Homework Exercises 9 - Probabilistic Method

9-1 *Crossing lemma.* Let G be a graph drawn on a plane such that no three edges coincide at a point except at vertices. Let $c(G)$, m and n be the number of edge intersections, the number of edges, and the number of vertices. If $m \geq 4n$, show that $c(G) \geqslant m^3/64n^2$.

Hint: Euler formula gives us a linear lower bound: $c(G) \geq m - 3n + 6$. Apply this lower bound to a random subgraph of G where each vertex is chosen by a probability of p for some appropriate value of p that you have to find it.

9-2 *Incidence problem.* Let P and L be a set of n points and a set of m lines. We say $p \in P$ is incident with $\ell \in L$ if p lies on ℓ. Show that the number of incidence is $O(m^{2/3}n^{2/3} + m + n)$.

Hint: Use Crossing lemma.