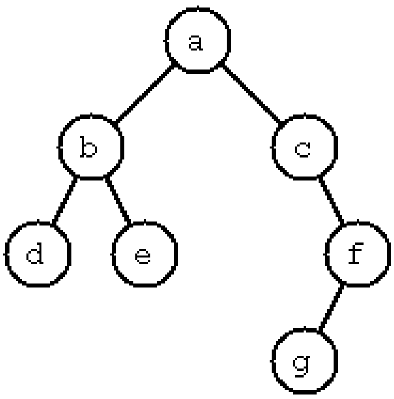
# Exercise 5 - Tree Traversal

**Deadline: 12th May, 2009**

5-1 (i) **Euler Tour**: A Euler tour of a connected, directed graph $G = (V_G, E_G)$ is a cycle that traverses each edge of the graph $G$ exactly once, although it may visit a vertex more than once. A graph $G$ has an Euler tour if $G$ is connected and in-degree($v$) = out-degree($v$) for every node $v \in V_G$.
Given an undirected tree $T = (V, E)$ and a root $r$ of this tree, one can create a bi-directional tree $T' = (V', E')$ such that $V' = V$ and for all $\{u, v\} \in E$, both directed edges $(u, v), (v, u) \in E'$. $E'$ does not contain any other edge. Clearly, an Euler tour exists for such a bi-directional tree that starts and ends at $r$. Show that such an Euler tour can be computed in $O(\log n)$ time using $O(n)$ processor EREW PRAM.
(*Hint*: For each node, define an arbitrary ordering of its neighbors. Using this, define for each edge, its successor in the Euler tour. Rank the resultant list.)

(ii) **Depth of a node**: In a tree, we define the depth of a node $v$ as its distance from the root. Given a rooted tree $T = (V, E)$ $(n := |V|)$, give a $O(\log n)$ time and $O(n)$ processor EREW PRAM algorithm to compute the depth $d(v)$ for all nodes $v \in V$.

(iii) **Parent of a node**: Given a rooted tree $T = (V, E)$, give a $O(\log n)$ time and $O(n)$ processor EREW PRAM algorithm to compute the parent $p(v)$ for all nodes $v \in V$.




(iv) **Number of Descendants**: Given a rooted tree $T = (V, E)$ $(n := |V|)$, compute for each node $v$ the number of nodes in the subtree rooted at $v$. For the tree in the figure above, the number of descendants of nodes are $\{a : 6, b : 2, c : 2, d : 0, e : 0, f : 1, g : 0\}$. Your algorithm should take $O(\log n)$ time and $O(n/\log n)$ processors with high probability.
(*Hint*: Reduce the problem to list ranking.)

(v) **Pre-order Traversal**: The pre-order traversal of a rooted tree $T = (V, E)$ $(n := |V|)$ consists of a traversal of the root $r$, followed by the preorder traversals of the subtrees of $r$ from left to right. For the tree in the figure above, the pre-order numbering of nodes is $\{a : 1, b : 2, c : 5, d : 3, e : 4, f : 6, g : 7\}$. Show how to obtain the preorder number of each node $v$ in $O(\log n)$ time (with $O(n)$ processors) on the EREW PRAM model.

(vi) **Post-order Traversal**: The post-order traversal of a rooted tree $T = (V, E)$ ($n := |V|$) consists of the post-order traversals of the subtrees of $r$ from left to right followed by a traversal of the root $r$. For the tree in the figure above, the post-order numbering of nodes is $\{a : 7, b : 3, c : 6, d : 1, e : 2, f : 5, g : 4\}$. Show how to obtain the post-order number of each node $v$ in $O(\log n)$ time (with $O(n)$ processors) on the EREW PRAM model.

(vii) **In-order Traversal**: Given a rooted binary tree $T = (V, E)$ (with $n := |V|$), root $r$, the in-order traversal of $T$ consists of the in-order traversal of the left subtree of $r$, followed by $r$, followed by the in-order traversal of the right subtree. For the tree in the figure above, the in-order numbering of nodes is $\{a : 4, b : 2, c : 5, d : 1, e : 3, f : 7, g : 6\}$. Develop an $O(\log n)$ time and $O(n)$ processor algorithm to assign each node of $T$ its inorder number.