Computational Geometry (Fall 2012)
Project 3, part B: Theoretical Questions
Deadline: 21st of December

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Question 4: Consider a regular n-gon and let P be its set of vertices. Describe the Voronoi diagram and the Delaunay triangulation of P.

Question 5 (optional): The concept of the Voronoi diagram of a pointset can be easily generalized to any dimension. For instance, for n points $p_1, \cdots, p_n$ in three dimensions, the Voronoi diagram of P is the decomposition of three-dimensional Euclidean space into n cells, $c_1, \cdots, c_n$, such that for any point $p \in c_i$, p_i is the closest point to p among the points $p_1, \cdots, p_n$. Also, similar to the two-dimensional case, each c_i is a convex region (i.e., the intersection of a number of halfspaces).

Prove that there exists a set P of n points in three-dimensions such that the total complexity of the Voronoi diagram of P is $\Omega(n^2)$.

Hint: First, figure out the main property of a Voronoi vertex and then show that there can be $\Omega(n^2)$ vertices. To do that, start by assuming that half of the points are on a single line.

Question 6:

A. Given a set S of n intervals, build a data structure such that given a query point p, one can *count* the intervals that contain q. Your data structure should use $O(n)$ space and should be able to answer queries in $O(\log n)$ time.

B. Use the segment tree paradigm to solve the following problem: given a set S of n axis-aligned rectangles in two dimensions, build a data structure such that given a query point p, one can *count* the rectangles that contain q efficiently. Your data structure should use $O(n \log n)$ space and should be able to answer queries in $O(\log^2 n)$ time.