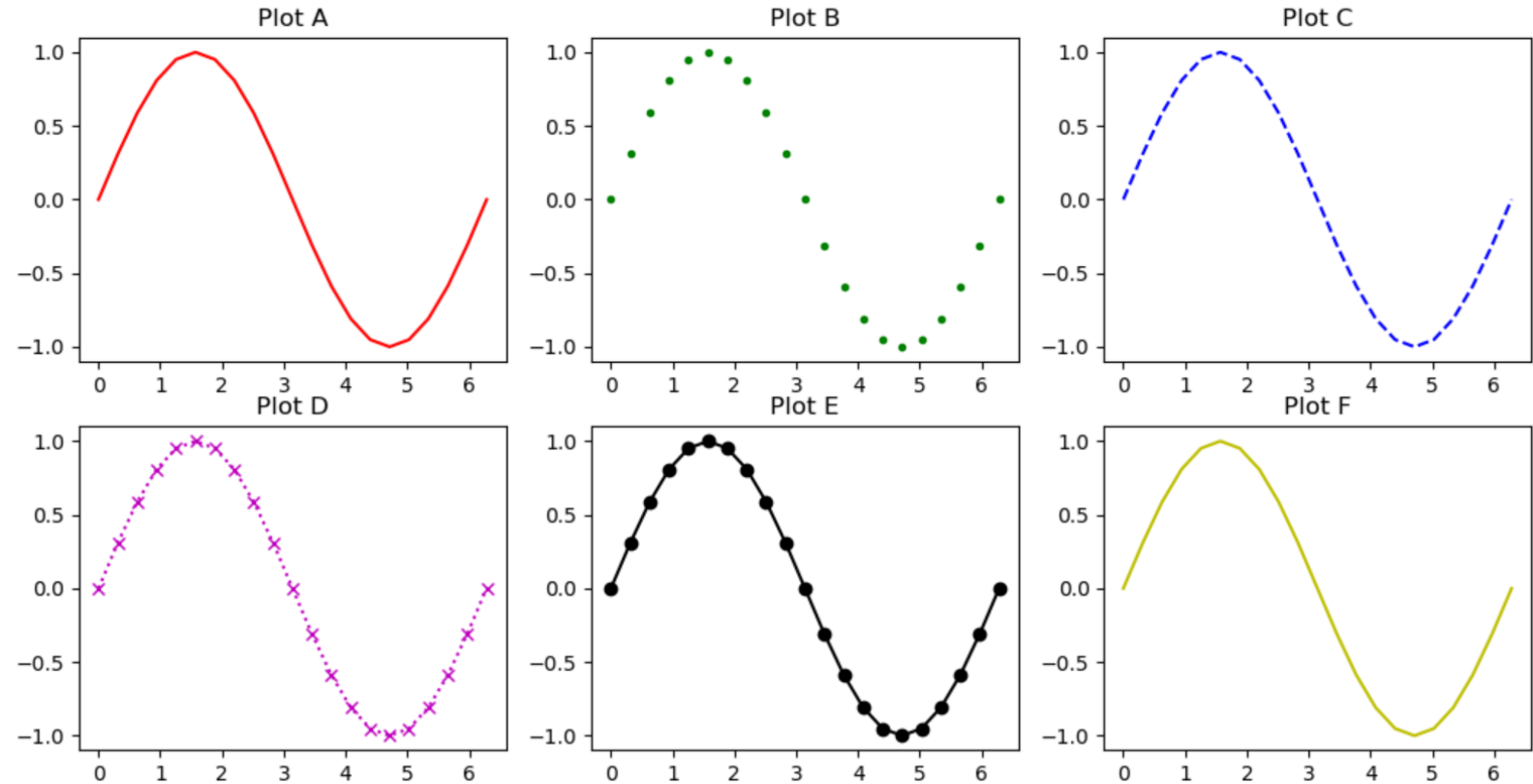
plt.subplot(2, 3, 5)
plt.plot(x, y, 'ko-')
plt.title('Plot E')

plt.subplot(2, 3, 6)
plt.plot(x, y, 'y')
plt.title('Plot F')

plt.suptitle('2 x 3 subplots', fontsize=16)
plt.show()
```

Subplots (2 rows, 3 columns)

2 x 3 subplots



matplotlib-subplots.py

```
import matplotlib.pyplot as plt
import math

x_min, x_max, n = 0, 2 * math.pi, 20
x = [x_min + (x_max - x_min) * i / n
      for i in range(n + 1)]
y = [math.sin(v) for v in x]

plt.subplot2grid((5, 5), (0,0),
                 rowspan=3, colspan=3)
plt.plot(x, y, 'r-')
plt.title('Plot A')

plt.subplot2grid((5, 5), (0,3),
                 rowspan=2, colspan=2)
plt.plot(x, y, 'g.')
plt.title('Plot B')

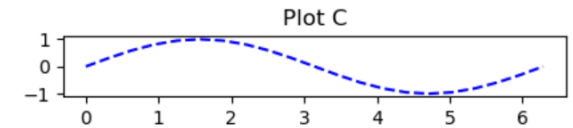
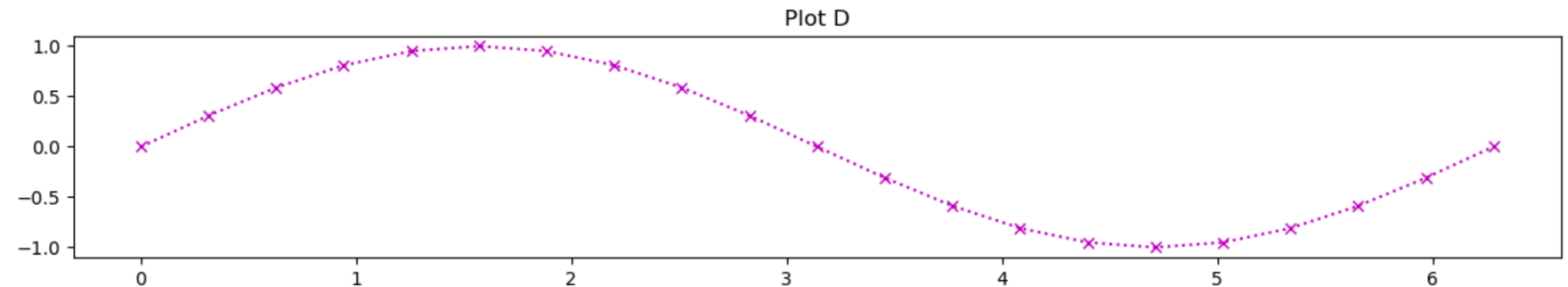
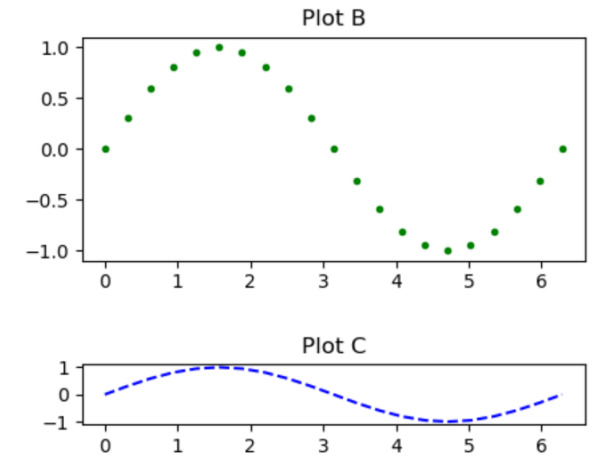
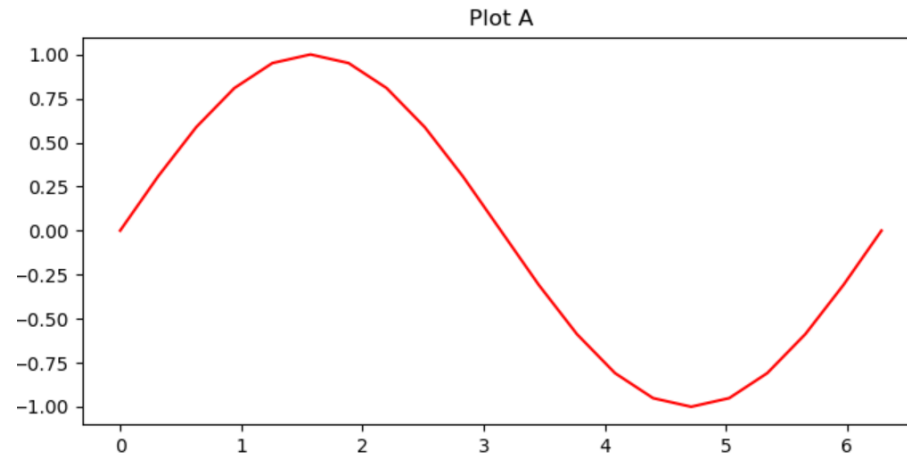
plt.subplot2grid((5, 5), (2,3),
                 rowspan=1, colspan=2)
plt.plot(x, y, 'b--')
plt.title('Plot C')

plt.subplot2grid((5, 5), (3,0),
                 rowspan=2, colspan=5)
plt.plot(x, y, 'm-x')
plt.title('Plot D')



plt.tight_layout()
plt.show()
```

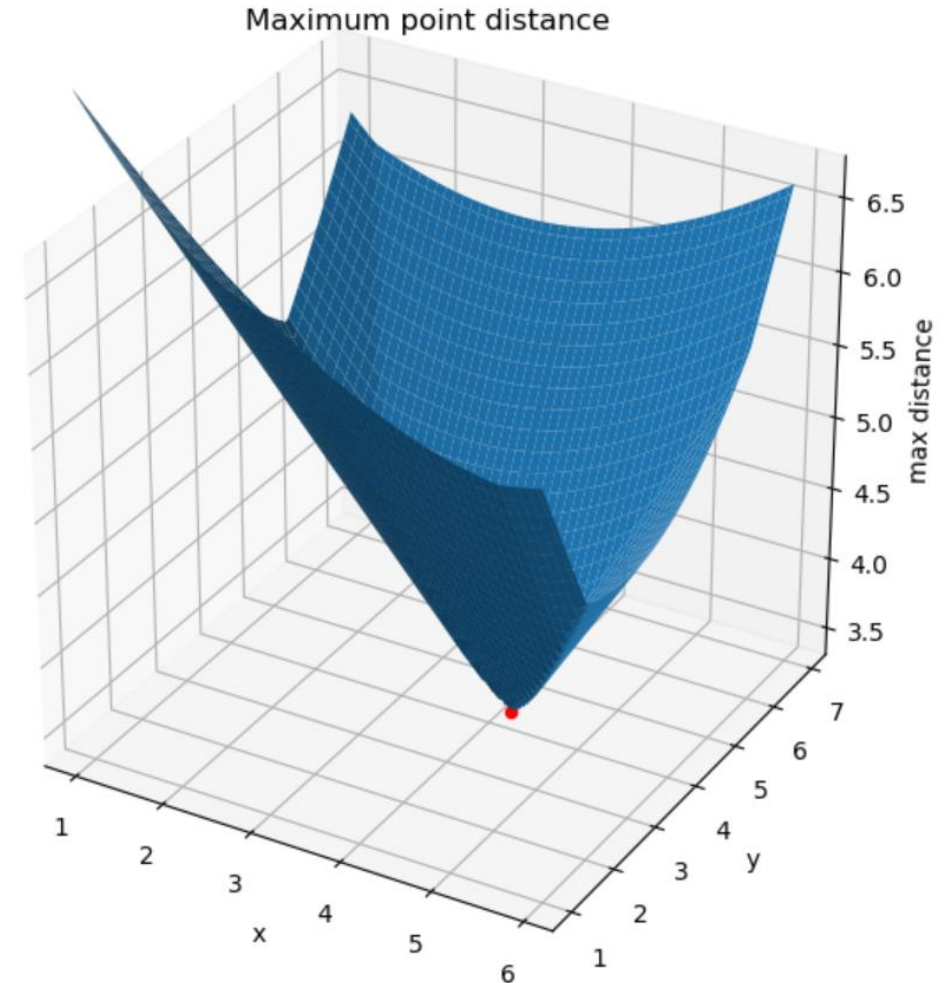
subplot2grid (5 x 5)

upper left corner

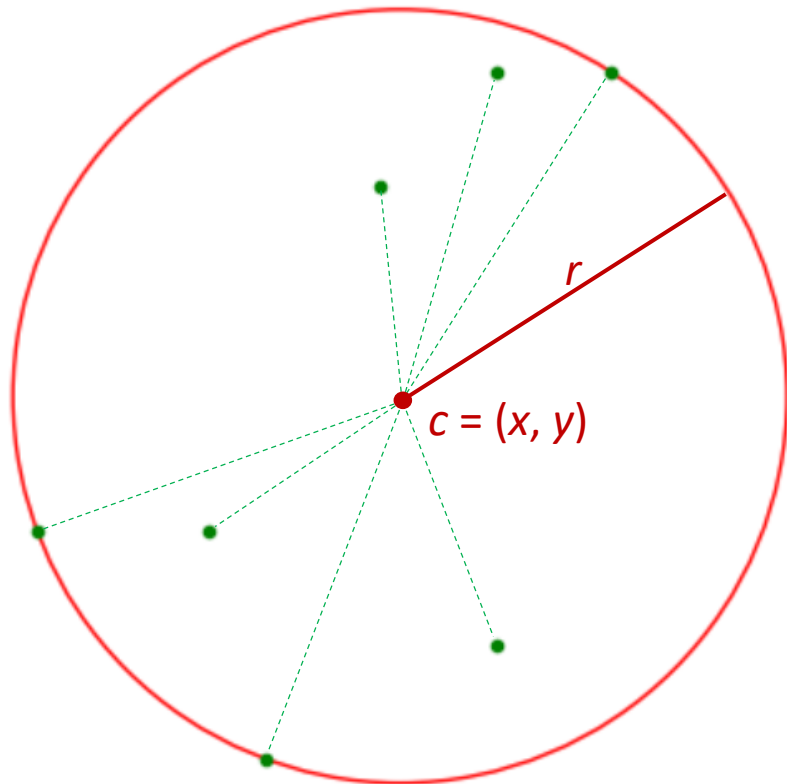



scipy.optimize.minimize

- Find point p minimizing function f
- Supports 13 algorithms – but no guarantee that result correct
- Knowledge about optimization will help you know what optimization algorithm to select and what parameters to provide for better results
-  **WARNING** 
Many solvers return the wrong value



Example: Minimum enclosing circle

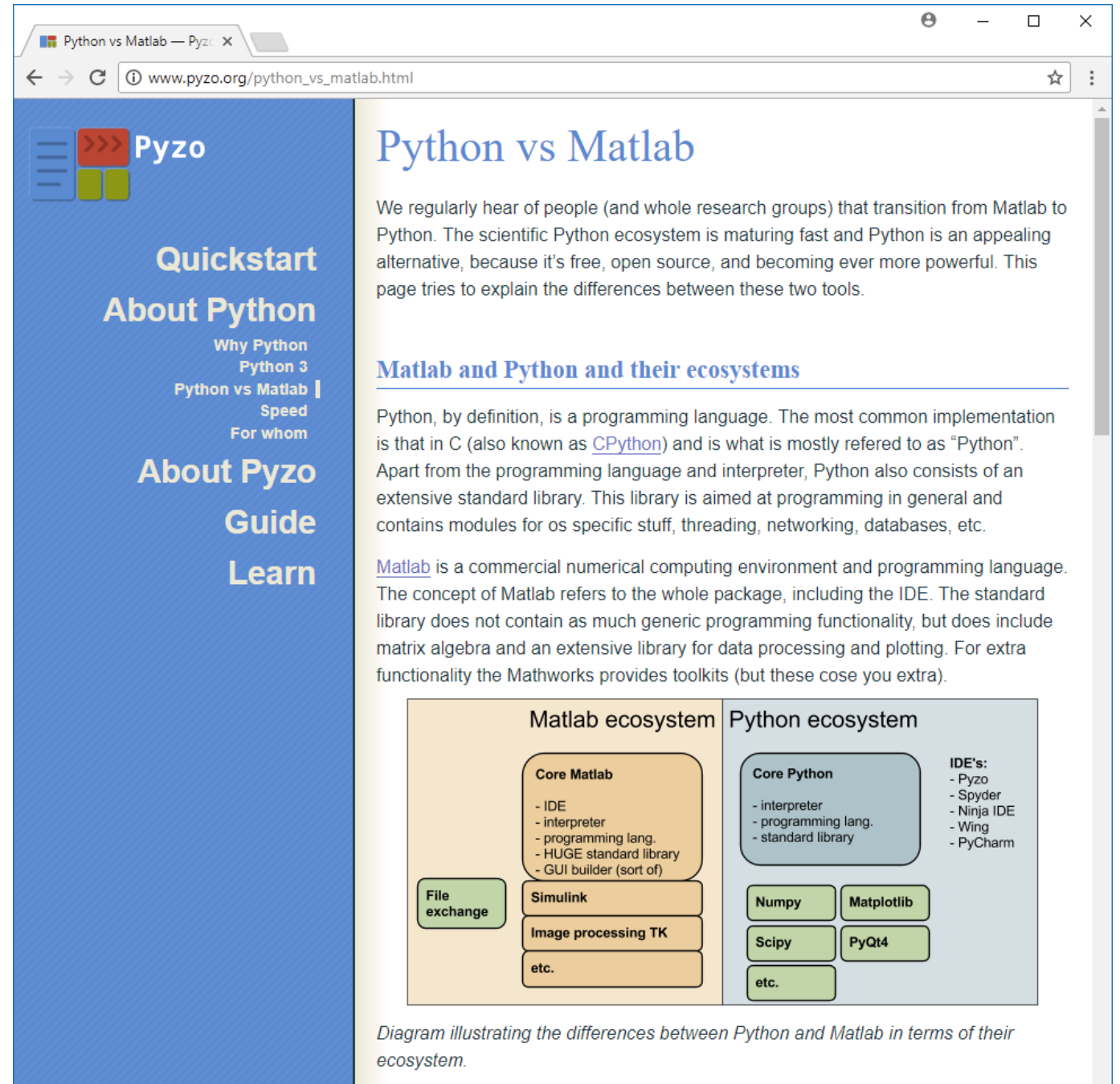


- Find c such that $r = \max_p |p - c|$ is **minimized**
- A solution is characterized by either 1) three points on circle, where the triangle contains the circle center 2) two opposite points on diagonal
- Try a standard numeric minimization solver
-  Computation involves **max** and \sqrt{x} , which can be hard for numeric optimization solvers

Python/scipy vs MATLAB

Some basic differences

- “end” closes a MATLAB block
- “;” at end of command avoids command output
- a(i) instead a[i]
- 1st element of a list a(1)
- a(i:j) includes both a(i) and a(j)



The screenshot shows a web browser window with the URL www.pyzo.org/python_vs_matlab.html. The page title is "Python vs Matlab". The left sidebar contains navigation links: "Quickstart", "About Python" (with sub-links: "Why Python", "Python 3", "Python vs Matlab", "Speed", "For whom"), "About Pyzo", "Guide", and "Learn". The main content area has the heading "Python vs Matlab" and a paragraph: "We regularly hear of people (and whole research groups) that transition from Matlab to Python. The scientific Python ecosystem is maturing fast and Python is an appealing alternative, because it's free, open source, and becoming ever more powerful. This page tries to explain the differences between these two tools." Below this is a section titled "Matlab and Python and their ecosystems" with two paragraphs. The first paragraph discusses Python's ecosystem, and the second discusses Matlab's ecosystem. At the bottom, there is a diagram comparing the two ecosystems.

Matlab ecosystem

- Core Matlab
 - IDE
 - interpreter
 - programming lang.
 - HUGE standard library
 - GUI builder (sort of)
- File exchange
- Simulink
- Image processing TK
- etc.

Python ecosystem

- Core Python
 - interpreter
 - programming lang.
 - standard library
- IDE's:
 - Pyzo
 - Spyder
 - Ninja IDE
 - Wing
 - PyCharm
- Numpy
- Matplotlib
- Scipy
- PyQt4
- etc.

Diagram illustrating the differences between Python and Matlab in terms of their ecosystem.

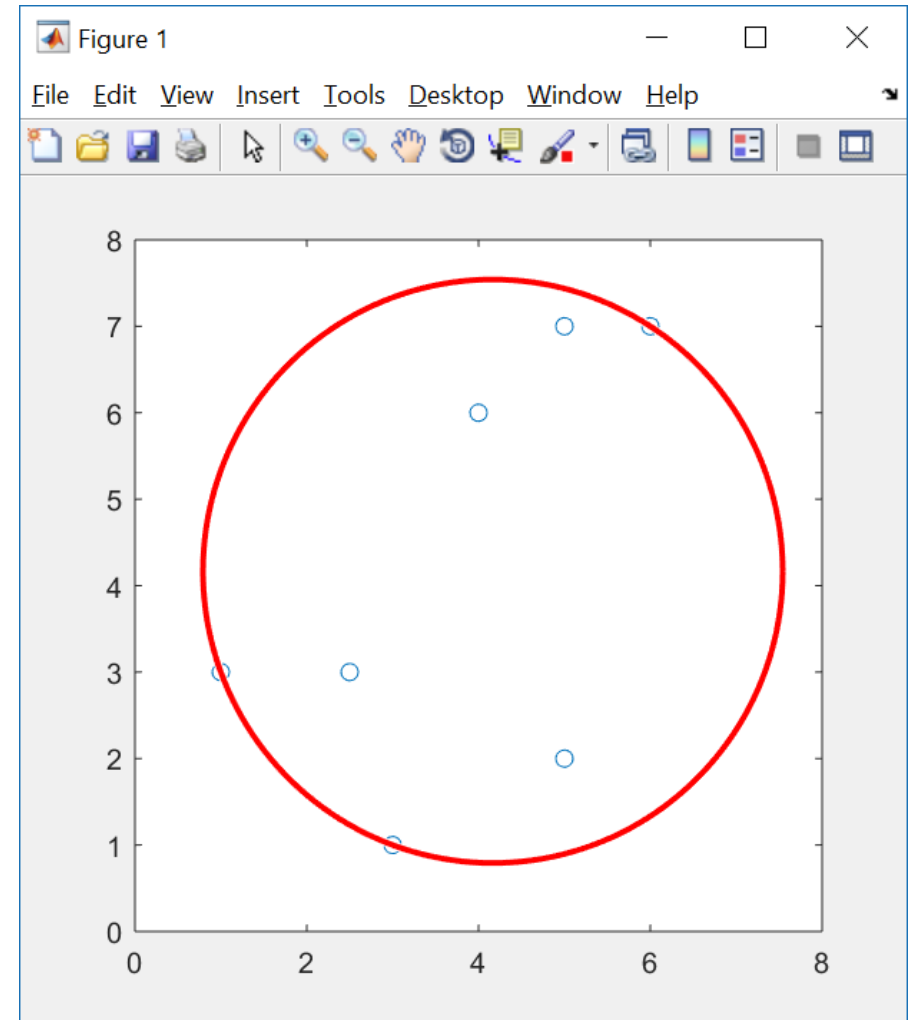
Minimum enclosing circle in MATLAB

enclosing_circle.m

```
% Minimum enclosing circle of a point set
% fminsearch uses the Nelder-Mead algorithm

global x y
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
c = fminsearch(@(x) max_distance(x), [0,0]);
plot(x, y, "o");
viscircles(c, max_distance(c));

function dist = max_distance(p)
    global x y
    dist = 0.0;
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)],
                               'euclidean'));
    end
end
end
```



Minimum enclosing circle in MATLAB (trace)

```
enclosing_circle_trace.m
```

```
global x y trace_x trace_y

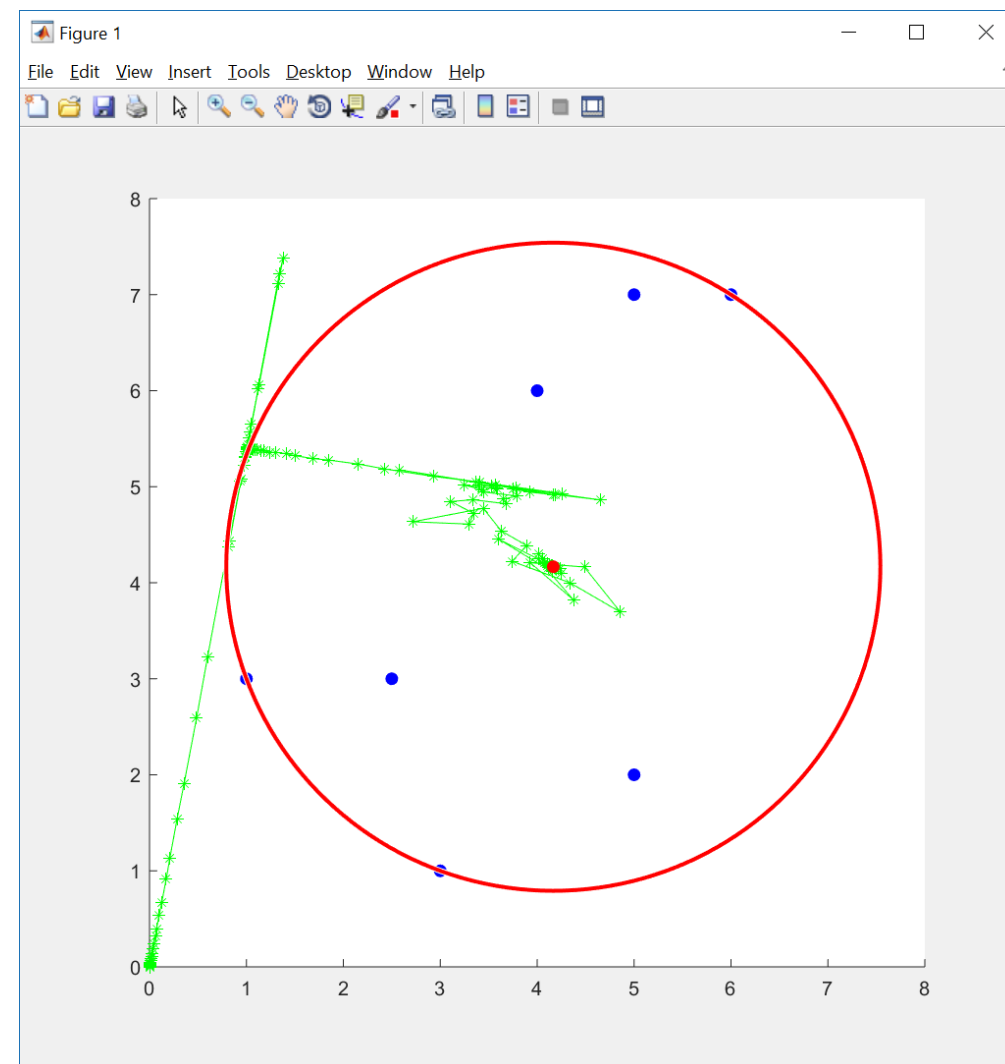
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
trace_x = [];
trace_y = [];

c = fminsearch(@(x) max_distance(x), [0,0]);

hold on
plot(x, y, "o", 'color', 'b', 'MarkerFaceColor', 'b');
plot(trace_x, trace_y, "*-", "color", "g");
plot(c(1), c(2), "o", 'color', 'r', 'MarkerFaceColor', 'r');
viscircles(c, max_distance(c), "color", "red");

function dist = max_distance(p)
    global x y trace_x trace_y
    trace_x = [trace_x, p(1)];
    trace_y = [trace_y, p(2)];

    dist = 0.0;
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)], 'euclidean' ));
    end
end
```



Minimum enclosing circle in Python

`enclosing_circle.py`

```
from math import sqrt
from scipy.optimize import minimize
import matplotlib.pyplot as plt } import modules

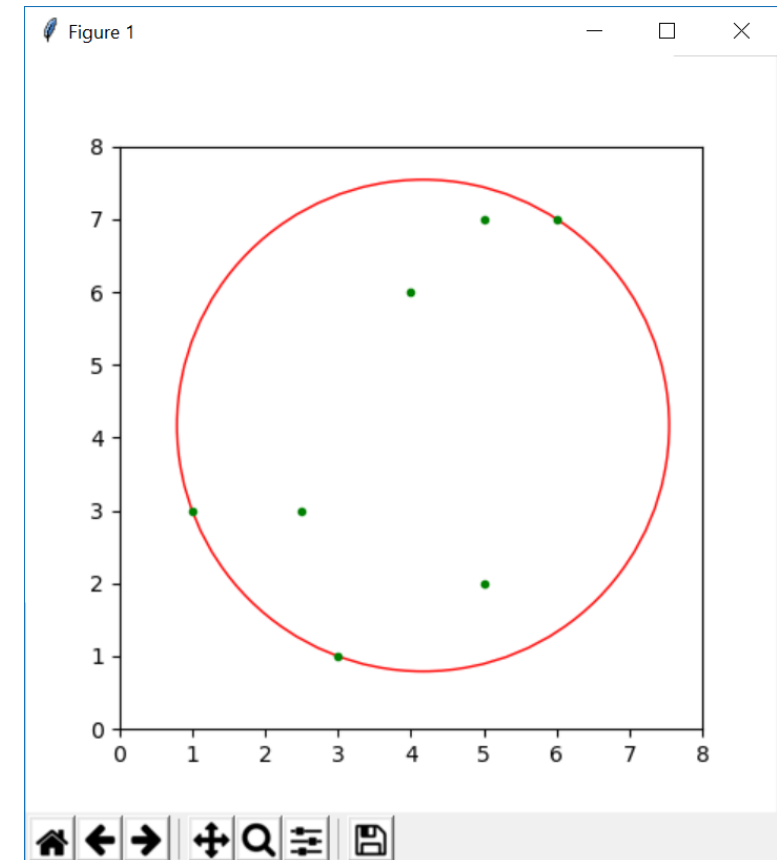
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]

def dist(p, q):
    return sqrt((p[0]-q[0])**2 + (p[1]-q[1])**2)
def max_distance(c):
    return max([dist(p, c) for p in zip(x, y)])

c = minimize(max_distance, [0.0, 0.0], \
             method="nelder-mead").x

ax = plt.gca()
ax.set_xlim((0, 8))
ax.set_ylim((0, 8))
ax.set_aspect("equal") } manually set axis (force circle inside plot)
plt.plot(x, y, "g.")
ax.add_artist(plt.Circle(c, max_distance(c), \
                       color="r", fill=False))

plt.show()
```



Minimum enclosing circle in Python (trace)

enclosing_circle_trace.py

```
from math import sqrt
from scipy.optimize import minimize
import matplotlib.pyplot as plt

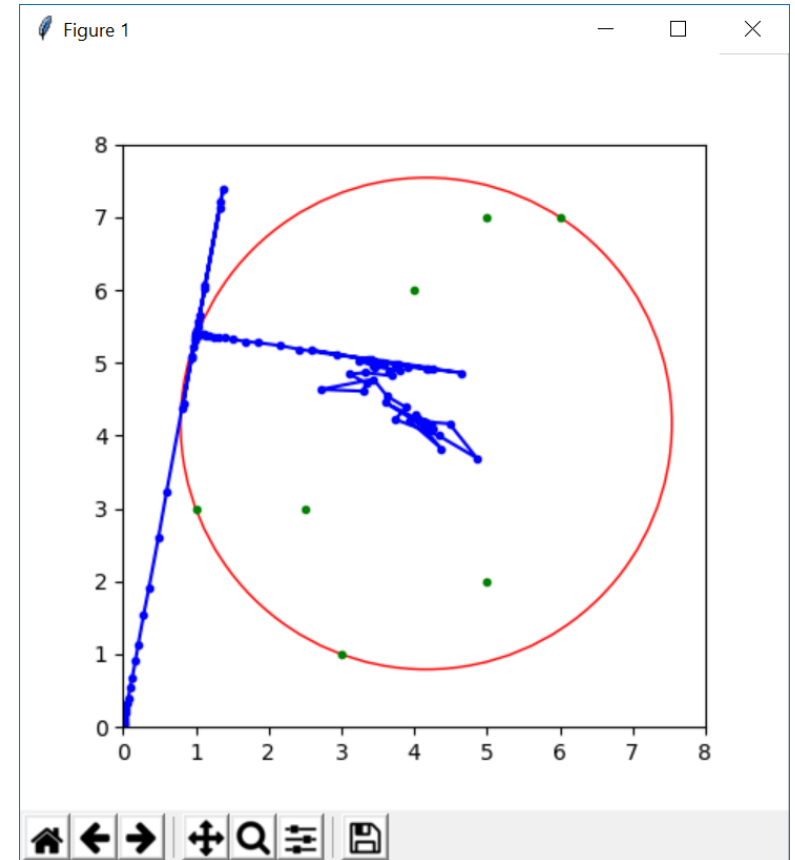
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]
trace = []

def dist(p, q):
    return sqrt((p[0]-q[0])**2 + (p[1]-q[1])**2)
def max_distance(c):
    trace.append(c)
    return max([dist(p, c) for p in zip(x, y)])

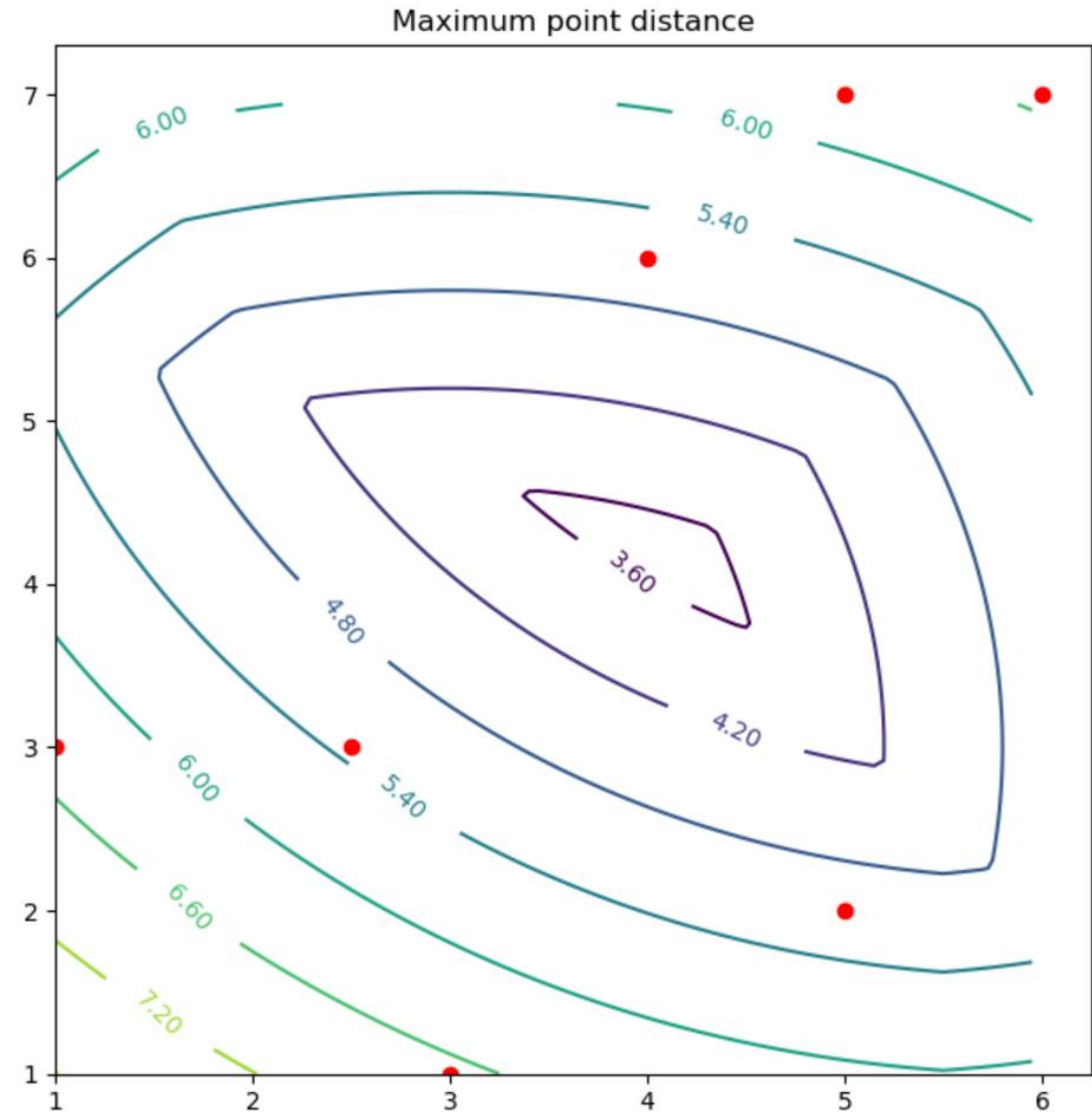
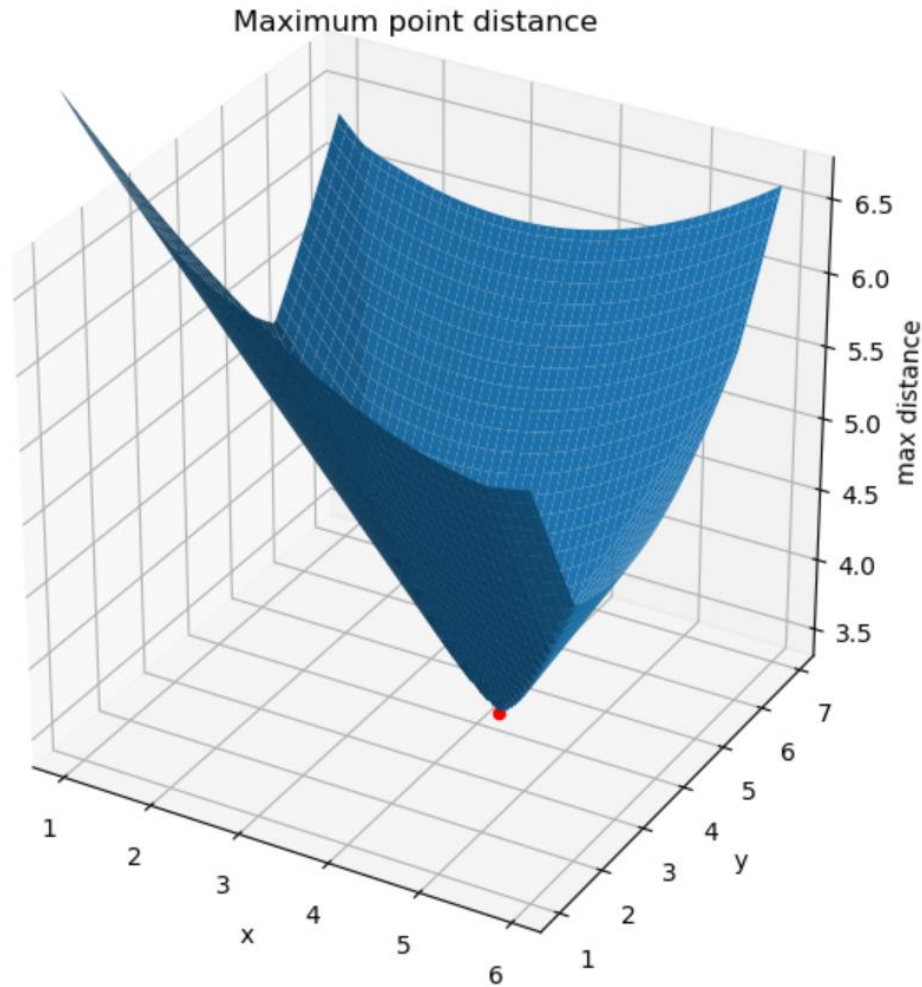
c = minimize(max_distance, [0.0, 0.0],
             method="nelder-mead").x

ax = plt.gca()
ax.set_xlim((0, 8))
ax.set_ylim((0, 8))
ax.set_aspect("equal")
plt.plot(x, y, "g.")
plt.plot(*zip(*trace), "b.-")
ax.add_artist(plt.Circle(c, max_distance(c),
                        color="r", fill=False))

plt.show()
```



Minimum enclosing circle – search space



scipy.minimize algorithms (without arguments)

