HUSK EKSAMENSTILMELDING

Frist: 15. februar

Algoritmer og Datastrukturer 1

Merge-Sort [CLRS, kapitel 2.3]
Heaps [CLRS, kapitel 6]

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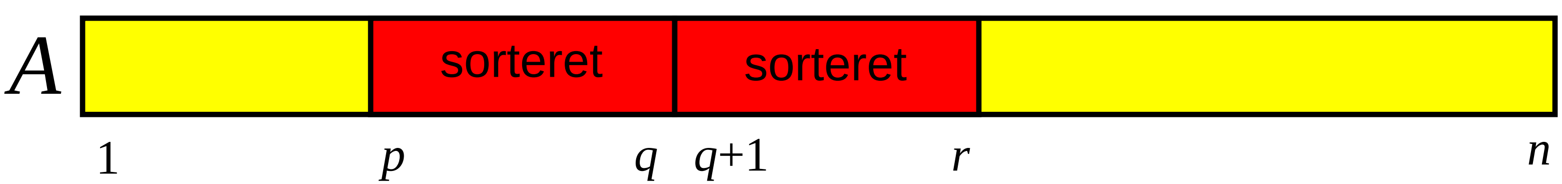
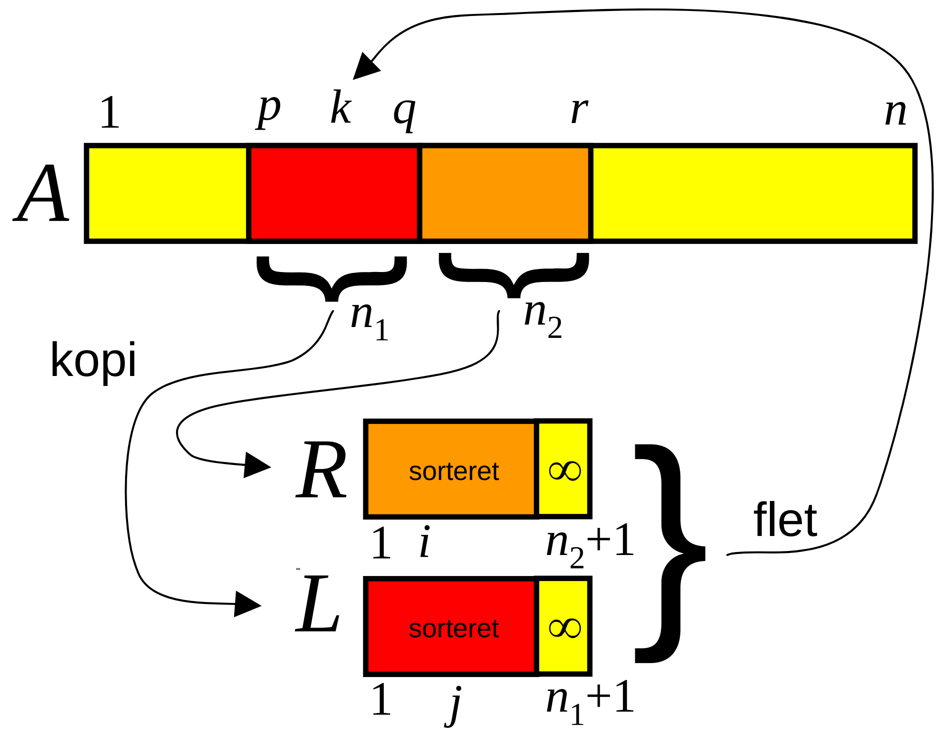
Merge-Sort
(Eksempel på Del-og-kombiner)

```
MERGE-SORT(A, p, r)
  if p < r
    then q ← ⌊(p + r)/2⌋
         MERGE-SORT(A, p, q)
         MERGE-SORT(A, q + 1, r)
         MERGE(A, p, q, r)
```



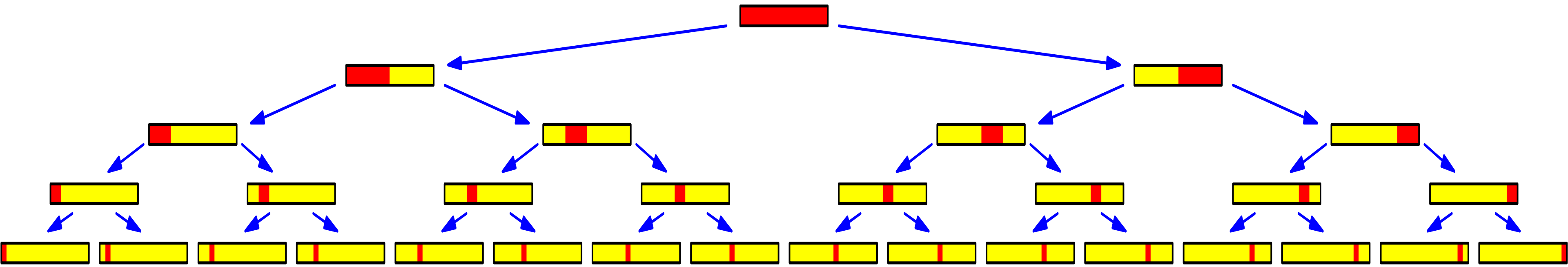
MERGE(A, p, q, r)

```
n1 ← q − p + 1
n2 ← r − q
create arrays L[1 .. n1 + 1] and R[1 .. n2 + 1]
for i ← 1 to n1
        do L[i] ← A[p + i − 1]
for j ← 1 to n2
        do R[j] ← A[q + j]
L[n1 + 1] ← ∞
R[n2 + 1] ← ∞
i ← 1
j ← 1
for k ← p to r
        do if L[i] ≤ R[j]
                then A[k] ← L[i]
                     i ← i + 1
                else  A[k] ← R[j]
                     j ← j + 1
```



Merge-Sort : Analyse

Rekursionstræet



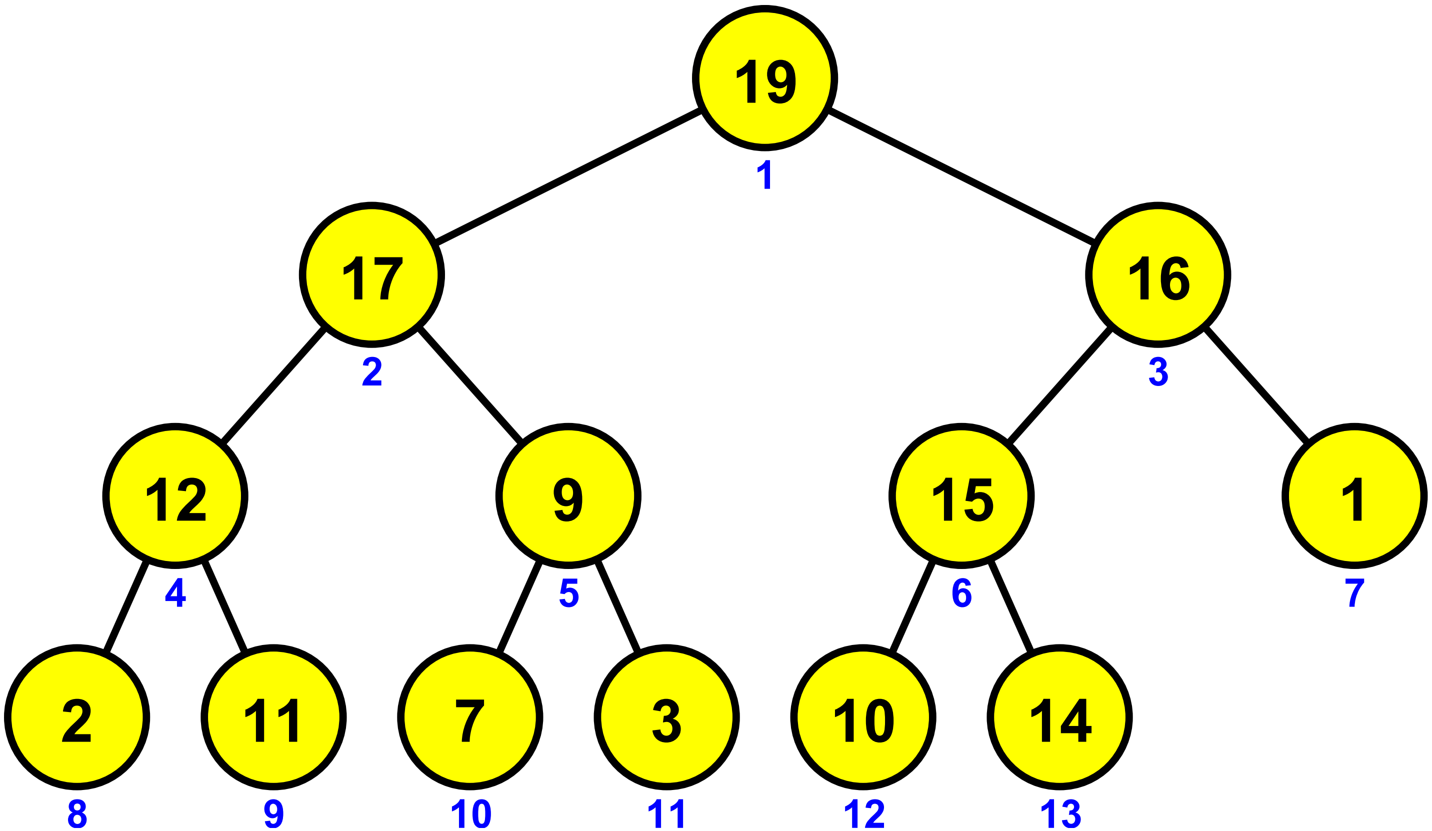
Observation

Samlet arbejde per lag er $O(n)$

Arbejde

$$O(n \cdot \#\,\text{lag}) = O(n \cdot \log_2 n)$$

Binær (Max-)Heap

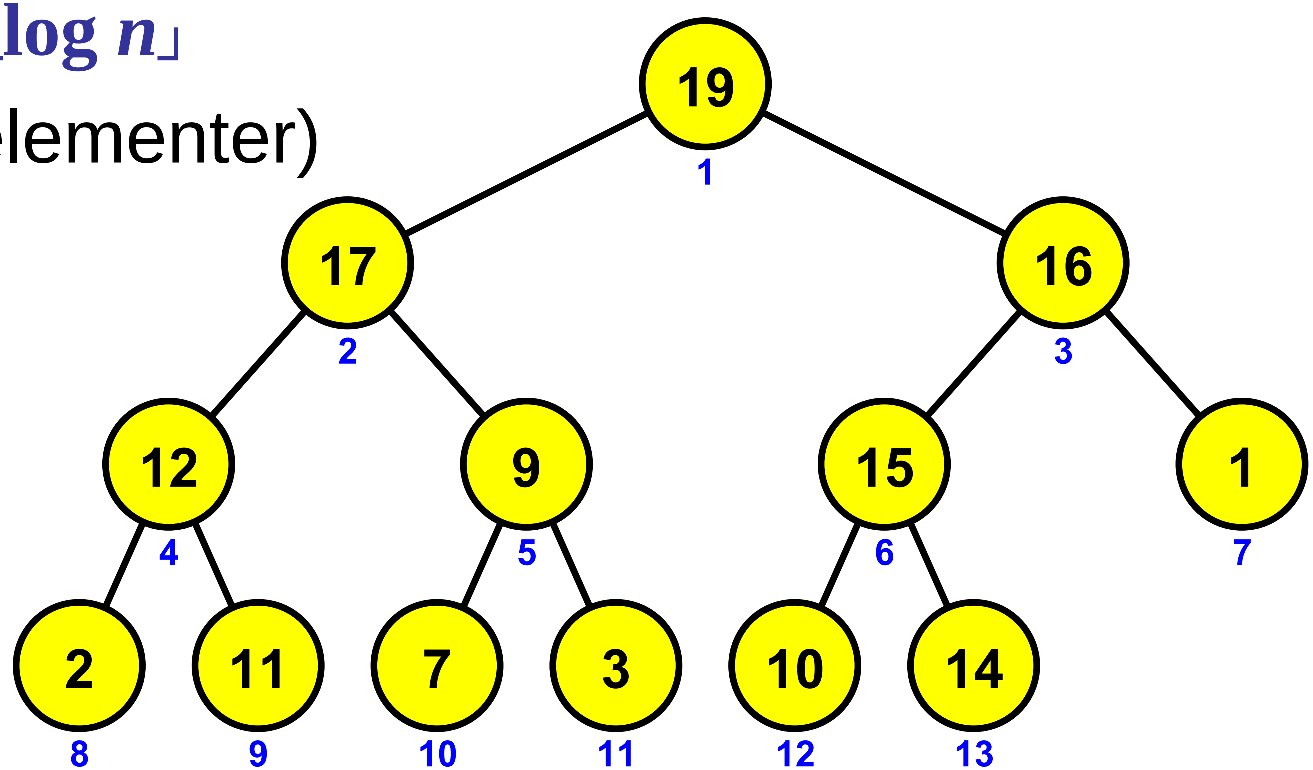


19	17	16	12	9	15	1	2	11	7	3	10	14
1	2	3	4	5	6	7	8	9	10	11	12	13

Williams, 1964

Max-heap : Egenskaber

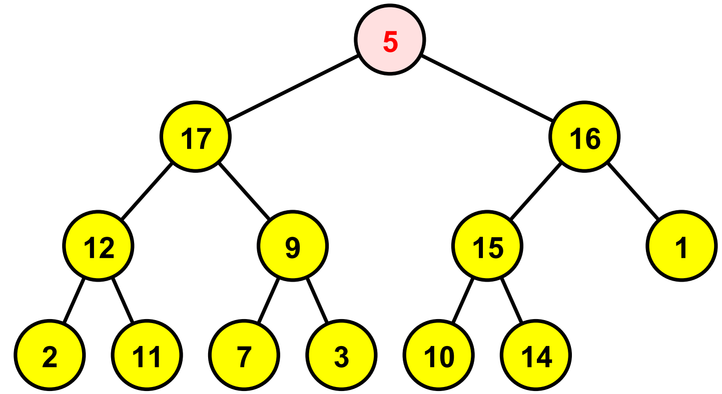
- Roden : knude **1**
- Børn til knude i : $\mathbf{2\boldsymbol{i}}$ og $\mathbf{2\boldsymbol{i}+1}$
- Faren til knude i : $\lfloor \boldsymbol{i} / 2 \rfloor$
- Dybde : $1 + \lfloor \log n \rfloor$
  (n = antal elementer)



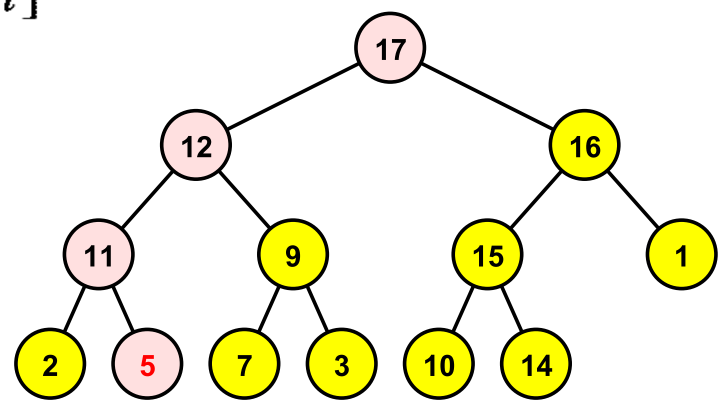
Max-Heapify

```
MAX-HEAPIFY(A, i)
l ← LEFT(i)
r ← RIGHT(i)
if l ≤ heap-size[A] and A[l] > A[i]
    then largest ← l
    else largest ← i
if r ≤ heap-size[A] and A[r] > A[largest]
    then largest ← r
if largest ≠ i
    then exchange A[i] ↔ A[largest]
         MAX-HEAPIFY(A, largest)
```

Tid O(log *n*)



Før



Efter

Heap-Sort

Floyd, 1964

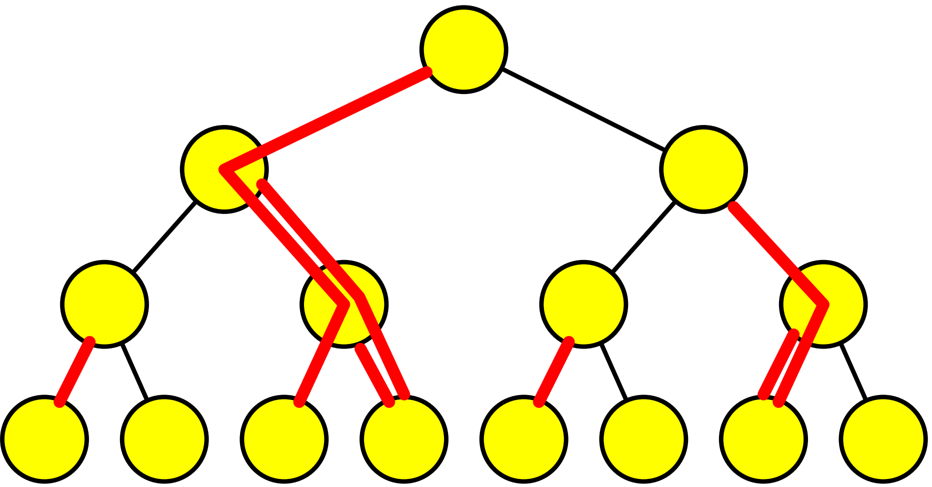
```
BUILD-MAX-HEAP(A)
heap-size[A] ← length[A]
for i ← ⌊length[A]/2⌋ downto 1
        do MAX-HEAPIFY(A, i)
```

Williams, 1964

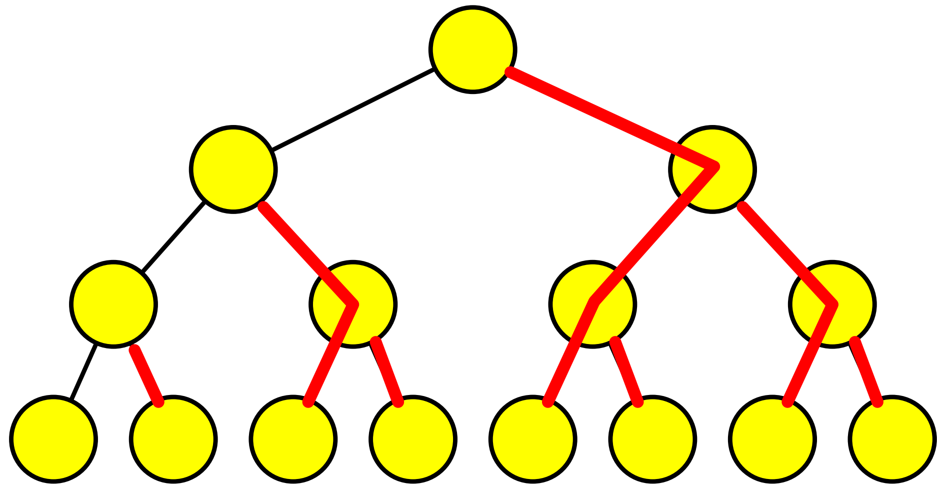
```
HEAPSORT(A)
BUILD-MAX-HEAP(A)
for i ← length[A] downto 2
        do exchange A[1] ↔ A[i]
           heap-size[A] ← heap-size[A] − 1
           MAX-HEAPIFY(A, 1)
```

Tid O($n \cdot \log n$)

Build-Max-Heap



Max-Heapify stierne (eksempel)



Ikke-overlappende stier med samme #kanter (højre, venstre, venstre...)

Tid for Build-Max-Heap

= Σ tid for Max-Heapify

= # røde kanter

≤ # røde kanter

= n **- dybde**

= $O(n)$

Tid $O(n)$

Sorterings-algoritmer

<table>
<tr><th>Algoritme</th><th>Worst-Case Tid</th></tr>
<tr><td>Heap-Sort</td><td rowspan="2">$O(n \cdot \log n)$</td></tr>
<tr><td>Merge-Sort</td></tr>
<tr><td>Insertion-Sort</td><td>$O(n^2)$</td></tr>
</table>

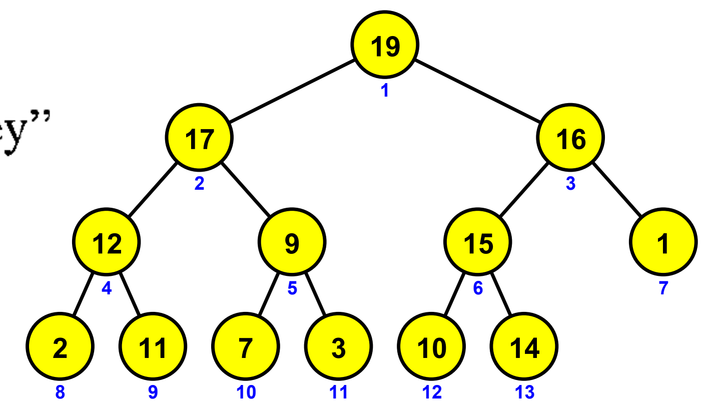
Max-Heap operationer

```
HEAP-MAXIMUM(A)
    return A[1]
```

```
MAX-HEAP-INSERT(A, key)
    heap-size[A] ← heap-size[A] + 1
    A[heap-size[A]] ← −∞
    HEAP-INCREASE-KEY(A, heap-size[A], key)
```

```
HEAP-EXTRACT-MAX(A)
    if heap-size[A] < 1
       then error "heap underflow"
    max ← A[1]
    A[1] ← A[heap-size[A]]
    heap-size[A] ← heap-size[A] − 1
    MAX-HEAPIFY(A, 1)
    return max
```

```
HEAP-INCREASE-KEY(A, i, key)
    if key < A[i]
       then error "new key is smaller than current key"
    A[i] ← key
    while i > 1 and A[PARENT(i)] < A[i]
        do exchange A[i] ↔ A[PARENT(i)]
           i ← PARENT(i)
```



Max-Heap operation

Operation	Worst-Case Tid
Max-Heap-Insert	$O(\log n)$
Heap-Extract-Max	
Max-Increase-Key	
Heap-Maximum	$O(1)$

n = aktuelle antal elementer i heapen

Prioritetskø

En **prioritetskø** er en abstrakt datastruktur der gemmer en mængde af **elementer** med tilknyttet **nøgle** og understøtter operationerne:

- **Insert**(S,x)
- **Maximum**(S)
- **Extract-Max**(S)

Maximum er med hensyn til de tilknyttede nøgler.

En mulig implementation af en prioritetskø er en heap.