Exercise 8 - Random Walks

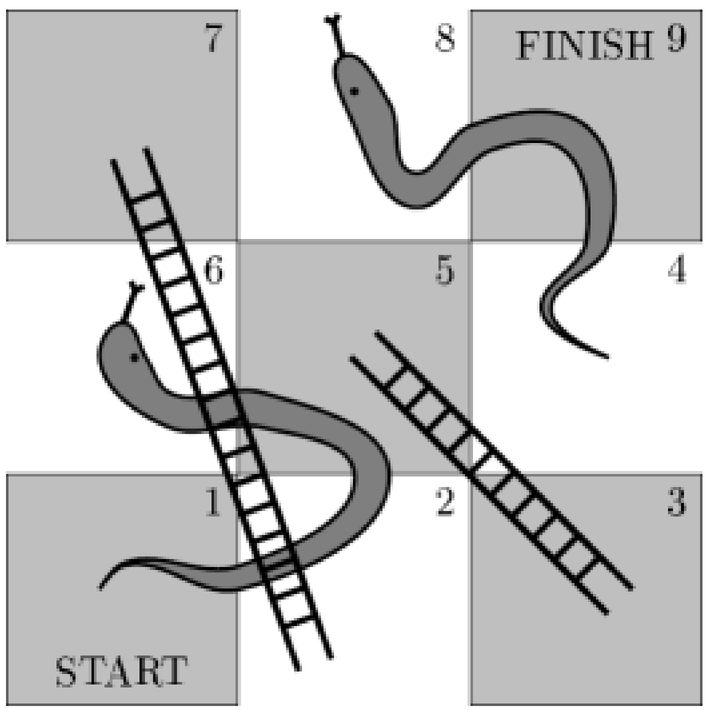
Deadline: 2nd June, 2009

8-1 An $n \times n$ irreducible matric P is said to be *stochastic* if all its entries are non-negative and for each row i, $\sum_j P_{ij} = 1$. It is said to be *doubly stochastic* if, in addition, $\sum_i P_{ij} = 1$.

(a) Show that for any stochastic matrix P, there exists an n-dimensional vector π such that $\pi P = \pi$.

(b) Suppose that the transition probability matrix P for a Markov chain is doubly stochastic. Show that the stationary distribution for this Markov chain is necessarily the uniform distribution.

8-2 A game of *Snakes and Ladders* is played on a board of nine squares:



At each turn a player tosses a fair coin and advances one or two places according to whether the coin lands heads or tails. If a player lands at the foot of a ladder, he immediately climbs to the top of the ladder. If a player lands on the head of a snake, he slides down to the tail of the snake. The game starts at square 1.

Allan, Morten, and Thomas bet 5 Madalgo Dollars on a game of Snakes and Ladders. Allan starts with heads, advances to square two, and climbs the ladder to square 7. Morten gets tails, advances to square three, and climbs the ladder to square 5. Claiming that the game is stupid, Thomas plays with a ball and hits Morten's Coke. He gets penalized with having to skip the first turn, i.e., he stays at square 1.

From the current position, who is most likely to win the game, i.e., what is the expected number of coin tosses for each of them to advance to square 9?