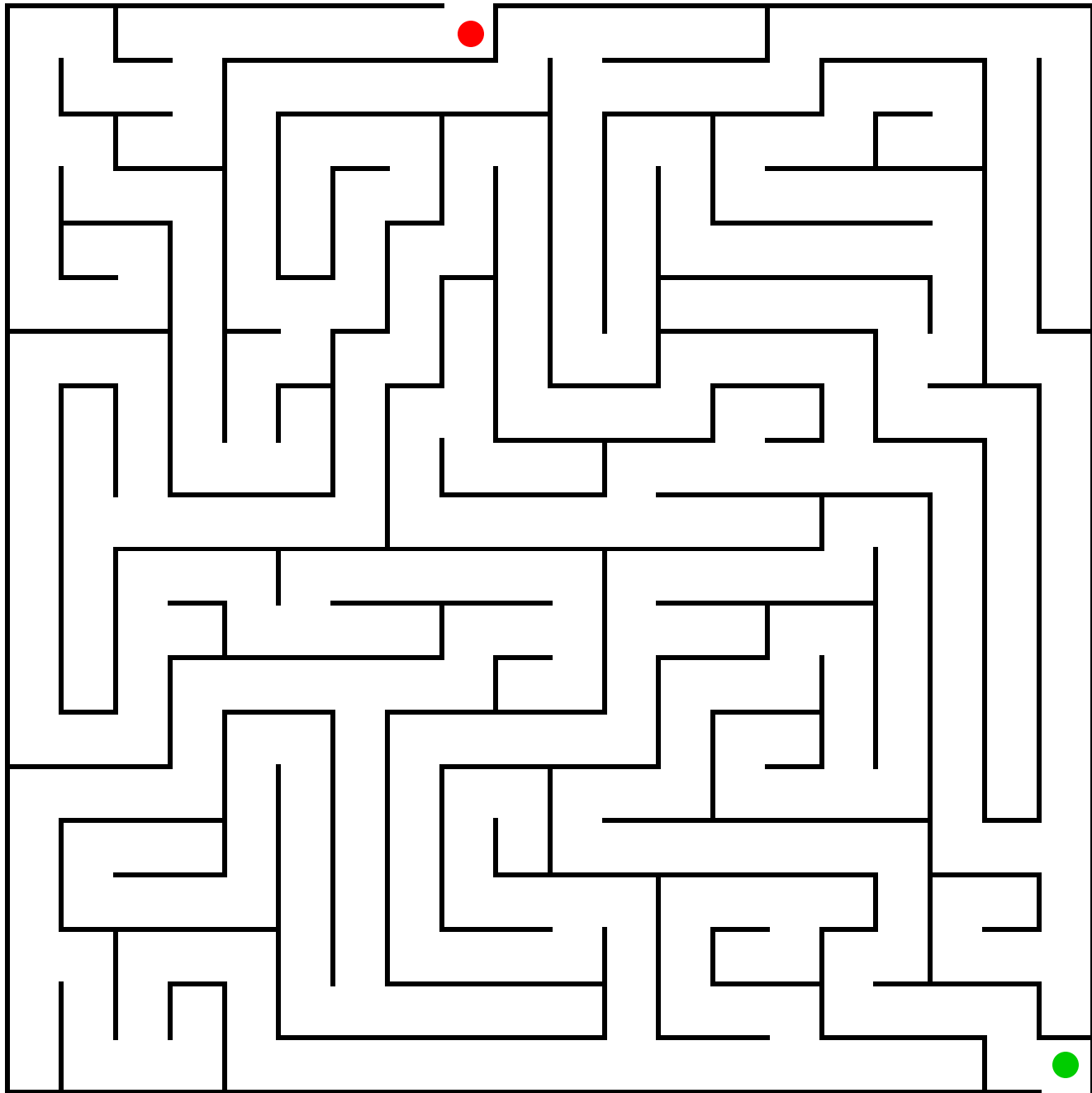


Grundlæggende Algoritmer og Datastrukturer

**Topologisk Sortering, Stærke Sammenhængskomponenter
[CLRS, kapitel 22.4-22.5]**



Dybde Først Søgning (DFS)

DFS(G)

```
1 for each vertex  $u \in G.V$ 
2    $u.color = WHITE$ 
3    $u.\pi = NIL$ 
4    $time = 0$ 
5 for each vertex  $u \in G.V$ 
6   if  $u.color == WHITE$ 
7     DFS-VISIT( $G, u$ )
```

$u.color$

WHITE = knuderne endnu ikke besøgt
GRAY = knuder på rekursionsstakken
BLACK = knuderne besøgt

$u.\pi$ = faderen til u i DFS træet

$u.d$ = "discover time" for u

$u.f$ = "finishing time" for u

DFS-VISIT(G, u)

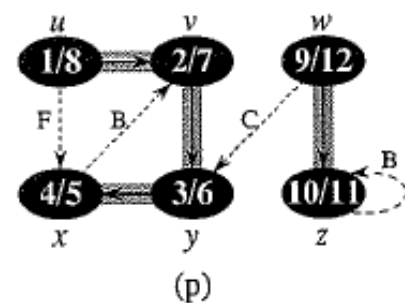
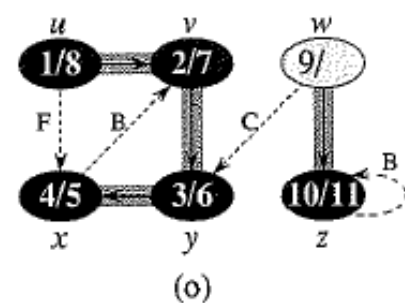
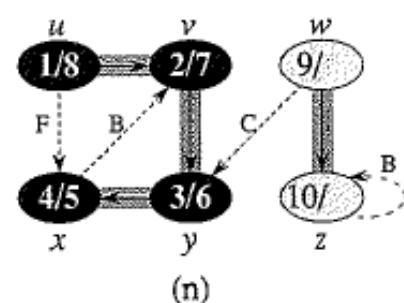
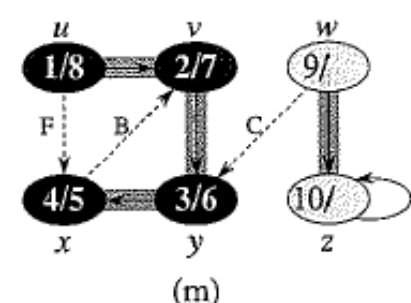
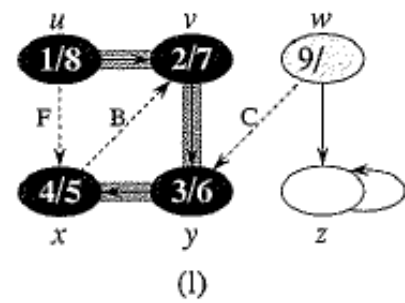
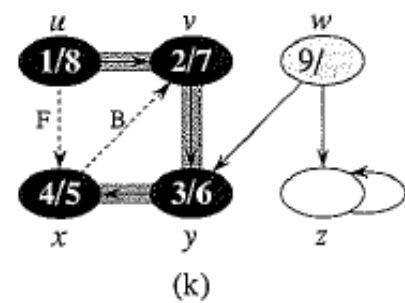
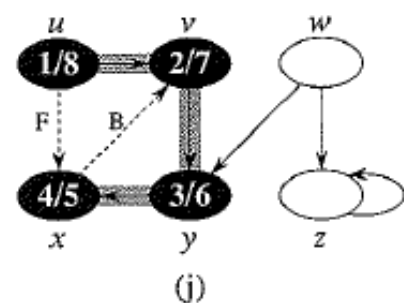
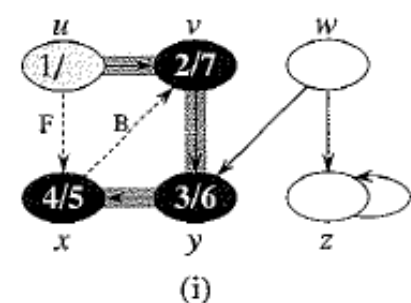
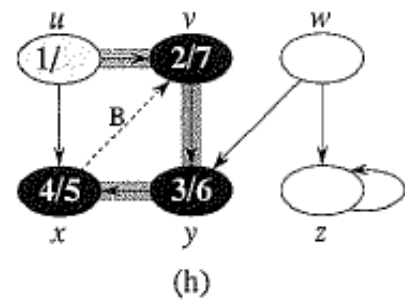
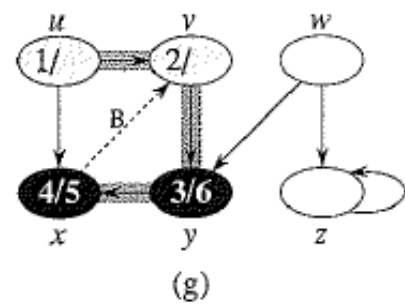
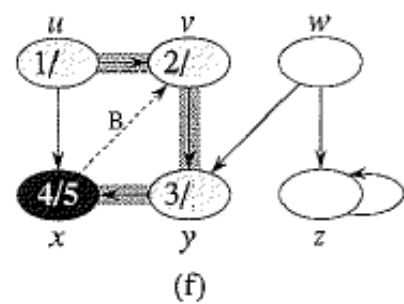
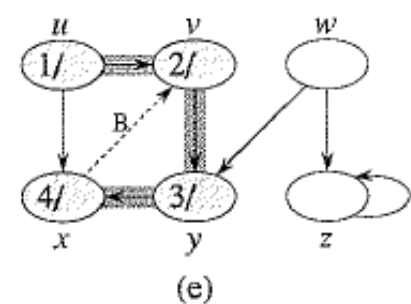
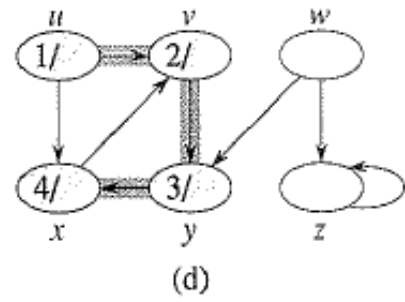
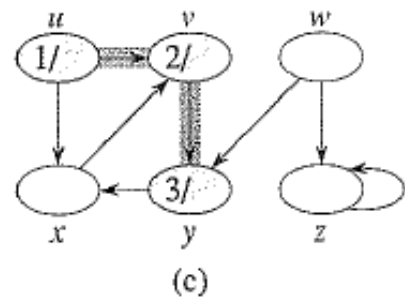
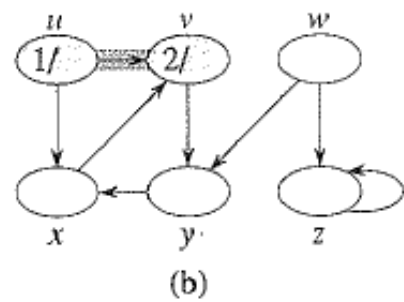
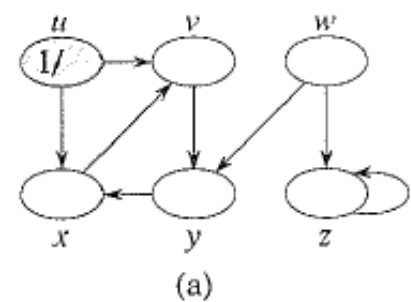
```
1  $time = time + 1$ 
2  $u.d = time$ 
3  $u.color = GRAY$ 
4 for each  $v \in G.Adj[u]$ 
5   if  $v.color == WHITE$ 
6      $v.\pi = u$ 
7     DFS-VISIT( $G, v$ )
8  $u.color = BLACK$ 
9  $time = time + 1$ 
10  $u.f = time$ 
```

// white vertex u has just been discovered

// explore edge (u, v)

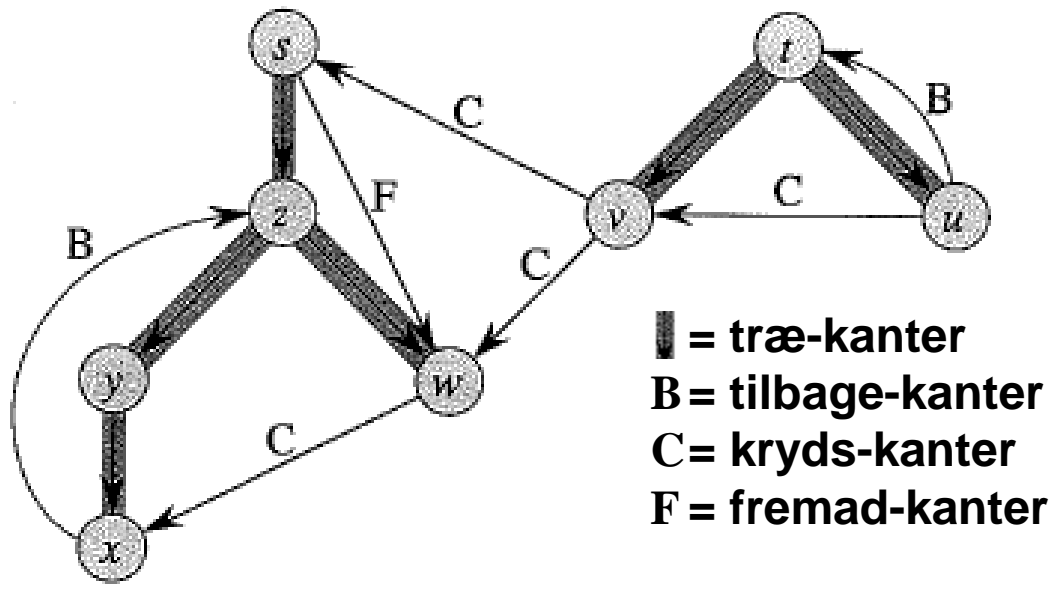
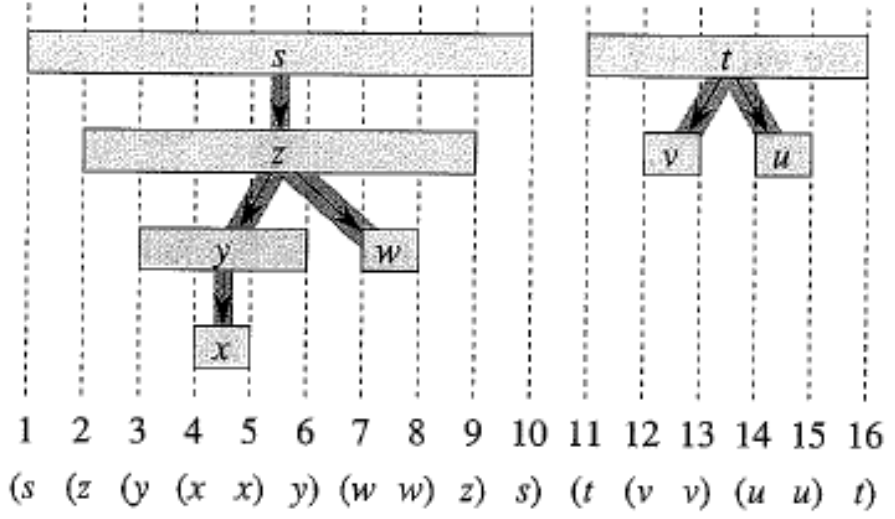
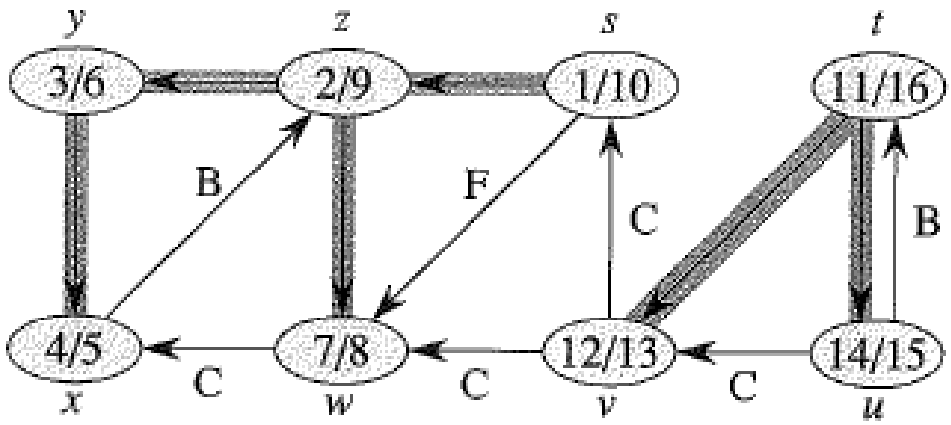
// blacken u ; it is finished

Tid $O(n+m)$



**Kan en knude have 13/17 som
DFS discover/finishing tider ?**

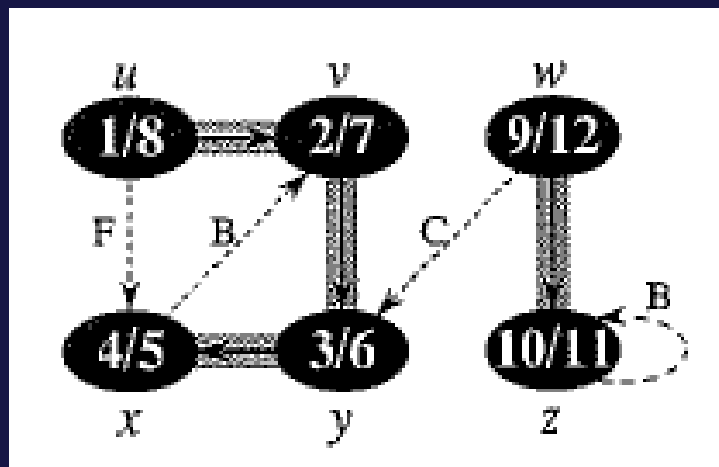
- a) Ja
- b) Nej
- c) Ved ikke



█ = træ-kanter
B = tilbage-kanter
C = kryds-kanter
F = fremad-kanter

Antag der findes sti $a \rightsquigarrow b$ i en orienteret graf.
Gælder der altid at $a.f > b.f$?

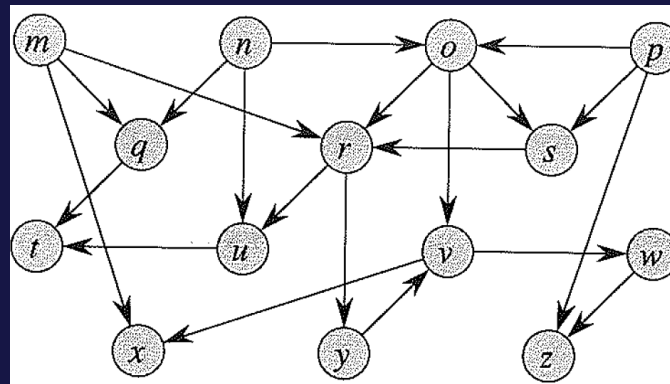
- a) Ja
- b) Nej
- c) Ved ikke



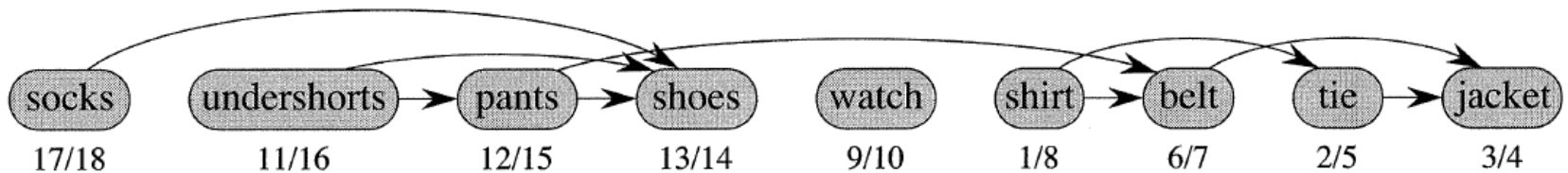
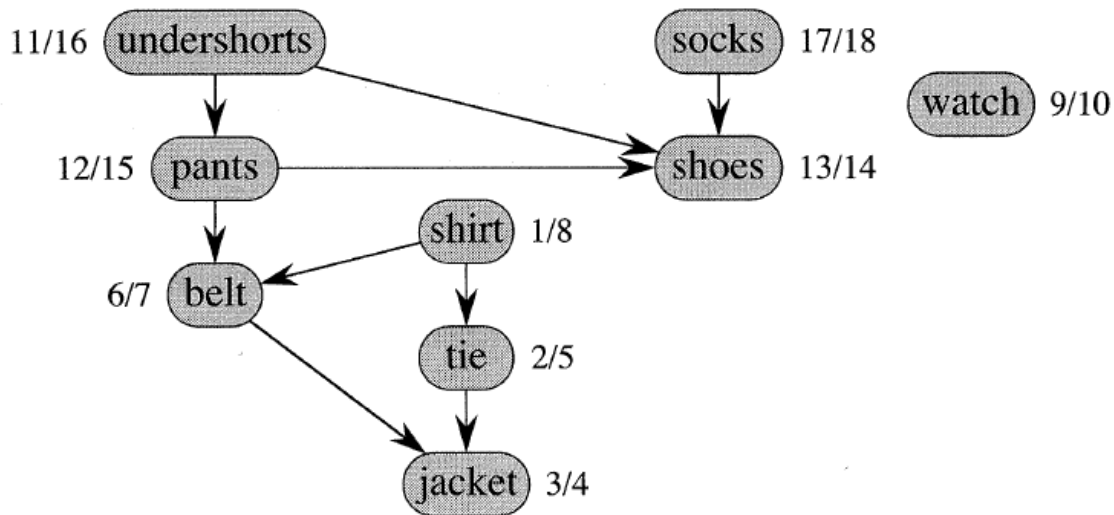
Antag der findes sti $a \rightsquigarrow b$ i en orienteret graf uden cykler.

Gælder der altid at $a.f > b.f$?

- a) Ja
- b) Nej
- c) Ved ikke



Acykliske Grafer: Topologisk Sortering



Alle kanter går fra venstre-mod-højre

Microsoft Excel - Copy of SheepFlock

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

H18 fx =B18*G18

	A	B	C	D	E	F	G	H	I
3	I. Description of animals in flock during the year.								
4	Ewes in flock:	700			[Green cells are those you can change.]				
5	Lambing rate:	4	times per	3	years =	1.33	times/year.		
6	Lambs weaned/lambing:	1.5	Days of lactation/lambing:		60				
7	Adult death loss per year:	3%	Days in lactation/year:		80				
8	Postweaning lamb loss:	2%	Lambs weaned per ewe per year:			2.0			
9	Ewe culling rate:	15%	Ram culling rate:		50%				
10	Rams/100 ewes:	1	(Only 1/3 of ewes bred per season under STAR system.)						Inventory
11			Weaning	Market	Final	Price	Value	or sale	
12		Number	wt, lb	wt, lb	wt, lb	\$/lb	per head	value	
13	Ewes	700			150	\$1.00	\$150	\$105,000	
14	Rams	8			200	\$2.00	\$400	\$3,200	
15	Ewe lamb rplcmnts	126	30		100	\$1.25	\$125	\$15,750	
16	Ram lamb rplcmnts	5	40		130	\$2.00	\$260	\$1,300	
17	Ewe lambs sold	560	30	70		\$1.10	\$77	\$43,120	
18	Ram lambs sold	681	40	70		\$1.10	\$77	\$52,437	
19	Cull ewes sold	105		150		\$0.30	\$45	\$4,725	
20	Cull rams sold	5		200		\$0.30	\$60	\$300	
21	Fleece weight per adult	708			6	\$0.30	\$1.80	\$1,274	
22							Inventory:	\$125,250	
23							Sales:	\$101,856	

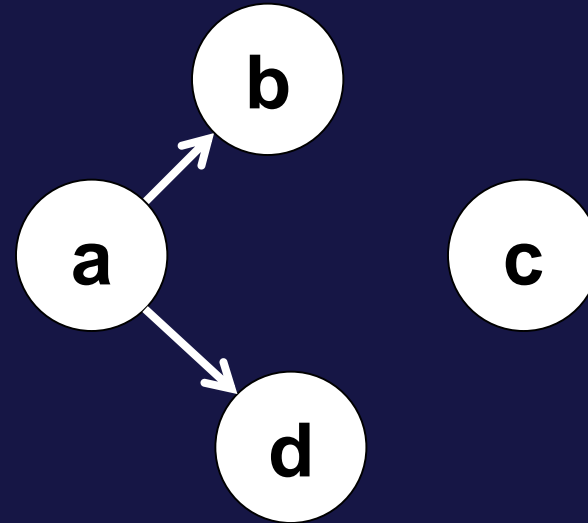
Sheep flock /

Ready

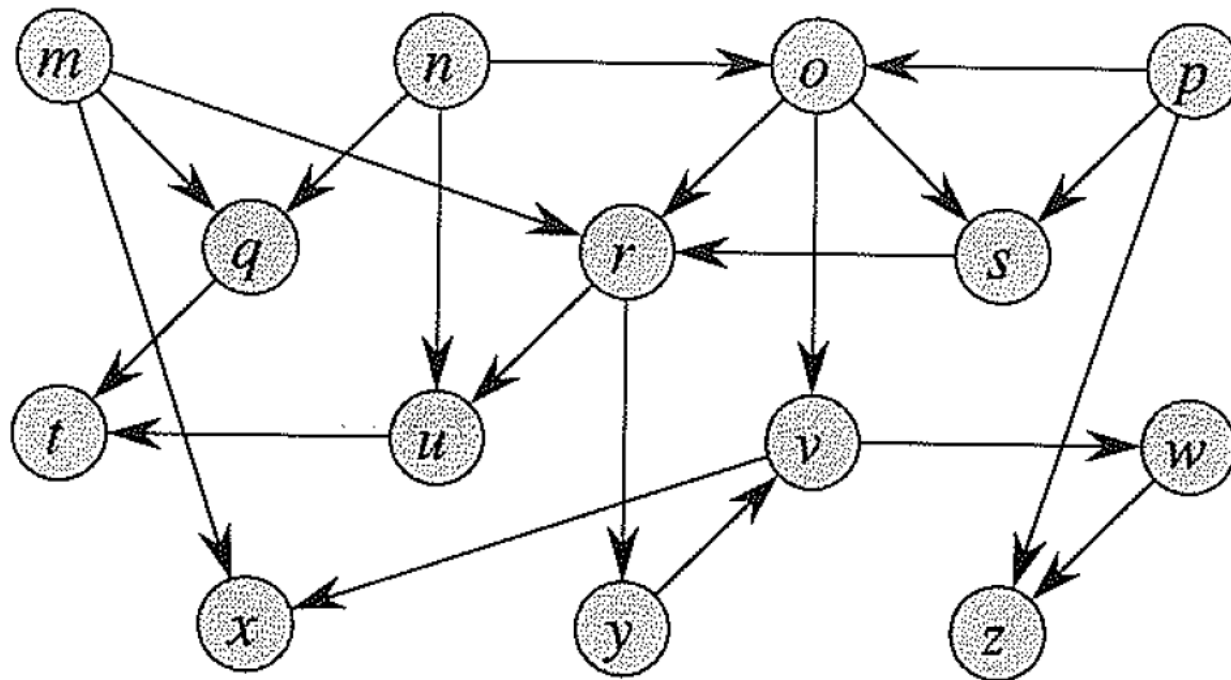
Topologisk sortering = en rækkefølge hvor vi kan beregne cellernes indhold

Hvor mange mulige måder kan man topologisk sortere grafen ?

- a) 1
- b) 2
- c) 3
- d) 4
- e) 5
- f) 6
- g) 7
- h) 8
- i) 9
- j) Ved ikke



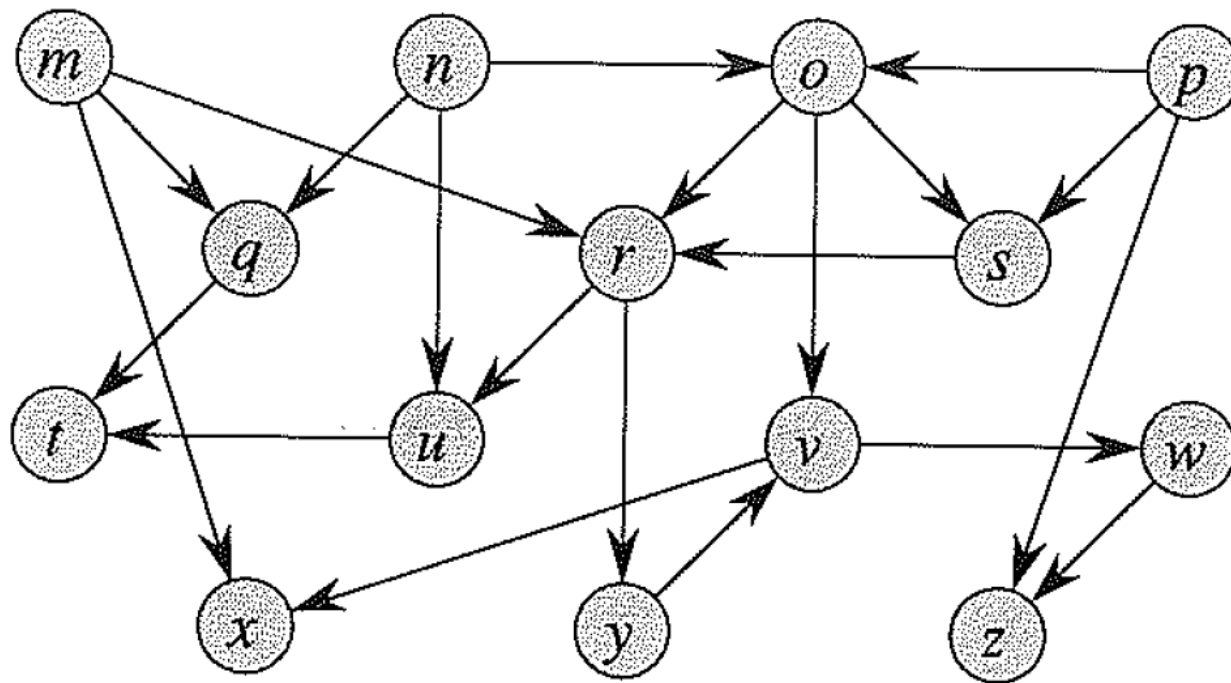
Topologisk Sortering (I)



Algoritme: Grådigt slet en knude med indgrad 0 (og udgående kanter), og tilføj knuden sidst i den topologiske orden

Tid $O(m+n)$

Topologisk Sortering (II)



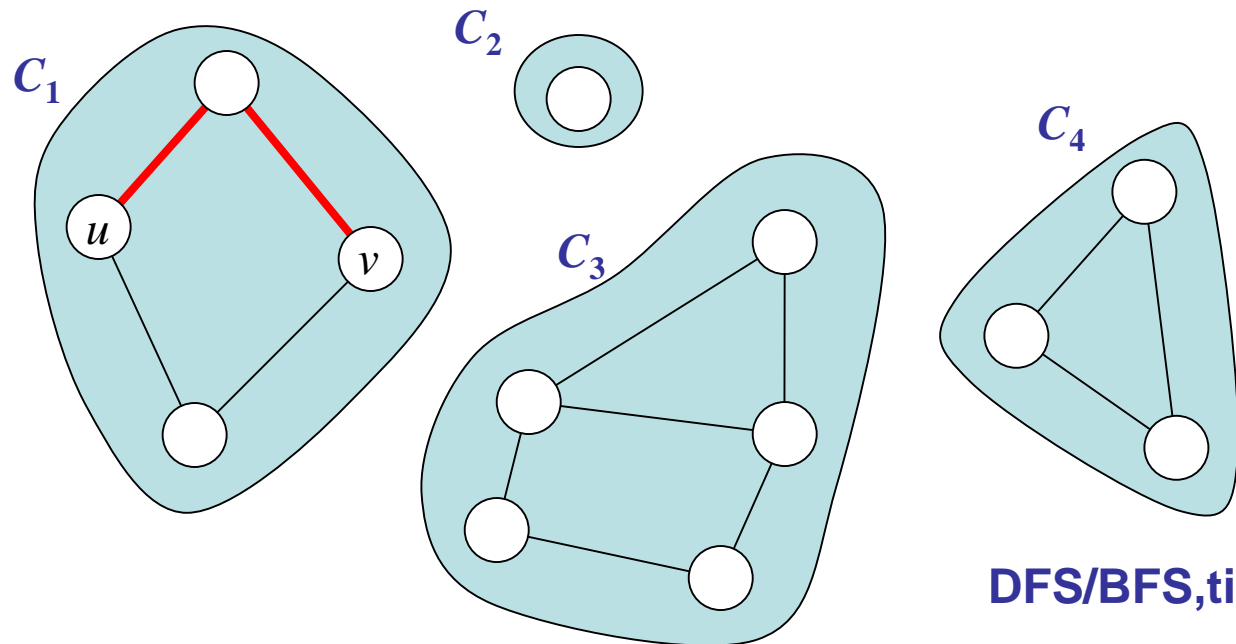
TOPOLOGICAL-SORT(G)

- 1 call $\text{DFS}(G)$ to compute finishing times $v.f$ for each vertex v
- 2 as each vertex is finished, insert it onto the front of a linked list
- 3 **return** the linked list of vertices

Tid $O(m+n)$

Sammenhængskomponenter

Opdeling af knuderne i en **uorienteret** graf i **komponenter** C_1, \dots, C_k , således at u og v er i C_i hvis og kun hvis der er en **sti** mellem u og v



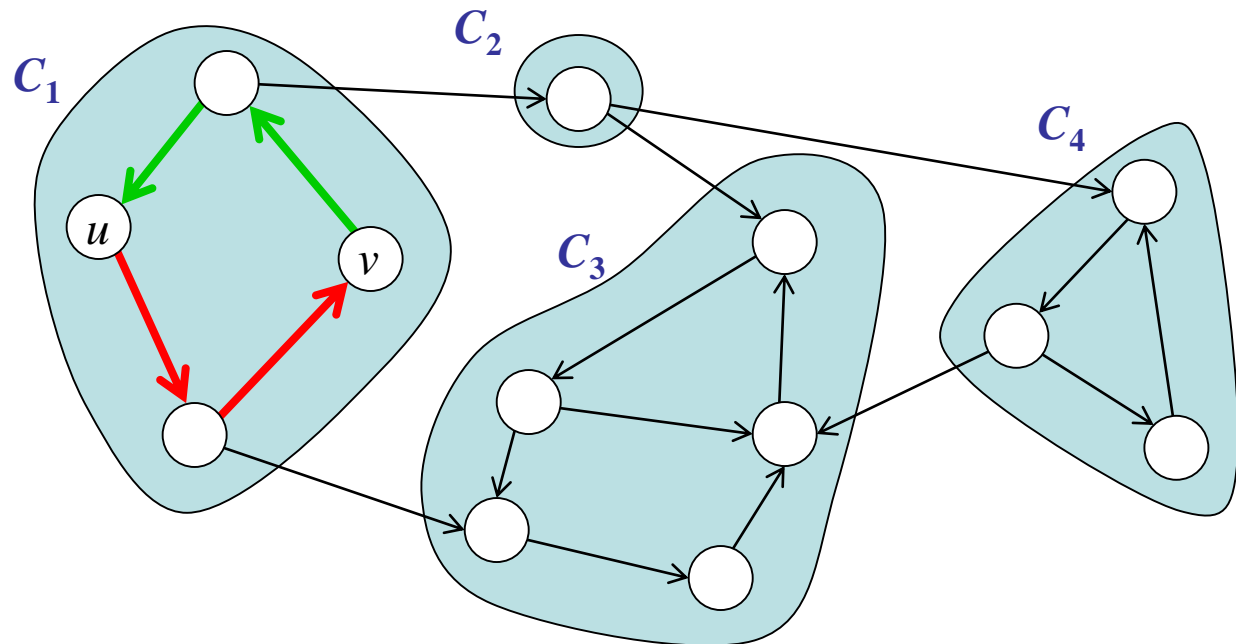
DFS/BFS, tid $O(m+n)$

Stærke Sammenhængskomponenter

Opdeling af knuderne i en **orienteret** graf i **komponenter** C_1, \dots, C_k , således at

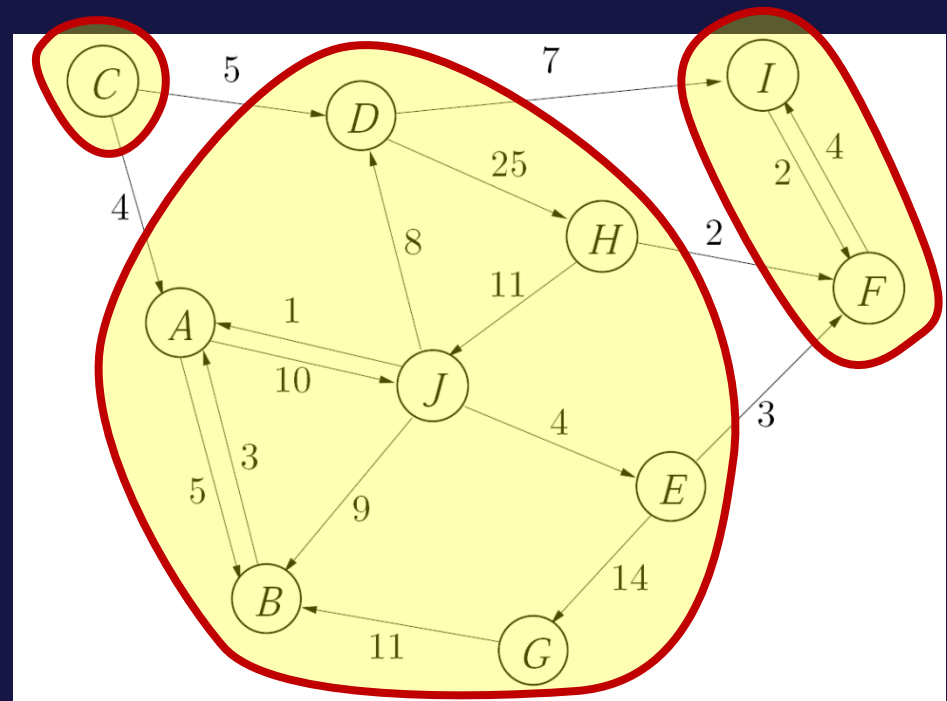
u og v er i C_i hvis og kun hvis der både er

- en **sti fra u til v** og
- en **sti fra v til u**



Antal stærke sammenhængskomponenter ?

- a) 1
- b) 2
- c) 3
- d) 4
- e) 5
- f) Ved ikke



Eksamensopgave 1(d), sommeren 2009
{A,B,D,E,G,H,J}, {C}, {F, I}

Stærke Sammenhængskomponenter

STRONGLY-CONNECTED-COMPONENTS (G)

- 1 call DFS(G) to compute finishing times $u.f$ for each vertex u
- 2 compute G^T
- 3 call DFS(G^T), but in the main loop of DFS, consider the vertices in order of decreasing $u.f$ (as computed in line 1)
- 4 output the vertices of each tree in the depth-first forest formed in line 3 as separate strongly connected component

DFS(G)

```

1 for each vertex  $u \in G.V$ 
2    $u.color = WHITE$ 
3    $u.\pi = NIL$ 
4  $time = 0$ 
5 for each vertex  $u \in G.V$ 
6   if  $u.color == WHITE$ 
7     DFS-VISIT( $G, u$ )

```

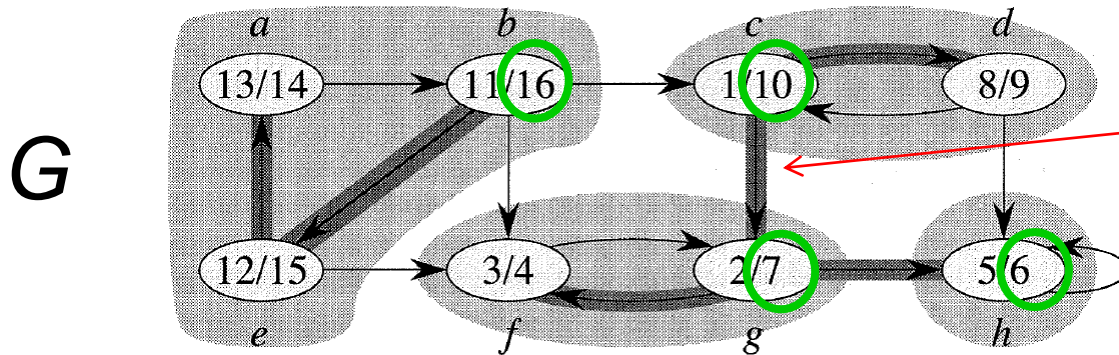
DFS-VISIT(G, u)

```

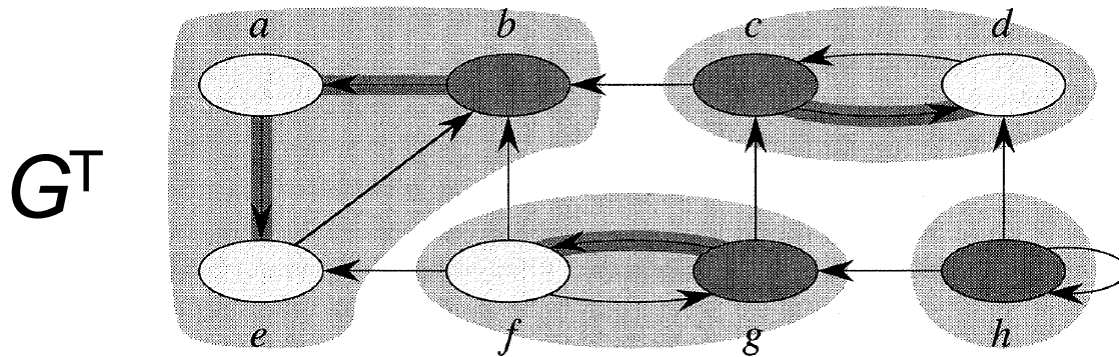
1  $time = time + 1$            // white vertex  $u$  has just been discovered
2  $u.d = time$ 
3  $u.color = GRAY$ 
4 for each  $v \in G.Adj[u]$      // explore edge  $(u, v)$ 
5   if  $v.color == WHITE$ 
6      $v.\pi = u$ 
7     DFS-VISIT( $G, v$ )
8  $u.color = BLACK$          // blacken  $u$ ; it is finished
9  $time = time + 1$ 
10  $u.f = time$ 

```

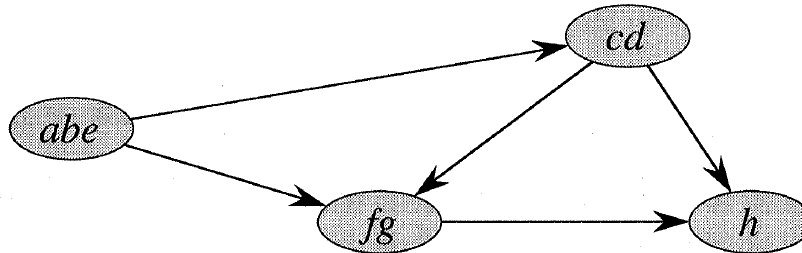
Stærke Sammenhængskomponenter



DFS trækanter mellem to stærke sammenhængskomponenter



De største finishing tider i hver komponent udgør en (omvendt) topologisk sortering af komponenterne



Tid $O(m+n)$