Computational Geometry (Fall 2012)
Project 3, part A: Theoretical Questions

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Question 1: Let H be the set of n lines in the plane. Consider the arrangement A formed by H. Prove that the total complexity of all the unbounded cells is linear.

Question 2: Consider a set P of n points in the plane. We say that a line ℓ is *free* if it is possible to continuously move ℓ into a vertical line without it crossing any points in P during this movement. Prove that given P, it is possible to build a data structure of linear size such that for any given query line ℓ we can decide whether it is free in $O(\log n)$ time.

Question 3: Consider an input set P of n weighted points in the plane, meaning, each point $p_i \in P$ is associated with a real-valued weight $w(p_i)$. Prove that it is possible to build a data structure of linear size such that given a query line ℓ, the point with the largest weight below ℓ can be found in $O(\log n)$ time.

To make it easier to tackle this problem, try to follow the following step by step approach.

1. Use duality and clearly state the dual problem.
2. For the dual problem, try to find the regions of the plane, $r_1, \ldots, r_t$, such that the answer throughout each region r_i, $1 \leq i \leq t$, is the same.
3. Show that $t = O(n)$ and then use it to build a data structure that can solve the problem.

Bonus!: Show that the total preprocessing time is $O(n \log n)$.

General hints: Remember to use the duality transformation. When using duality, clearing state what is the dual problem before solving the dual problem.