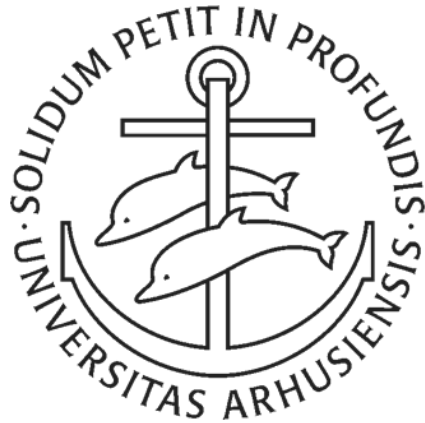


Algoritmer og Datastrukturer 2

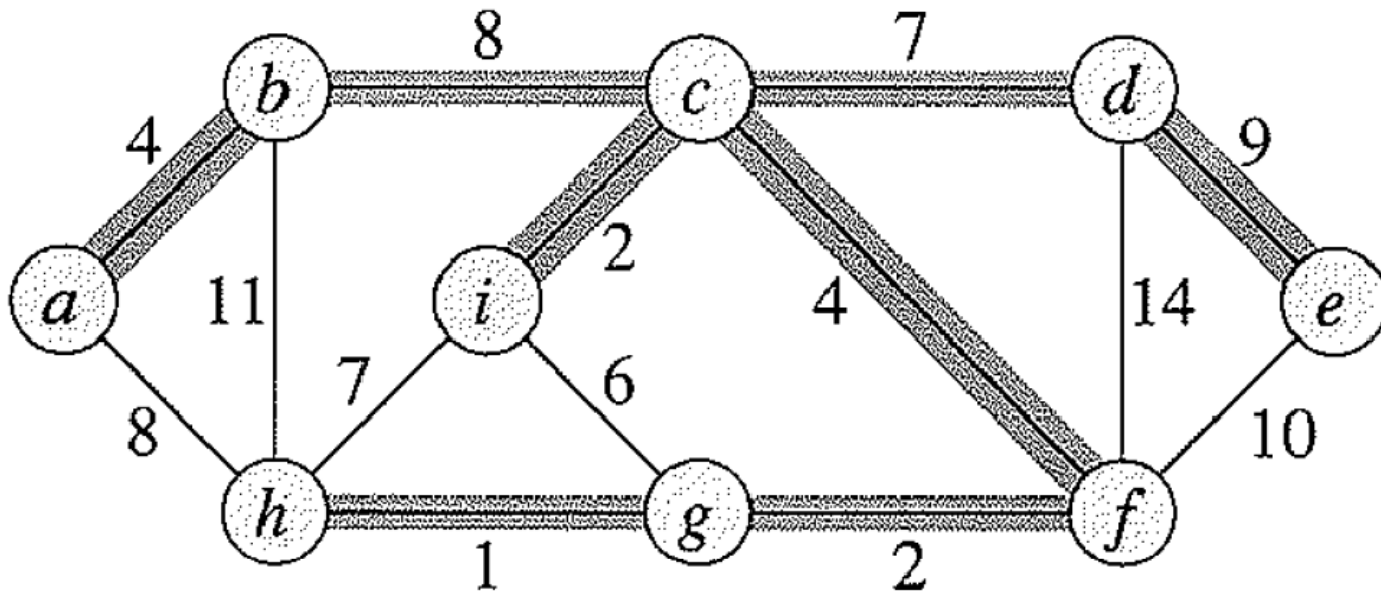
Minimum Udspændende Træer [CLRS, kapitel 23]



Gerth Stølting Brodal

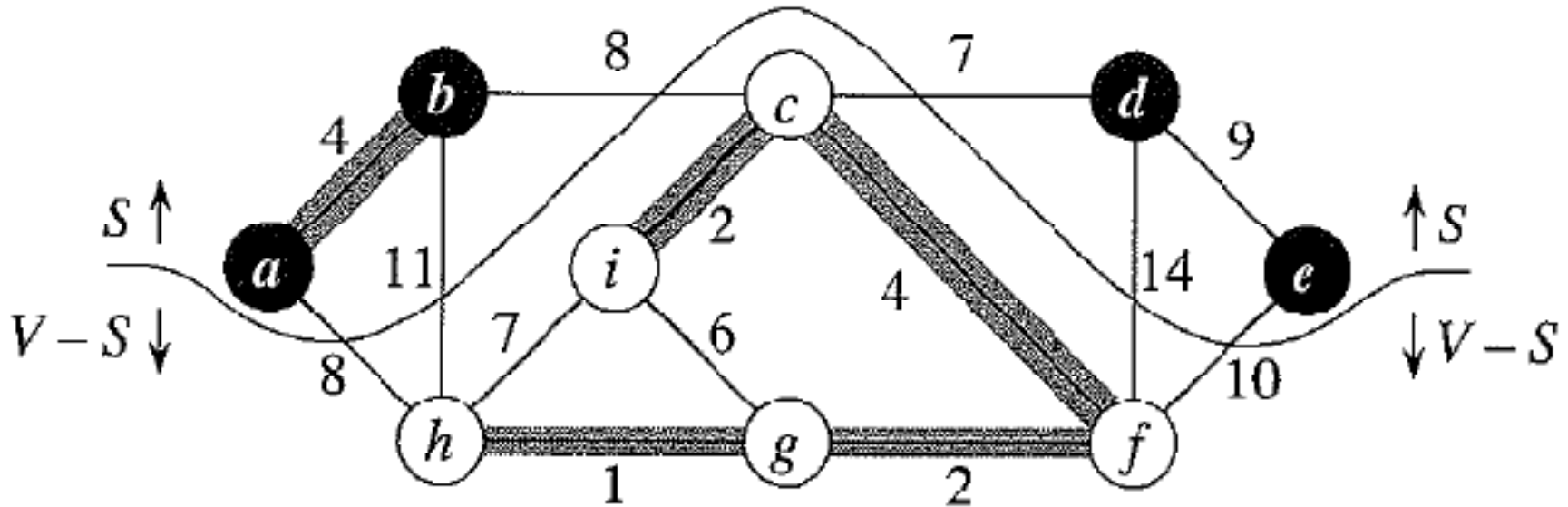
Aarhus Universitet

Minimum Udspændende Træer: Eksempel



Problem: Find et udspændende træ for en sammenhængende graf således at summen af kanterne er mindst mulig

Minimum Udspændende Træer: Snit



Sætning Hvis alle vægte er forskellige, så gælder der for ethvert snit at den **letteste kant** der krydser snitter er med i et minimum udspændende træ

Minimum Udspændende Træer: Grådig general algoritme

GENERIC-MST(G, w)

- 1 $A \leftarrow \emptyset$
- 2 **while** A does not form a spanning tree
- 3 **do** find an edge (u, v) that is safe for A
- 4 $A \leftarrow A \cup \{(u, v)\}$
- 5 **return** A

Kruskall's Algoritme

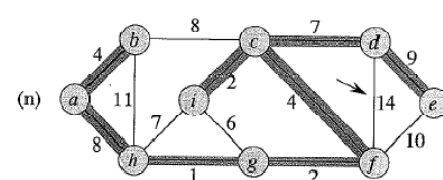
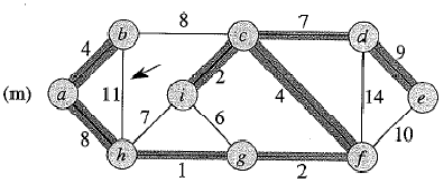
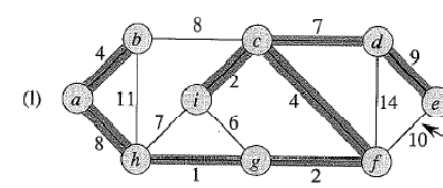
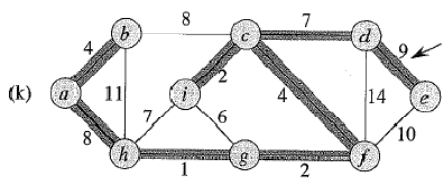
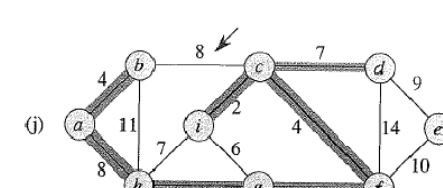
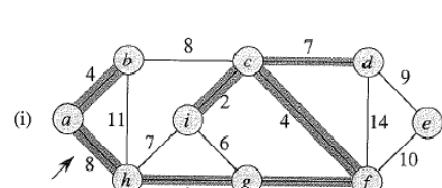
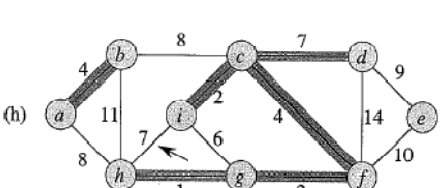
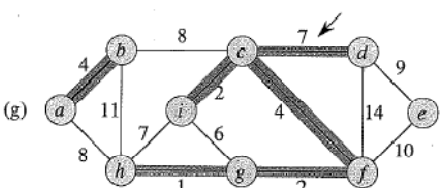
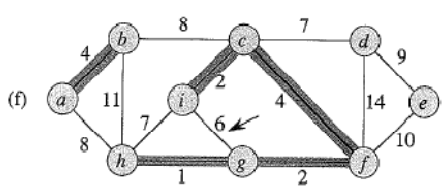
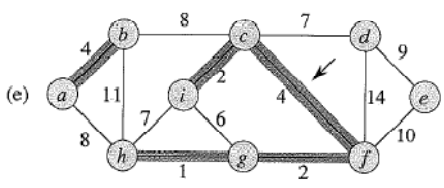
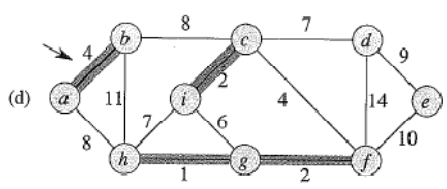
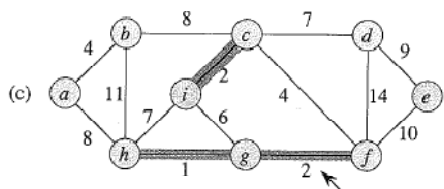
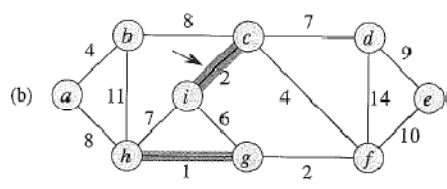
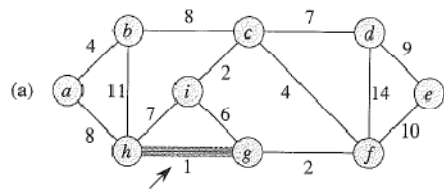
MST-KRUSKAL(G, w)

```
1   $A \leftarrow \emptyset$ 
2  for each vertex  $v \in V[G]$ 
3      do MAKE-SET( $v$ )
4  sort the edges of  $E$  into nondecreasing order by weight  $w$ 
5  for each edge  $(u, v) \in E$ , taken in nondecreasing order by weight
6      do if FIND-SET( $u$ )  $\neq$  FIND-SET( $v$ )
7          then  $A \leftarrow A \cup \{(u, v)\}$ 
8              UNION( $u, v$ )
9  return  $A$ 
```

flaskehals i algoritmen

Tid $O(m \cdot \log n)$

Kruskal's Algorithm: Eksempel



Prim's Algorithm

MST-PRIM(G, w, r)

```
1  for each  $u \in V[G]$ 
2      do  $key[u] \leftarrow \infty$ 
3      do  $\pi[u] \leftarrow \text{NIL}$ 
4   $key[r] \leftarrow 0$ 
5   $Q \leftarrow V[G]$ 
6  while  $Q \neq \emptyset$ 
7      do  $u \leftarrow \text{EXTRACT-MIN}(Q)$ 
8         for each  $v \in \text{Adj}[u]$ 
9             do if  $v \in Q$  and  $w(u, v) < key[v]$ 
10                 then  $\pi[v] \leftarrow u$ 
11                     $key[v] \leftarrow w(u, v)$ 
```

flaskehals i algoritmen

Tid $O(m \cdot \log n)$

Prim's Algorithm: Eksempel

