

Algoritmer og Datastrukturer 1

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Elementære Datastrukturer [CLRS, kapitel 10]



[CLRS, Del 3] : Datastrukturer

Oprethold en struktur for en
dynamisk mængde data

Abstrakte Datastrukturer for Mængder

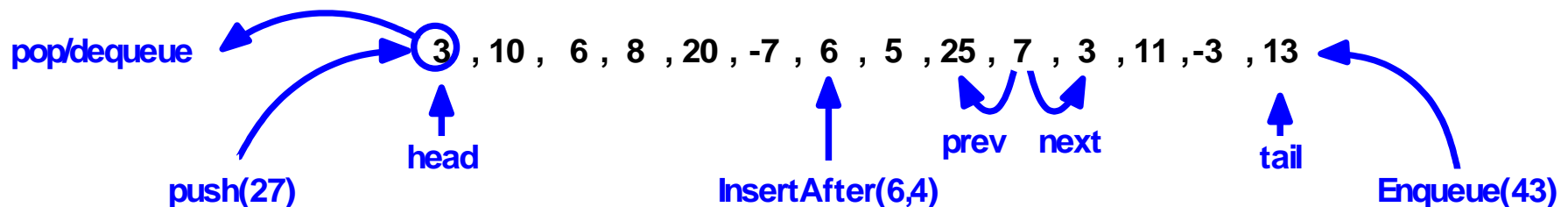
-Min-prioritetskø
-Max-prioritetskø
- Ordbog

Forespørgsel	Minimum(S)	pointer til element	●		
	Maximum(S)	pointer til element		●	
	Search(S, x)	pointer til element			●
	Member(S, x)	TRUE eller FALSE			
	Successor(S, x)	pointer til element			
	Predecessor(S, x)	pointer til element			
Opdateringer	Insert(S, x)	pointer til element	●	●	●
	Delete(S, x)	-			●
	DeleteMin(S)	element	●		
	DeleteMax(S)	element		●	
	Join(S_1, S_2)	mængde S			
	Split(S, x)	mængder S_1 og S_2			

Abstrakte Datastrukturer for Lister

-Stak -KØ

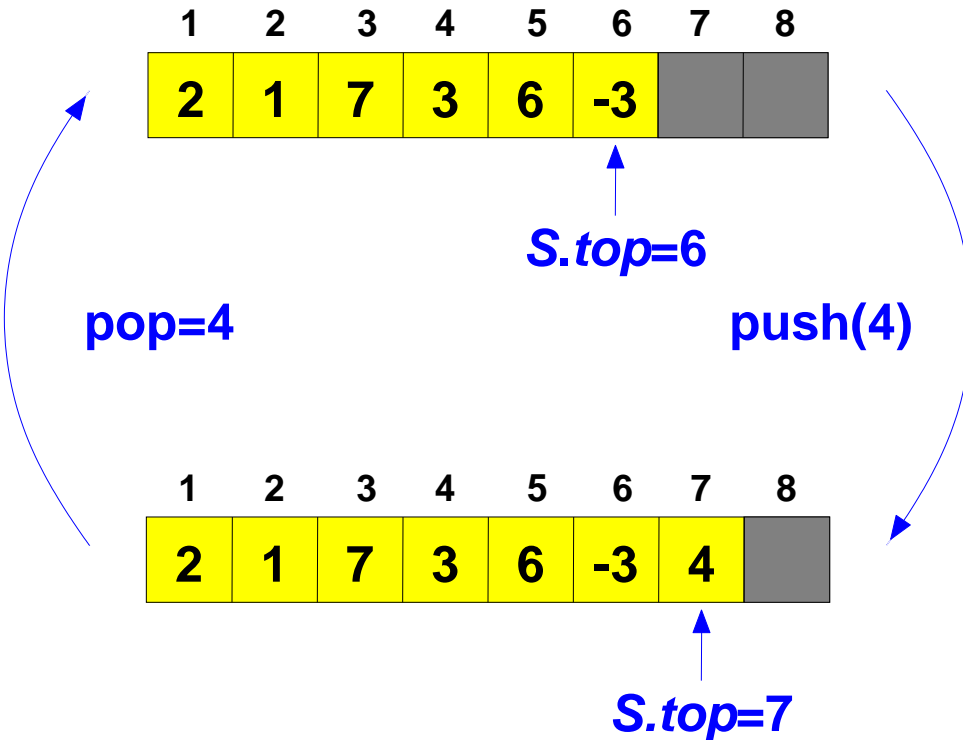
Forespørgsel	Empty(S)	TRUE eller FALSE	●	●
	Head(S), Tail(S)	pointer til element		
	Next(S, x), Prev(S, x)	pointer til element		
	Search(S, x)	pointer til element		
Opdateringer	Push(S, x)	-	●	
	Pop/Dequeue(S)	element	●	●
	Enqueue(S, x)	-		●
	Delete(S, x)	Element		
	InsertAfter(S, x, y)	pointer til element		





Stak

Stak : Array Implementation



STACK-EMPTY(S)

```
1  if  $S.top == 0$ 
2      return TRUE
3  else return FALSE
```

PUSH(S, x)

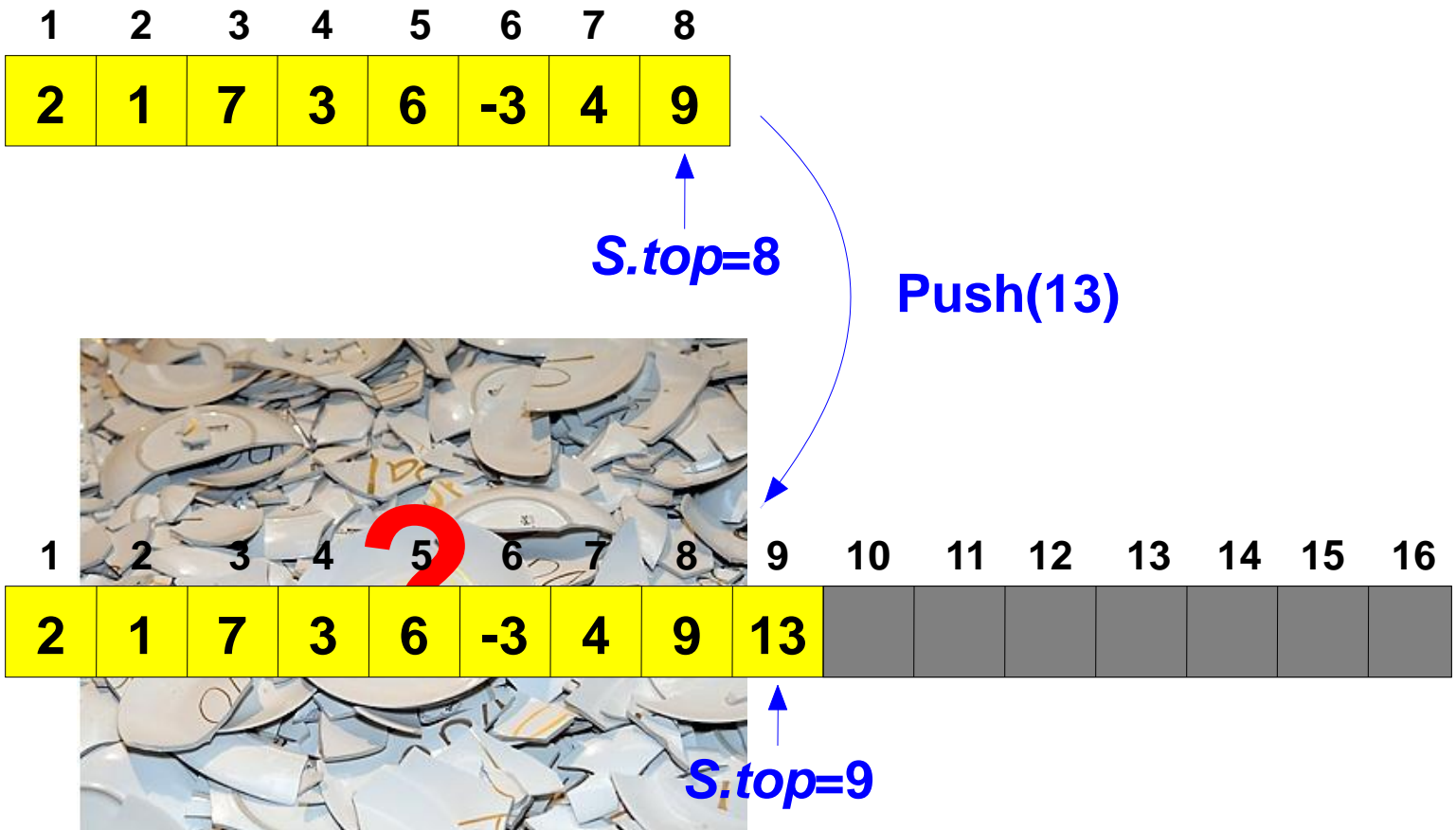
```
1   $S.top = S.top + 1$ 
2   $S[S.top] = x$ 
```

POP(S)

```
1  if STACK-EMPTY( $S$ )
2      error "underflow"
3  else  $S.top = S.top - 1$ 
4      return  $S[S.top + 1]$ 
```

Stack-Empty, Push, Pop : $O(1)$ tid

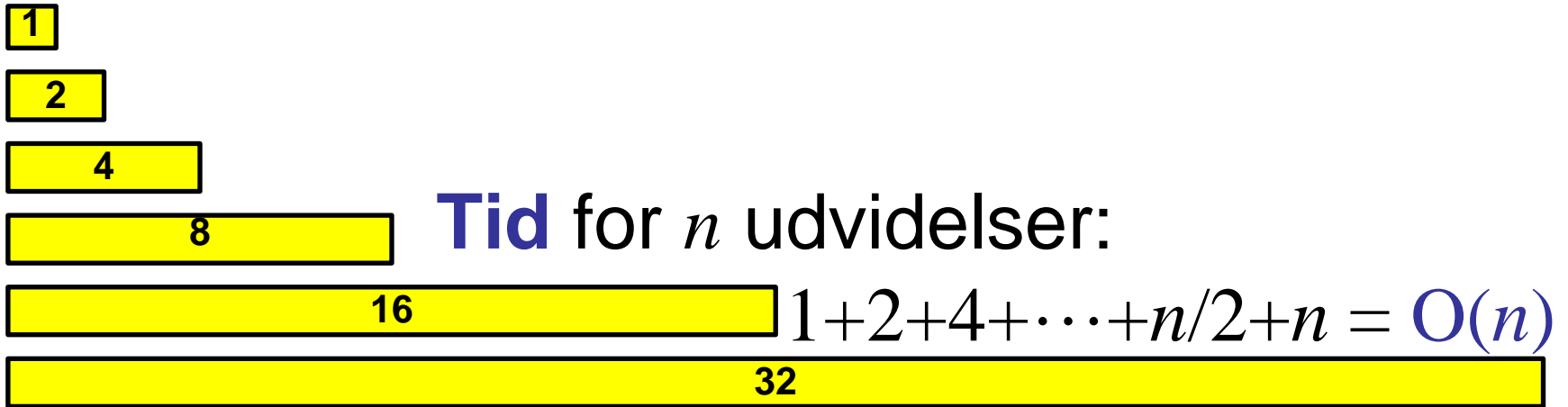
Stak : Overløb



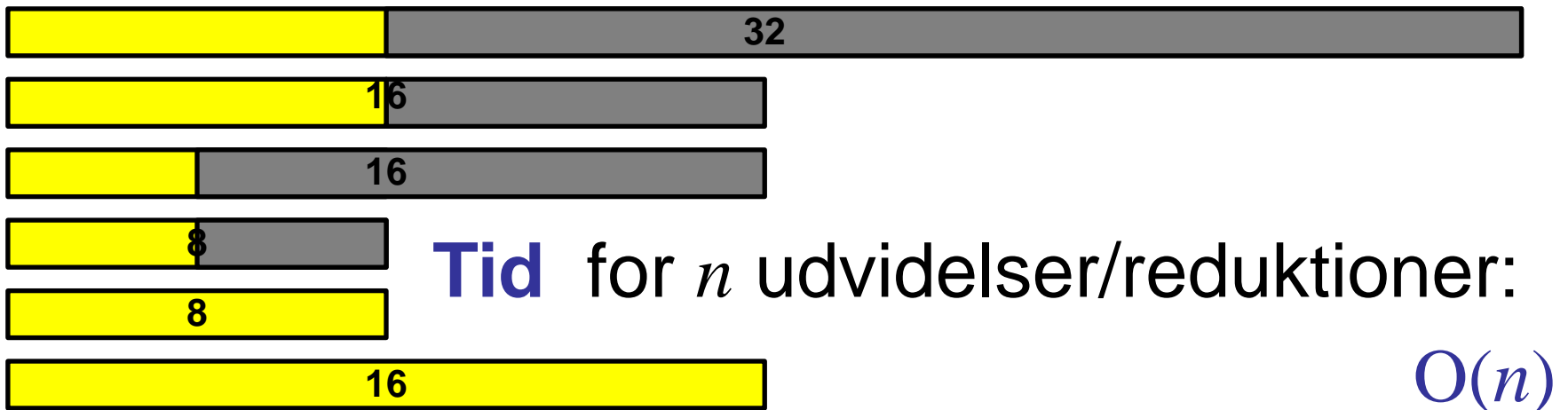
Array fordobling : $O(n)$ tid

Array Fordobling

Fordoble arrayet når det er fuld



Halver arrayet når det er $<1/4$ fyldt



Array Fordobling + Halvering

– en generel teknik

Tid for n udvidelser/reduktioner er $O(n)$

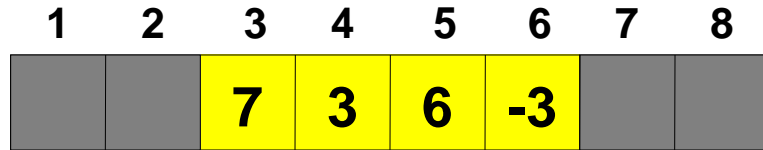
Plads $\leq 4 \cdot$ aktuelle antal elementer

Array implementation af Stak:
 n push og pop operationer tager $O(n)$ tid



Kø

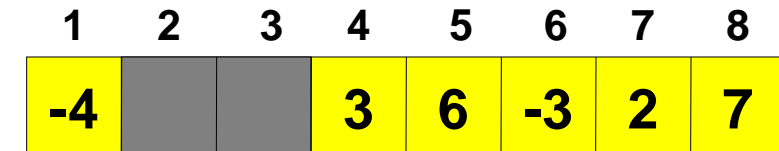
Kø : Array Implementation



$Q.head=3$

$Q.tail=7$

Enqueue(2)
Enqueue(7)
Enqueue(-4)
Dequeue = 7



$Q.tail=2$

$Q.head=4$

ENQUEUE(Q, x)

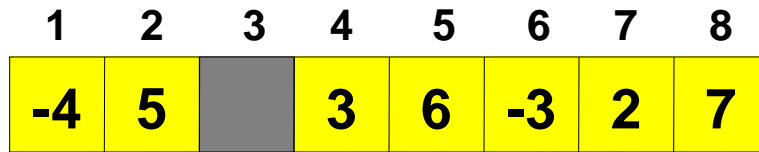
```
1  $Q[Q.tail] = x$   
2 if  $Q.tail == Q.length$   
3      $Q.tail = 1$   
4 else  $Q.tail = Q.tail + 1$ 
```

DEQUEUE(Q)

```
1  $x = Q[Q.head]$   
2 if  $Q.head == Q.length$   
3      $Q.head = 1$   
4 else  $Q.head = Q.head + 1$   
5 return  $x$ 
```

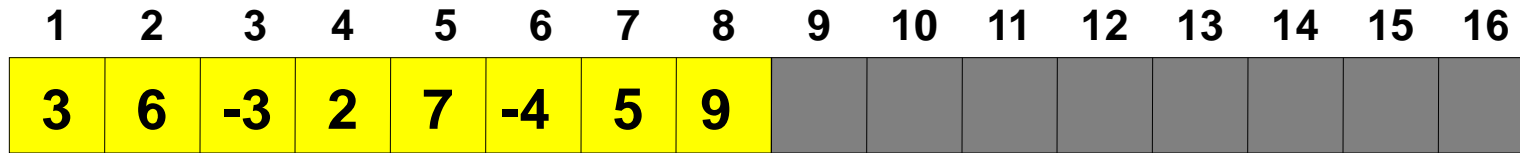
Enqueue, dequeue : $O(1)$ tid

Kø : Array Implementation



$Q.tail=3$ $Q.head=4$

Enqueue(9)



$Q.head=1$

$Q.tail=9$

Empty : $Q.tail=Q.head$?

Overløb : array fordobling/
halvering

Array implementation af Kø:

n enqueue og dequeue operationer tager $O(n)$ tid

Arrays (med Fordobling/Halvering)

Stak	Push(S, x)	$O(1)^*$
	Pop(S)	$O(1)^*$
Kø	Enqueue(S, x)	$O(1)^*$
	Dequeue(S)	$O(1)^*$

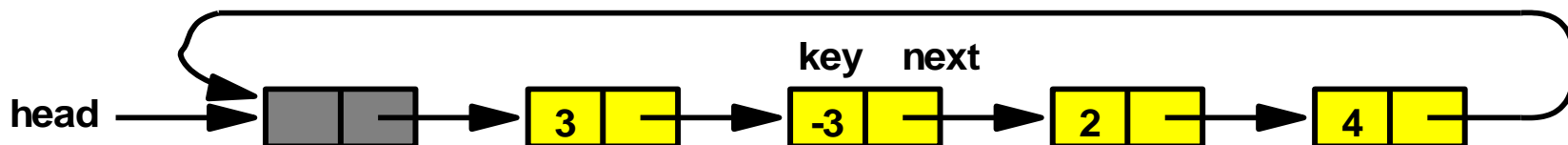
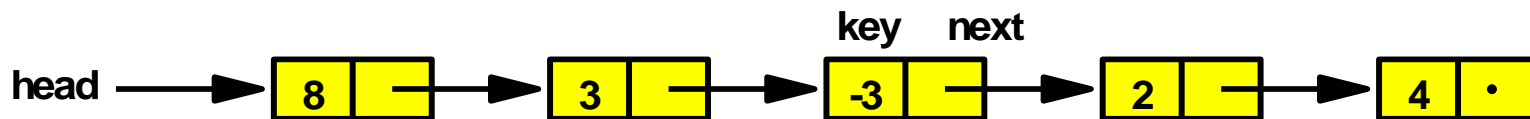
* Worst-case uden fordobling/halvering
Amortiseret ([CLRS, Kap. 17]) med fordobling/halvering



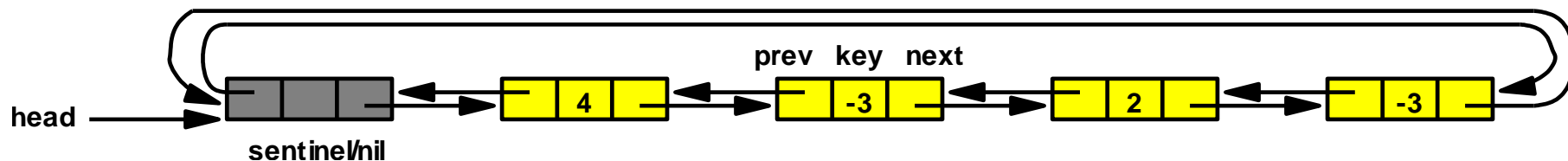
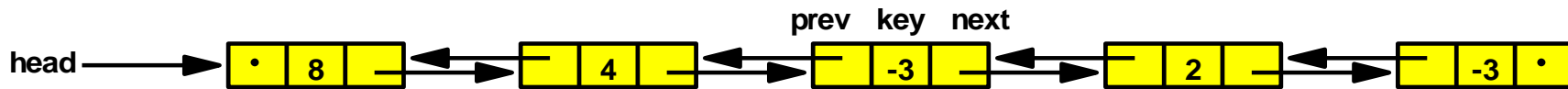
Kædede lister

Kædede Lister

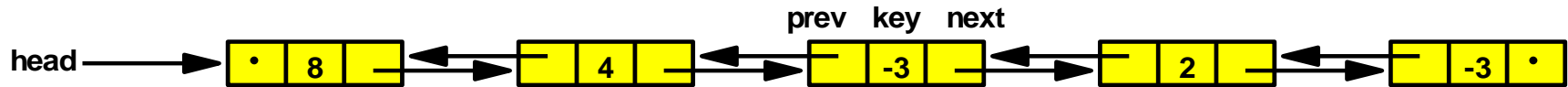
Enkelt kædede (ikke-cyklisk og cyklisk)



Dobbelt kædede (ikke-cyklisk og cyklisk)



Dobbelt Kædede Lister



LIST-SEARCH(L, k)

```
1  $x = L.head$ 
2 while  $x \neq \text{NIL}$  and  $x.key \neq k$ 
3      $x = x.next$ 
4 return  $x$ 
```

LIST-INSERT(L, x)

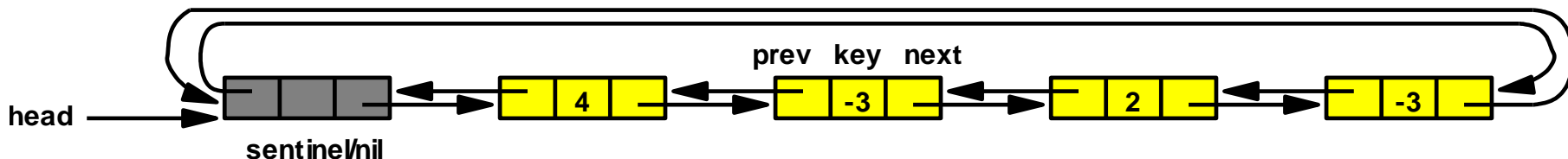
```
1  $x.next = L.head$ 
2 if  $L.head \neq \text{NIL}$ 
3      $L.head.prev = x$ 
4  $L.head = x$ 
5  $x.prev = \text{NIL}$ 
```

LIST-DELETE(L, x)

```
1 if  $x.prev \neq \text{NIL}$ 
2      $x.prev.next = x.next$ 
3 else  $L.head = x.next$ 
4 if  $x.next \neq \text{NIL}$ 
5      $x.next.prev = x.prev$ 
```

List-Search	$O(n)$
List-Insert	$O(1)$
List-Delete	$O(1)$

Dobbelt Kædede Cykliske Lister



LIST-SEARCH'(L, k)

- 1 $x = L.nil.next$
- 2 **while** $x \neq L.nil$ and $x.key \neq k$
- 3 $x = x.next$
- 4 **return** x

LIST-INSERT'(L, x)

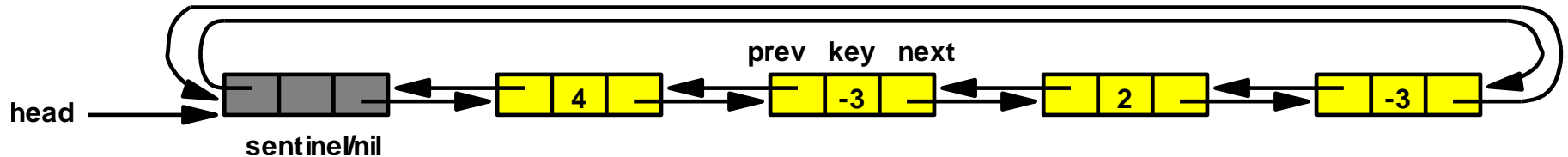
- 1 $x.next = L.nil.next$
- 2 $L.nil.next.prev = x$
- 3 $L.nil.next = x$
- 4 $x.prev = L.nil$

LIST-DELETE'(L, x)

- 1 $x.prev.next = x.next$
- 2 $x.next.prev = x.prev$

List-Search'	$O(n)$
List-Insert'	$O(1)$
List-Delete'	$O(1)$

Dobbelt Kædede Cykliske Lister



Stak	Push(S, x)	$O(1)$
	Pop(S)	$O(1)$
Kø	Enqueue(S, x)	$O(1)$
	Dequeue(S)	$O(1)$

Dancing Links

Donald E. Knuth, Stanford University

My purpose is to discuss an extremely simple technique that deserves to be better known. Suppose x points to an element of a doubly linked list; let $L[x]$ and $R[x]$ point to the predecessor and successor of that element. Then the operations

$$L[R[x]] \leftarrow L[x], \quad R[L[x]] \leftarrow R[x] \quad (1)$$

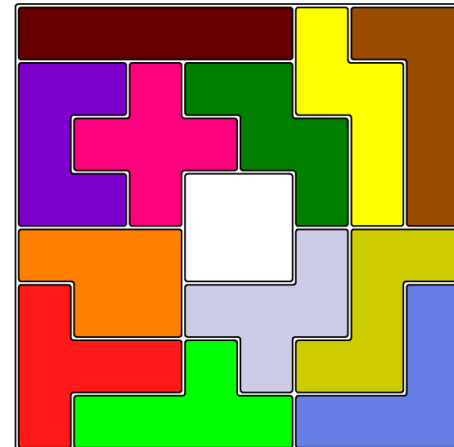
remove x from the list; every programmer knows this. But comparatively few programmers have realized that the subsequent operations

$$L[R[x]] \leftarrow x, \quad R[L[x]] \leftarrow x \quad (2)$$

will put x back into the list again.



Donald E. Knuth (1938-)



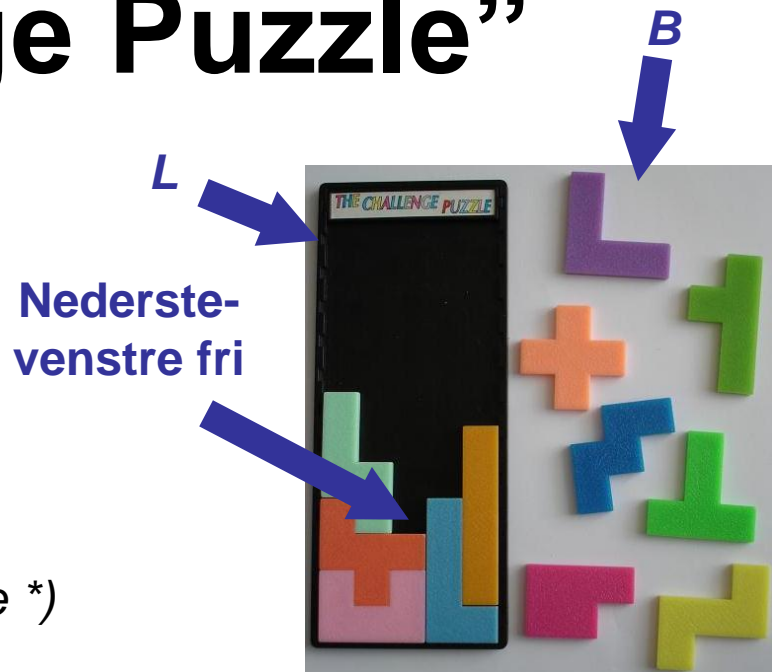
“The Challenge Puzzle”



”The Challenge Puzzle”

L := Tomt bræt
 B := Alle brikker
Solve(L, B)

```
procedure Solve(Delløsning  $L$ , Brikker  $B$ )  
  for alle  $b$  i  $B$   
    for alle orienteringer af  $b$  (* max 8 forskellige *)  
      if  $b$  kan placeres i nederste venstre fri then  
        fjern  $b$  fra  $B$   
        indsæt  $b$  i  $L$   
        if  $|B|=0$  then  
          rapporter  $L$  er en løsning  
        else  
          Solve( $L, B$ )  
      fi  
    slet  $b$  fra  $L$   
  genindsæt  $b$  i  $B$   
fi
```



Før



Efter

”The Challenge Puzzle”

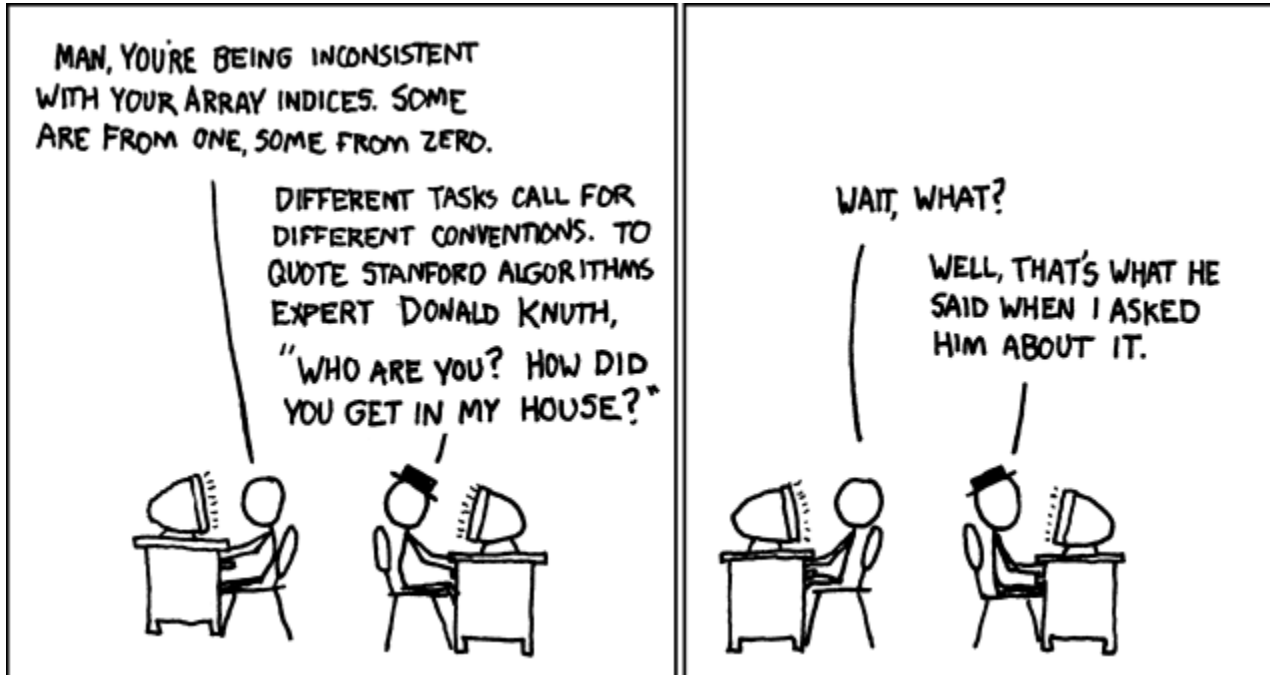


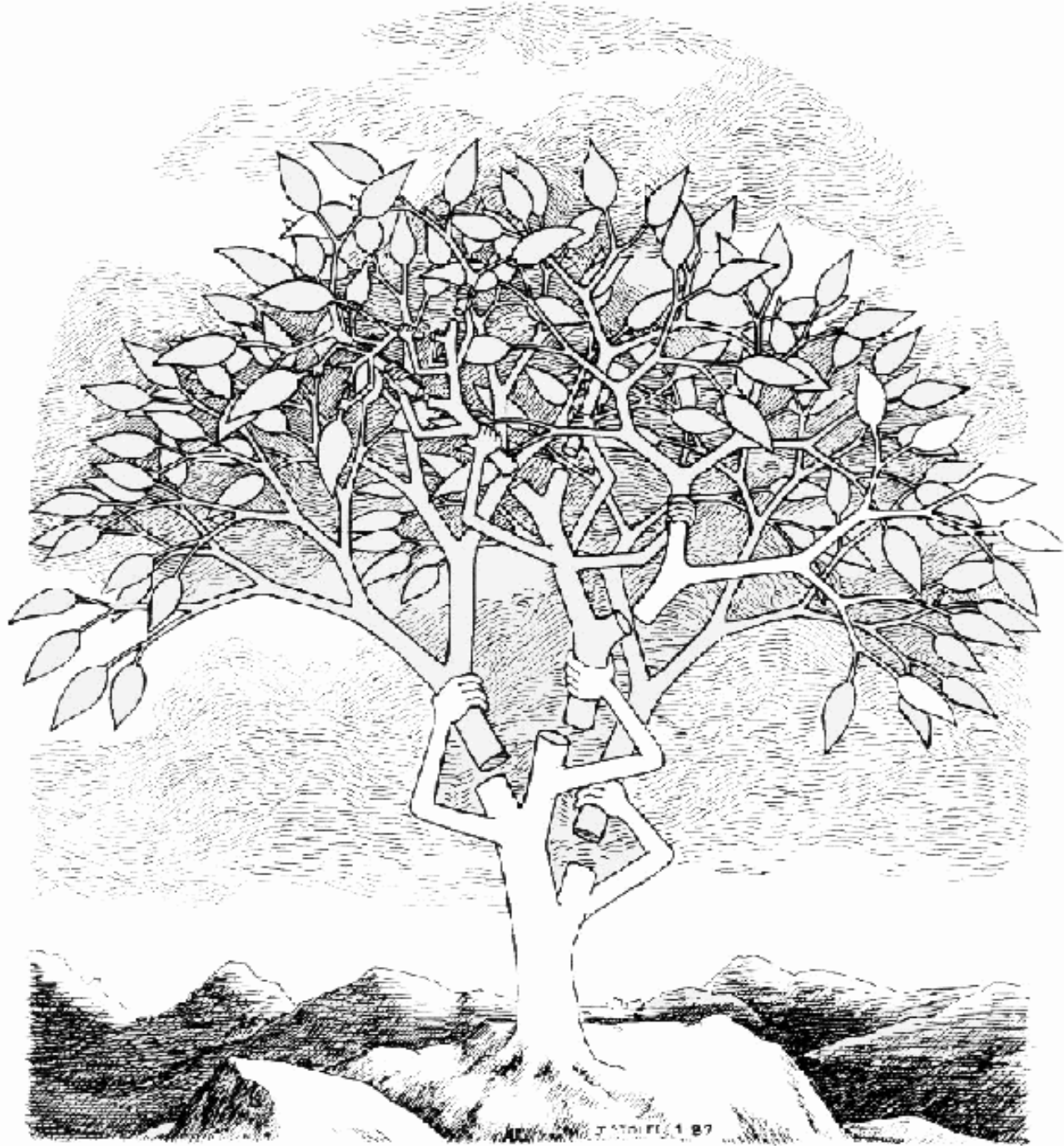
4.040 løsninger

Solve placerer

8.387.259 brikker

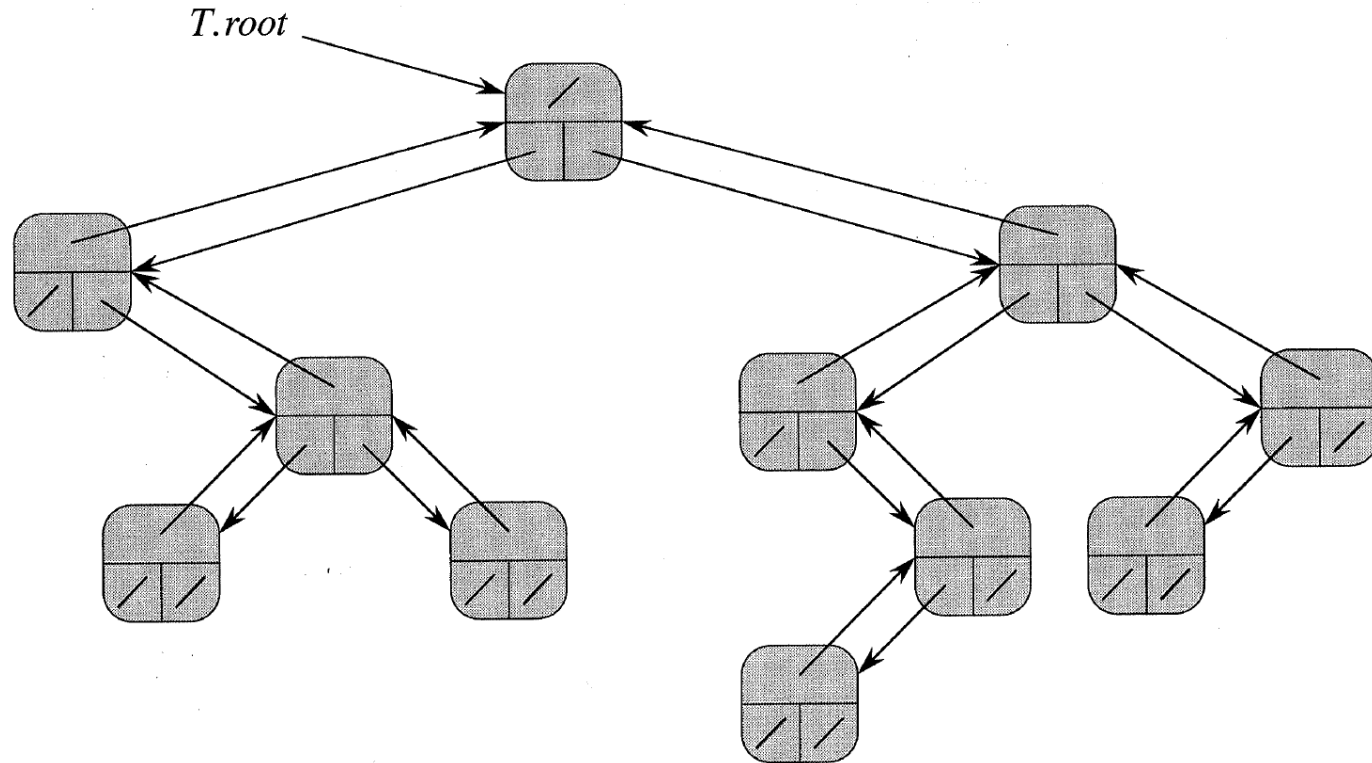
Donald Knuth





(Jorge Stolfi)

Binær Træ Repræsentation



Felter: **Left, right, parent**

