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Instability of Networks with Quasi-Static Link Capacities *

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Abstract

A *packet-switched* network is *stable* if the number of packets in the network remains bounded at all times. A very natural question that arises in the context of stability properties of such networks is how the dynamical change of network link capacities precisely affects these properties.

In this work, we embark on a systematic study of this question adopting the *Adversarial Queueing Theory* framework, where an adversary controls rates of packet injections and determines packet paths. In addition, the power of the adversary is enhanced to include manipulation of *link capacities*. However, in doing so, the adversary may use only two possible (integer) values, namely 1 and $C > 1$; moreover, the capacity changes are not abrupt: once a link capacity is set to a value, it maintains this value for a period of time proportional to the number of packets in the system at the time of setting the link capacity to the value. We call this the *Adversarial, Quasi-Static Queueing Theory* model.

Within this model, we obtain a comprehensive collection of results in the form of instability bounds on injection rate of the adversary for various greedy protocols:

- Allowing the dynamical changing of link capacities of a network that uses LIS (*Longest-in-System*) protocol for contention-resolution may result in dropping its instability bound. This is shown through a construction of an LIS network of just ten nodes which becomes unstable at rates $\rho > \sqrt{2} - 1$ for large enough values of C. This represents the current record for the instability bound on the injection rate of the adversary for LIS over models of Adversarial Queueing Theory with dynamic capacities.
- The combination of dynamically changing link capacities with the use of compositions of protocols for contention-resolution on network queues suffices to drop the instability bound of a network to a substantially low value. This is shown through novel, yet simple and natural, combinatorial constructions of networks on which a composition of greedy protocols are running. In particular, we show that the composition of LIS with any of SIS (*Shortest-in-System*), NTS (*Nearest-to-Source*) and FTG (*Furthest-to-Go*) protocols is unstable at rates $\rho > \sqrt{2} - 1$ for large enough values of C. These represent the first results of instability bounds on injection rate for compositions of greedy protocols over models of Adversarial Queueing Theory with dynamic capacities.
- How much can the instability bound of network subgraphs that are forbidden for stability be affected by the dynamical changing of link capacities? Through improved combinatorial constructions of networks and executions, we present instability bounds for all *directed subgraphs* that are known to be *forbidden* for stability on networks running a certain greedy protocol. These bounds are lower than their counterparts for the classical Adversarial Queueing Theory model.

1 Introduction

1.1 Motivation and Framework

Objectives. We are interested in the behavior of *packet-switched networks* in which packets arrive dynamically at the *nodes* and they are routed in discrete time steps across the *links*. Recent years have witnessed a vast amount of work on analyzing packet-switched networks under *non-probabilistic* assumptions (rather than stochastic ones); We work within a model of *worst-case* continuous packet arrivals, originally proposed by Borodin *et al.* [4] and termed *Adversarial Queueing Theory* to reflect the assumption of an *adversarial* way of packet generation and path determination.

A major issue that arises in such a setting is that of *stability*– will the number of packets in the network remain bounded at all times? The answer to this question may depend on the *rate* of injecting packets into the network, the *capacity* of the links, which is the rate at which a link forwards outgoing packets, and the *protocol* that is used when more than one packet wants to cross a given link in a single time step to resolve the conflict. The underlying goal of our study is to establish stability and instability properties of networks and protocols when packets are injected by an adversary (rather than by an oblivious randomized process) and capacities are chosen by the same adversary in a dynamic way.

Model of Quasi-Static Capacities. Most studies of packet-switched networks assume that one packet can cross a network link (an edge) in a single time step. This assumption is well motivated when we assume that all links are identical. However, a packet-switched network can contain different types of links, which is common especially in large-scale networks like Internet. Then, it is well motivated to assign a capacity to each link. Furthermore, if each link capacity takes on values in the two-valued set of integers $\{1, C\}$ for $C > 1$, C takes on large values and each value stays for a long time, then we can consider approximately as a link failure the assigning of unit capacity to a link, while the assigning of capacity C to a link can be considered as the proper service rate. Therefore, the study of stability behavior of networks and protocols under our model of quasi-static capacities can be considered as an approximation of the fault-tolerance of a network where links can temporarily fail (zero capacity). This subfield of study was initiated by Borodin *et al.* in [5]. Note that we continue to assume uniform packet sizes.

Stability. Roughly speaking, a protocol P is *stable* [4] on a network $\mathcal{G}$ against an adversary $\mathcal{A}$ of rate ρ if there is a constant B (which may depend on $\mathcal{G}$ and $\mathcal{A}$) such that the number of packets in the system is bounded at all times by B. We say that *a protocol* P *is universally stable* [4] if it is stable against every adversary of rate less than 1 and on every network. We also say that *a network* $\mathcal{G}$ *is universally stable* [4] if every greedy protocol is stable against every

Protocol name	Which packet it advances:	**US**
Shortest-In-System (SIS)	The most recently injected packet	$\surd$ [1, Theorem 2.3]
Longest-In-System (LIS)	The least recently injected packet	$\surd$ [1, Theorem 2.5]
Furthest-To-Go (FTG)	The furthest packet from its destination	$\surd$ [1, Theorem 2.7]
Nearest-To-Source (NTS)	The nearest packet to its origin	$\surd$ [1, Theorem 2.8]
Nearest-To-Go-Using-LIS (NTG-U-LIS)	The nearest packet to its destination or the same as LIS for tie-breaking	**X** [2, Lemma 7]

Table 1: Greedy protocols considered in this paper. (**US** stands for universally stable)

adversary of rate less than 1 on $\mathcal{G}$. We say *forbidden subgraphs* for network stability [2, 8] any graph obtained by replacing any edge of the graphs $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4, \mathcal{U}_1$ and $\mathcal{U}_2$ (see Figure 5) by disjoint directed paths.

Greedy Protocols. We consider five *greedy* contention-resolution protocols– ones that always advance a packet across a queue (but one packet at each discrete time step) whenever there resides at least one packet in the queue (see Table 1).

1.2 Contribution

We define here the *weakest* possible adversary of dynamically changing *capacities* of network *links* in the context of Adversarial Queueing Theory where the adversary may set link capacities to any of two integer values 1 and C ($C > 1$ is a parameter called high capacity).[1] Moreover, once a link changes its capacity value, the model we use requires that the value stays fixed for at least a constant proportion of the number of packets in the system at the time when the capacity was last set. We call this the *Adversarial, Quasi-Static Queueing Theory* model. In this framework, we consider four protocols LIS, SIS, FTG, NTS; all four were shown universally stable in the standard model of Adversarial Queueing Theory.

- We construct a simple LIS network of only 10 nodes that is unstable at rates $\rho > \sqrt{2} - 1$ for large enough values of C. This result is the first one that presents an instability bound less than $\frac{1}{2}$ for a small-size network. Till now instability bounds of $\frac{1}{2}$ or less have been proved only on parameterized networks. To show this, we use an adversarial construction that sets properly the capacities of various networks edges to 1 for specified time intervals in order to accumulate packets.

[1] In the classical Adversarial Queueing Theory only one capacity value is available to the adversary.

- We then consider networks where different protocols may run on their nodes (heterogeneous networks, Internet). Thus, we prove that the composition of LIS with any of SIS, NTS and FTG is unstable at rates $\rho > \sqrt{2} - 1$ (for large enough values of C). To show this, we provide interesting combinatorial constructions of networks, for each queue of which we specify the contention-resolution protocol to be used.

- Finally, we examine the impact on network stability of dynamically changing network link capacities presenting bounds for the instability bounds of all the *directed subgraphs* that are *forbidden* for stability. Through improved combinatorial constructions of networks and executions we present adversarial constructions that improve the state-of-the-art instability bound induced by certain known forbidden subgraphs on networks running a certain greedy protocol. More specifically we improve the instability bound of the six simple subgraphs in Figure 5 that have been proved in [2] to be forbidden subgraphs for the universal stability of networks.

1.3 Related Work

Adversarial Queueing Model. Adversarial Queueing Theory and corresponding stability and instability issues, received a lot of interest and attention (see, e.g., [1, 2, 7, 9, 11, 13]). The universal stability of SIS, LIS, NTS and FTG protocols was established by Andrews *et al.* [1]. Also, several greedy protocols such as Nearest-To-Go (NTG) have been proved unstable at arbitrary small rates of injection in [13].

Stability in Heterogeneous Networks. The subfield of study of the stability properties of compositions of universally stable protocols has been opened recently by Koukopoulos *et al.* [9, 11, 10] where lower bounds of 0.683, 0.519 and 0.5 on the injection rate that guarantee instability for the composition pairs LIS-SIS, LIS-NTS and LIS-FTG were presented.

Instability of Forbidden Subgraphs. In [2, Theorems 1, 2], a characterization for directed networks universal stability is given when the packets follow simple paths (paths do not contain repeated edges) that are pure (simple paths do not contain repeated vertices) and simple paths that are not pure (simple paths contain repeated vertices).[2] According to this characterization, a directed network graph is not pure simple path universally stable if and only if it does not contain as subgraphs any of the extensions of the subgraphs $\mathcal{U}_1$ or $\mathcal{U}_2$ [2, Theorem 1]; it is pure simple path universally stable if and only if it does not contain as subgraphs any of the extensions of the subgraphs $\mathcal{S}_1$ or $\mathcal{S}_2$ or $\mathcal{S}_3$ or $\mathcal{S}_4$ [2, Theorem 1] (see Figure 5). For purpose of completeness and comparison, we summarize, in Table 2, all results that are shown in this work

[2] Corresponding characterization for the stability of undirected networks was shown in [1, Theorem 3.16].

	Apply to:	Instability (AQM)	Instability (AQSQM)
$\mathcal{S}_1$	pure s.p.	$\rho > 0.87055$ [2, Lemma 8]	$\rho > 0.8191$ [Thm. 5.1]
$\mathcal{S}_2$	pure s.p.	$\rho > 0.84089$ [2, Lemma 9]	$\rho > 0.8191$ [Thm. 5.2)
$\mathcal{S}_3$	pure s.p.	$\rho > 0.84089$ [2, Lemma 10]	$\rho > 0.8191$ [Thm. 5.3]
$\mathcal{S}_4$	pure s.p.	$\rho > 0.84089$ [2, Lemma 11]	$\rho > 0.8191$ [Thm. 5.4]
$\mathcal{U}_1$	not pure s.p.	$\rho > 0.84089$ [2, Lemma 7]	$\rho > 0.794$ [Thm. 5.5]
$\mathcal{U}_2$	not pure s.p.	$\rho > 0.84089$ [2, Lemma 7]	$\rho > 0.755$ [Thm. 5.6]

Table 2: Instability bounds of forbidden subgraphs in AQM vs. AQSQM. We denote **AQM** the Adversarial Queueing Theory Model, **AQSQM** the Adversarial Quasi-Static Queueing Theory Model and *s.p.* the simple path.

and in [2] that provide instability bounds on the injection rate for the forbidden subgraphs ($\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$, $\mathcal{S}_4, \mathcal{U}_1, \mathcal{U}_2$).

Stability Issues in Dynamic Networks. Borodin *et al.* in [5] studied for the first time the impact on stability when the edges in a network can have capacities and slowdowns. They proved that the universal stability of networks is preserved under this varying context. Also, it was shown that many well-known universally stable protocols (SIS, NTS, FTG) do maintain their universal stability when either the link capacity or slowdown is changing dynamically, whereas the universal stability of LIS is not preserved. More specifically Borodin *et al.* in [5, Theorem 1] presented for the first time an instability bound of $\rho > \frac{C}{2C-1} > 0.5$ for LIS protocol.

1.4 Road Map

The rest of this paper is organized as follows. Section 2 presents model definitions. Section 3 presents our instability bound for LIS. Section 4 demonstrates instability bounds for protocol compositions. Section 5 shows instability bounds for forbidden subgraphs. We conclude, in Section 6, with a discussion of our results and some open problems.

2 Definitions and Preliminaries

The model definitions are patterned after those in [4, Section 3], adjusted to reflect the fact that edge capacities may vary arbitrarily as in [5, Section 2], but we address the weakest possible model of changing capacities. We consider that a communication network is modelled by a directed graph $\mathcal{G} = (V, E)$. Each node $u \in V$ represents a communication switch, and each edge $e \in E$ represents a link between two switches. In each node, there is a buffer (queue) associated with each outgoing link.

Time proceeds in discrete time steps. Buffers store packets that are injected into the network with a route, which is a simple directed path in $\mathcal{G}$. A *packet* is an atomic entity that resides at a buffer at the end of any step. It must travel along paths in the network from its *source* to its *destination,* both of which are nodes in the network. When a packet is injected, it is placed in the buffer of the first link on its route. When a packet reaches its destination, we say that it is *absorbed*. During each step, a packet may be sent from its current node along one of the outgoing edges from that node. Edges can have different integer capacities, which may or may not vary over time. Denote $\mathsf{C}_e(t)$ the *capacity* of edge e at time step t. That is, we assume that edge e is capable of simultaneously transmitting up to $\mathsf{C}_e(t)$ packets at time t.

Let $C > 1$ be an integer parameter. We demand that for all e and for all t $C_e(t) \in \{1, C\}$ (i.e. each edge capacity can get only two values, high and low). We also demand for each edge e that $C_e(t)$ stays at some value for a continuous period of time at least equal to $f(\rho, C)s$ time steps, where s is the number of packets in the system at the time when the capacity was last set and $f(\rho, C)$ is a function of the injection rate ρ of the adversary in the network and the high link capacity C. We call this the *Adversarial, Quasi-Static Queueing Theory Model*. This model is the weakest possible of models implied by [5].

Any packets that wish to travel along an edge e at a particular time step but are not sent wait in a queue for edge e. The *delay* of a packet is the number of steps spent by the packet while waiting in queues. At each step, an em adversary generates a set of requests. A *request* is a *path* specifying the route followed by a packet.[3] We say that the adversary generates a set of packets when it generates a set of requested paths. Also, we say that a packet p *requires* an edge e at time t if the edge e lies on the path from its position to its destination at time t. We restrict our study to the case of *non-adaptive* routing, where the path traversed by each packet is fixed at the time of injection, so that we are able to focus on queueing rather than routing aspects of the problem. (See [3] for an extension of the adversarial model to the case of *adaptive* routing.) There are no computational restrictions on how the adversary chooses its requests in any given time step.

Fix any arbitrary positive integer $w \geq 1$. For any edge e of the network and any sequence

[3]In this work, it is assumed, as it is common in packet routing, that all such paths are simple paths with no overlapping edges. However, in Section 5 we consider two different kinds of simple paths: simple paths that do not contain repeated vertices (pure simple paths) and simple paths that contain repeated vertices (not pure simple paths).

of w consecutive time steps, define $N(w, e)$ to be the number of paths injected by the adversary during the time interval of w consecutive time steps that traverse edge e. For any constant ρ, $0 < \rho \leq 1$, a (w, ρ)-*adversary* is an adversary that injects packets subject to the following *load condition*: For every edge e and for every sequence τ of w consecutive time steps, $N(\tau, e) \leq \rho \sum_{t \in \tau} \mathsf{C_e}(t)$. We say that a (w, ρ)-adversary injects packets at rate ρ with *window size* w. The assumption that $\rho \leq 1$ ensures that it is not necessary a priori that some edge of the network is congested (that happens when $\rho > 1$).

In order to formalize the behavior of a network under the adversarial, quasi-static queueing theory model, we use the notions of *system* and *system configuration*. A triple of the form $\langle \mathcal{G}, \mathcal{A}, \mathsf{P} \rangle$ where $\mathcal{G}$ is a network, $\mathcal{A}$ is an adversary and P is the used protocol on the network queues is called a system. The execution of the system proceeds in global time steps numbered $0, 1, \ldots$. Each time-step is divided in two sub-steps. In the first sub-step, one packet is sent from each non-empty buffer over its corresponding link. In the second sub-step, packets are received by the nodes at the other end of the links; they are absorbed (eliminated) if that node is their destination, and otherwise they are placed in the buffer of the next link on their respective routes. New packets are injected in the second sub-step.

In every time step t, the current configuration C^t of a system $\langle \mathcal{G}, \mathcal{A}, \mathsf{P} \rangle$ is a collection of sets $\{S_e^t : e \epsilon \mathcal{G}\}$, such that S_e^t is the set of packets waiting in the queue of the edge e at the end of step t. If the current system configuration is C^t, we obtain the system configuration C^{t+1} for the next time step as follows: (i) Addition of new packets to some of the sets S_e^t, each of which has an assigned path in $\mathcal{G}$, and (ii) for each non-empty set S_e^t deletion of a single packet $p \epsilon S_e^t$ and its insertion into the set S_f^{t+1} where f is the edge following e on its assigned path (if e is the last edge on the path of p, then p is not inserted into any set.) A time evolution of the system for a $((w, \rho))$-adversary is a sequence of such configurations $C^1, C^2, \ldots$.

A *contention-resolution* protocol specifies, for each pair of an edge e and a time step, which packet among those waiting at the tail of edge e will be moved along edge e. A *greedy* contention-resolution protocol always specifies some packet to move along edge e if there are packets waiting to use edge e. In this work, we restrict attention to deterministic, greedy contention-resolution protocols. In particular, we consider:

- SIS (*Shortest-in-System*) gives priority to the most recently injected packet into the network;
- LIS (*Longest-in-System*) gives priority to the least recently injected packet into the network;

- FTG (*Furthest-to-Go*) gives priority to the packet that has to traverse the larger number of edges to its destination;

- NTS (*Nearest-to-Source*) gives priority to the packet that has traversed the smallest number of edges from its origin;

- NTG-U-LIS (*Nearest-To-Go-Using-LIS*) gives priority to the nearest packet to its destination or the least recently injected packet for tie-breaking.

All these contention-resolution protocols require some tie-breaking rule in order to be unambiguously defined. In this work, whenever we are proving a positive result, we assume that the adversary can break the tie arbitrarily; for proving a negative result, we can assume any well-determined tie breaking rule for the adversary.

In the adversarial constructions we study here for proving instability, we split time into *phases*. In each phase, we study the evolution of the *system configuration* by considering corresponding *time rounds*. For each phase, we inductively prove that the number of packets of a specific subset of queues in the system increases in order to guarantee instability. This inductive argument can be applied repeatedly, thus showing instability.

Furthermore, we assume that there is a sufficiently large number of packets s_0 in the initial system configuration. This will imply instability results for networks with an *empty* initial configuration, as established by Andrews *et al.* [1, Lemma 2.9]. For simplicity, and in a way similar to that in [1] and in works following it, we omit floors and ceilings from our analysis, and we sometimes count time steps and packets only roughly. This may only result to loosing small additive constants, while it implies a gain in clarity.

3 Instability Bound for LIS

In this section we present a lower bound on injection rate that guarantees instability for an LIS network. Consider the network $\mathcal{N}$ (see Figure 1). We show:

Theorem 3.1 *Let $\rho' = \sqrt{2} - 1 + \epsilon$ with $\epsilon > 0$ and $C = C(\rho') > 1$. There is a network $\mathcal{N}$ and an adversary $\mathcal{A}$ of rate ρ that can change the link capacities of $\mathcal{N}$ between the two integer values 1 and $C(\rho')$ such that the system $\langle \mathcal{N}, \mathcal{A}, \mathsf{LIS} \rangle$ is unstable for every $\rho > \rho'$. When $C(\rho') \to \infty$ the system $\langle \mathcal{N}, \mathcal{A}, \mathsf{LIS} \rangle$ is unstable for $\rho > \sqrt{2} - 1$.*

Proof: The construction of the adversary $\mathcal{A}$ is broken into phases.

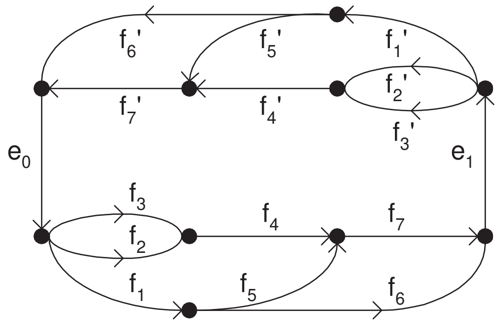

Figure 1: The network $\mathcal{N}$.

Inductive Hypothesis: At the beginning of phase j (suppose j is even), there are s_j packets that are queued in the queues $f_1^{'}, f_4^{'}, f_5^{'}, f_7^{'}$ (in total) requiring to traverse the edges e_0, f_2, f_4.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets that are queued in the queues f_1, f_4, f_5, f_7 (in total) requiring to traverse the edges $e_1, f_2^{'}, f_4^{'}$.

We will construct an adversary $\mathcal{A}$ such that the induction step holds. The main ideas of the construction of $\mathcal{A}$ are (a) the careful tuning of the duration of each round of every phase j (as a function of the high capacity C, the injection rate ρ and the number of packets in the system at the beginning of phase j, s_j) to maximize the growth of the packet population in the system and, (b) the careful setting of the capacities of some edges to 1 for specified time intervals in order to accumulate packets. Proving that the induction step holds, we ensure that the inductive hypothesis will hold at the beginning of phase $j+1$ for the symmetric edges with an increased value of s_j, $s_{j+1} > s_j$.

From the inductive hypothesis, initially, there are s_j packets (called $S-flow$) in the queues $f_1^{'}, f_4^{'}, f_5^{'},\ f_7^{'}$ requiring to traverse the edges e_0, f_2, f_4. In order to prove the induction step, it is assumed that there is a set S with a large enough number of $|S| = s_j$ packets in the initial system configuration.

During phase j the adversary plays three rounds of injections as follows:

- **Round 1:** This round lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round the edges $f_1^{'}, f_2^{'}, f_4^{'},\ f_5^{'},\ f_7^{'}, e_0, f_1,\ f_5, f_7, e_1$ have high capacity C, while all the other edges have unit capacity. The adversary injects a set X of $|X| = \rho C|T_1|$ packets in queue e_0 wanting to traverse the edges $e_0, f_1,\ f_5, f_7, e_1, f_2^{'}, f_4^{'}$ and a set S_1 of $|S_1| = \rho|T_1|$ packets in queue f_2 wanting to traverse the edges f_2, f_4.

Evolution of the system configuration. S packets delay X packets in e_0 and S_1 packets in f_2 because S packets are longer time in the system than X and S_1 packets. At the same time, S packets are delayed in f_2 due to the unit capacity of the edge f_2. The remaining S packets in f_2 at the end of this round are $|S^{'}| = |S| - |T_1|$ packets. The packets of set S that manage to traverse the edge f_2 continue traversing their remaining path and they are absorbed. Therefore, the number of packets in queue f_2 at the end of this round requiring to traverse the edges f_2, f_4 is a set S_2 of $|S_2| = |S^{'}| + |S_1|$ packets.

- **Round 2:** It lasts $|T_2| = \frac{|S_2|}{C}$ time steps.

  Adversary's behavior. During this round the edges $f_2, f_4, f_7,\ e_0, e_1, f_2^{'},\ f_4^{'}$ have high capacity C, while all the other edges have unit capacity. The adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue f_4 requiring to traverse the edges $f_4, f_7, e_1, f_2^{'},\ f_4^{'}$.

  Evolution of the system configuration. The packets of set Y are delayed by the packets of the set S_2 in queue f_4 because S_2 packets are longer time in the system than Y packets. S_2 packets traverse the edge f_4 and they are absorbed. At the same time, X packets are delayed in queue f_1 due to its unit capacity. Therefore, the remaining packets of X in queue f_1 is a set $|X^{'}|$ of $|X^{'}| = |X| - |T_2|$ packets.

- **Round 3:** It lasts $|T_3| = \frac{|X^{'}|}{C}$ time steps.

  Adversary's behavior. During this round the edges $f_1, f_6, e_1,\ f_2^{'}, f_4^{'}$ have high capacity C, while all the other edges have unit capacity. The adversary injects a set Z of $|Z| = \rho C|T_3|$ packets in queue f_1 requiring to traverse the edges $f_1, f_6, e_1, f_2^{'}, f_4^{'}$.

  Evolution of the system configuration. $X^{'}$ packets delay Z packets in queue f_1 because they are longer time in the system. At the same time $X^{'}$ packets are delayed in f_5 due to the unit capacity of the edge f_5. Therefore, the remaining packets of $X^{'}$ in queue f_5 is a set $|X^{''}|$ of $|X^{''}| = |X^{'}| - |T_3|$ packets. Moreover, Y packets are delayed in f_4 due to the unit capacity of the edge f_4 during this round. Therefore, the remaining packets of Y in queue f_4 is a set $|Y^{'}|$ of $|Y^{'}| = |Y| - |T_3|$ packets.

  Note that during this round $|K| = 2|T_3|$ packets arrive in queue f_7 from queues f_4, f_5. However, the edge f_7 has unit capacity and the duration of this round is $|T_3|$ time steps. Consequently, $|L| = |T_3|$ packets will remain in queue f_7 at the end of this round requiring to traverse the edges $f_7, e_1, f_2^{'}, f_4^{'}$. Therefore, the number of packets in queues f_1, f_4, f_5, f_7 requiring to traverse the edges $e_1, f_2^{'}, f_4^{'}$ at the end of this round is $s_{j+1} = |X^{''}| + |Y^{'}| + |Z| + |L| = \rho s_j - \frac{1+\rho}{C} s_j + \frac{2-\rho}{C^2} s_j + \frac{\rho-1}{C^3} s_j + \rho s_j + \frac{\rho^2-2\rho}{C} s_j + \frac{1}{C^2} s_j + \frac{\rho-1}{C^3} s_j + \rho^2 s_j - \frac{\rho}{C} s_j + \frac{\rho-\rho^2}{C^2} s_j + \frac{\rho}{C} s_j - \frac{1}{C^2} s_j + \frac{1-\rho^2}{C^3} s_j$.

In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^2[1 + \frac{1}{C} - \frac{1}{C^2}] + \rho[2 - \frac{3}{C} + \frac{1}{C^3}] + [-\frac{1}{C} + \frac{2}{C^2} - \frac{1}{C^3}] > 1$. Initially, note that the following inequalities hold:

– $1 + \frac{1}{C} - \frac{1}{C^2} < 1 + \frac{1}{C}$,

– $2 - \frac{3}{C} + \frac{1}{C^3} < 2$

– $1 + \frac{1}{C} - \frac{2}{C^2} + \frac{1}{C^3} > 1 - \frac{2}{C^2}$.

Therefore, the inequality becomes $\rho^2[1 + \frac{1}{C}] + 2\rho > 1 - \frac{2}{C^2}$. Thus, it suffices to be shown that $\rho^2[C^2 + C] + 2\rho C^2 > C^2 - 2$. This inequality holds for ρ larger than the largest root ρ_1 of the polynomial $\rho^2[C^2 + C] + 2\rho C^2 - (C^2 - 2)$. The largest root of the polynomial is $\rho_1 = \frac{-2C^2 + \sqrt{4C^4 + 4(C^2-2)(C^2+C)}}{2C(C+1)} = \frac{\sqrt{2C^2+C-2-\frac{2}{C}}}{C+1} - \frac{C}{C+1}$. But, $\frac{\sqrt{2C^2+C-2-\frac{2}{C}}}{C+1} < \sqrt{\frac{2C}{C+1}}$. Therefore, $\rho_1 < \sqrt{\frac{2C}{C+1}} - \frac{C}{C+1}$. Thus, it holds that $\rho > \sqrt{\frac{2C}{C+2}} - \frac{C}{C+2} > \rho_1$.

When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, the inequality $\rho^2[1 + \frac{1}{C} - \frac{1}{C^2}] + \rho[2 - \frac{3}{C} + \frac{1}{C^3}] + [-\frac{1}{C} + \frac{2}{C^2} - \frac{1}{C^3}] > 1$ becomes $\rho^2 + 2\rho - 1 > 0$ which holds for $\rho > \sqrt{2} - 1$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > \sqrt{2} - 1$ the system is unstable. If we replace ρ' with $\sqrt{2} - 1 + \epsilon$ at the inequality $\rho' > \sqrt{\frac{2C}{C+1}} - \frac{C}{C+1}$, we estimate a $C(\rho') > 1$ such that $s_{j+1} > s_j$. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > \sqrt{2} - 1$. ■

4 Instability Bounds for Protocol Compositions

In this section we demonstrate instability bounds for protocol compositions. First, we show an instability bound for the composition of protocols LIS and SIS on network $\mathcal{G}_1$ (Figure 2). The edges $e_0, e_1, f_1, f_1', f_3, f_3'$ of $\mathcal{G}_1$ use LIS protocol, while the rest edges of $\mathcal{G}_1$ use SIS protocol.

Theorem 4.1 *Let $\rho' = \sqrt{2} - 1 + \epsilon$ with $\epsilon > 0$ and $C = C(\rho') > 1$. There is a network $\mathcal{G}_1$ and an adversary $\mathcal{A}_1$ of rate ρ that can change the link capacities of $\mathcal{G}_1$ between the two integer values 1 and $C(\rho')$ such that the system $\langle \mathcal{G}_1, \mathcal{A}_1, \mathsf{LIS}, \mathsf{SIS} \rangle$ is unstable for every $\rho > \rho'$. When $C(\rho') \to \infty$ the system $\langle \mathcal{G}_1, \mathcal{A}_1, \mathsf{LIS}, \mathsf{SIS} \rangle$ is unstable for $\rho > \sqrt{2} - 1$.*

Proof: The construction of the adversary $\mathcal{A}_1$ is broken into phases.

Inductive Hypothesis: At the beginning of phase j (suppose j is even), there are s_j packets that are queued in the queues f_1', f_4', f_5', f_6' (in total) requiring to traverse the edges e_0, f_2, f_3, f_4.

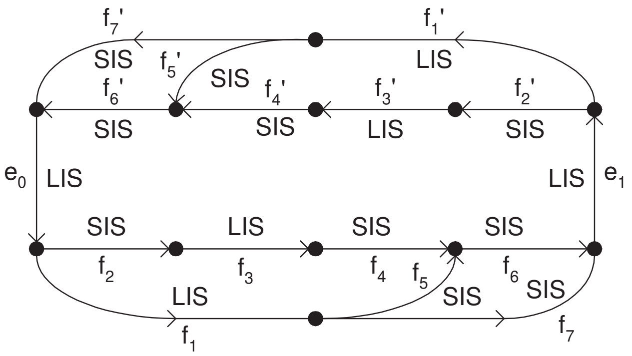

Figure 2: The network $\mathcal{G}_1$.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets that are queued in the queues f_1, f_4, f_5, f_6 (in total) requiring to traverse the edges $e_1, f_2^{'}, f_3^{'}, f_4^{'}$.

We will construct an adversary $\mathcal{A}_1$ such that the induction step holds. Proving that the induction step holds, we ensure that the inductive hypothesis will hold at the beginning of phase $j+1$ for the symmetric edges with an increased value of s_j, $s_{j+1} > s_j$. From the inductive hypothesis, initially, there are s_j packets (called $S-flow$) in the queues $f_1^{'}, f_4^{'}, f_5^{'}, f_6^{'}$ requiring to traverse the edges e_0, f_2, f_3, f_4. In order to prove the induction step, it is assumed that there is a set S with a large enough number of $|S| = s_j$ packets in the initial system configuration.

During phase j, the adversary plays three rounds of injections as follows:

- **Round 1:** This round lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round the edge f_2 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set X of $|X| = \rho C|T_1|$ packets in queue e_0 wanting to traverse the edges $e_0, f_1,\ f_5, f_6, e_1, f_2^{'}, f_3^{'}, f_4^{'}$ and a set S_1 of $|S_1| = \rho|T_1|$ packets in queue f_2 wanting to traverse the edge f_2.

  Evolution of the system configuration. S packets delay X packets in queue e_0 that uses LIS protocol because S packets are longer time in the system. At the same time, S packets are delayed in queue f_2 that uses SIS protocol due to S_1 packets which are shorter time in the system and the unit capacity of the edge f_2. The remaining packets of the set S in f_2 at the end of this round is a set S_2 of $|S_2| = |S| - (|T_1| - |S_1|)$ packets. The S packets that manage to traverse the edge f_2 traverse their remaining path and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|S_2|}{C}$ time steps.

Adversary's behavior. During this round the edge f_1 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue f_3 requiring to traverse the edges $f_3, f_4, f_6, e_1, f_2^{'}, f_3^{'}, f_4^{'}$.

Evolution of the system configuration. S_2 delay Y packets in queue f_3 that uses LIS protocol because S_2 packets are longer time in the system. S_2 packets traverse the edge f_3 and they are absorbed. At the same time, the packet set X is delayed in queue f_1 due to the unit capacity of the edge f_1. Therefore, the remaining packets of X in queue f_1 is a set $|X^{'}|$ of $|X^{'}| = |X| - |T_2|$ packets.

- **Round 3:** It lasts $|T_3| = \frac{|X^{'}|}{C}$ time steps.

  Adversary's behavior. During this round the edges f_4, f_5, f_6 have unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Z of $|Z| = \rho C|T_3|$ packets in queue f_1 requiring to traverse the edges $f_1, f_7, e_1, f_2^{'}, f_3^{'}, f_4^{'}$. Also, it injects a set S_3 of $|S_3| = \rho|T_3|$ packets in queue f_4 wanting to traverse the edge f_4, a set S_4 of $|S_4| = \rho|T_3|$ packets in queue f_5 wanting to traverse the edge f_5 and a set S_5 of $|S_5| = \rho|T_3|$ packets in queue f_6 wanting to traverse the edge f_6.

  Evolution of the system configuration. The $X^{'}$ packets delay the Z packets in queue f_1 that uses LIS protocol because they are longer time in the system. At the same time $X^{'}$ packets are delayed in queue f_5 that uses SIS protocol due to the unit capacity of the edge f_5 during this round and the S_4 packets that are shorter time in the system than $X^{'}$ packets. Therefore, the remaining packets of $X^{'}$ in queue f_5 is a set $|X^{''}|$ of $|X^{''}| = |X^{'}| - (|T_3| - |S_4|)|$. Moreover Y packets are delayed in queue f_4 that uses SIS protocol due to the unit capacity of the edge f_4 during this round and the S_3 packets that are shorter time in the system than Y packets. Therefore, the remaining packets of Y in queue f_4 is a set $|Y^{'}|$ of $|Y^{'}| = |Y| - (|T_3| - |S_3|)$ packets.

  Note that during this round $|K| = 2|T_3| - |S_3| - |S_4|$ packets arrive in queue f_6 from queues f_4, f_5. However, the edge f_6 has unit capacity and uses SIS protocol that gives priority to the S_5 packets. Furthermore, the duration of this round is $|T_3|$ time steps. Consequently, the number of packets that remain in queue f_6 at the end of this round requiring to traverse the edges $f_6, e_1, f_2^{'}, f_3^{'}, f_4^{'}$ is $|L| = |K| + |S_5| - |T_3|$. Therefore, the number of packets in queues f_1, f_4, f_5, f_6 requiring to traverse the edges $e_1, f_2^{'}, f_3^{'}, f_4^{'}$ at the end of this round is $s_{j+1} = |X^{''}| + |Y^{'}| + |Z| + |L| = \rho s_j + \frac{\rho^2-\rho-1}{C} s_j + 2\frac{1-\rho}{C^2} s_j + \frac{-\rho^2+2\rho-1}{C^3} s_j + \rho s_j + \frac{2\rho^2-2\rho}{C} s_j + \frac{1-\rho}{C^2} s_j + \frac{-\rho^2+2\rho-1}{C^3} s_j + \rho^2 s_j - \frac{\rho}{C} s_j + \frac{\rho-\rho^2}{C^2} s_j + \frac{\rho-\rho^2}{C} s_j + 2\frac{\rho-1}{C^2} s_j + \frac{\rho^2-2\rho+1}{C^3} s_j$.

  In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^2[1 + \frac{2}{C} - \frac{1}{C^2} - \frac{1}{C^3}] + \rho[2 - \frac{3}{C} - \frac{1}{C^2} + \frac{2}{C^3}] + [-\frac{1}{C} + \frac{2}{C^2} - \frac{1}{C^3}] > 1$. Initially, note that the following inequalities hold:

$-\ 1+\frac{2}{C}-\frac{1}{C^2}-\frac{1}{C^3}<1+\frac{2}{C}$,

$-\ 2-\frac{3}{C}-\frac{1}{C^2}+\frac{2}{C^3}<2$

$-\ 1+\frac{1}{C}-\frac{2}{C^2}+\frac{1}{C^3}>1-\frac{2}{C^2}$.

Therefore, the inequality becomes $\rho^2[1+\frac{2}{C}]+2\rho>1-\frac{2}{C^2}$. Thus, it suffices to be shown that $\rho^2[C^2+2C]+2\rho C^2>C^2-2$. This inequality holds for ρ larger than the largest root ρ_1 of the polynomial $\rho^2[C^2+2C]+2\rho C^2-(C^2-2)$. The largest root of the polynomial is $\rho_1=\frac{-2C^2+\sqrt{4C^4+4(C^2-2)(C^2+2C)}}{2C(C+2)}=\frac{\sqrt{2C^2+2C-2-\frac{4}{C}}}{C+2}-\frac{C}{C+2}$. But, $\frac{\sqrt{2C^2+2C-2-\frac{4}{C}}}{C+2}<\sqrt{\frac{2C}{C+2}}$. Therefore, $\rho_1<\sqrt{\frac{2C}{C+2}}-\frac{C}{C+2}$. Thus, it holds that $\rho>\sqrt{\frac{2C}{C+2}}-\frac{C}{C+2}>\rho_1$.

When $C\to\infty$, it holds that $\frac{1}{C^k}\to 0$ for all $k\geq 1$. Then, the inequality $\rho^2[1+\frac{2}{C}-\frac{1}{C^2}-\frac{1}{C^3}]+\rho[2-\frac{3}{C}-\frac{1}{C^2}+\frac{2}{C^3}]+[-\frac{1}{C}+\frac{2}{C^2}-\frac{1}{C^3}]>1$ becomes $\rho^2+2\rho-1>0$ which holds for $\rho>\sqrt{2}-1$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho)=f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho)=0$, then $\lim_{C\to\infty}\rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho>\sqrt{2}-1$ the system is unstable. If we replace ρ' with $\sqrt{2}-1+\epsilon$ at the inequality $\rho'>\sqrt{\frac{2C}{C+2}}-\frac{C}{C+2}$, we estimate a $C(\rho')>1$ such that $s_{j+1}>s_j$. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho>\sqrt{2}-1$. ■

Then, we show an instability bound for the composition of protocols LIS and NTS on network $\mathcal{G}_2$ (Figure 3). Network $\mathcal{G}_2$ is the same with network $\mathcal{G}_1$ that is used for proving the instability of the composition of protocols LIS and SIS (Theorem 4.1). But, the edges $f_2, f_2^{'}, f_4, f_4^{'}, f_5, f_5^{'}, f_6, f_6^{'},\ f_7, f_7^{'}$ of $\mathcal{G}_2$ use NTS protocol, while the rest edges of $\mathcal{G}_2$ use LIS protocol.

Theorem 4.2 *Let $\rho'=\sqrt{2}-1+\epsilon$ with $\epsilon>0$ and $C=C(\rho')>1$. There is a network $\mathcal{G}_2$ and an adversary $\mathcal{A}_2$ of rate ρ that can change the link capacities of $\mathcal{G}_2$ between the two integer values 1 and $C(\rho')$ such that the system $\langle\mathcal{G}_2,\mathcal{A}_2,\mathsf{LIS},\mathsf{NTS}\rangle$ is unstable for every $\rho>\rho'$. When $C(\rho')\to\infty$ the system $\langle\mathcal{G}_2,\mathcal{A}_2,\mathsf{LIS},\mathsf{NTS}\rangle$ is unstable for $\rho>\sqrt{2}-1$.*

Proof: We break the construction of the adversary $\mathcal{A}_2$ into phases.

Inductive Hypothesis: At the beginning of phase j (suppose j is even), there are s_j packets that are queued in the queues $f_1^{'}, f_4^{'}, f_5^{'}, f_6^{'}$ (in total) requiring to traverse the edges e_0, f_2, f_3, f_4.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1}>s_j$ packets that are queued in the queues f_1, f_4, f_5, f_6 (in total) requiring to traverse the edges $e_1, f_2^{'}, f_3^{'}, f_4^{'}$.

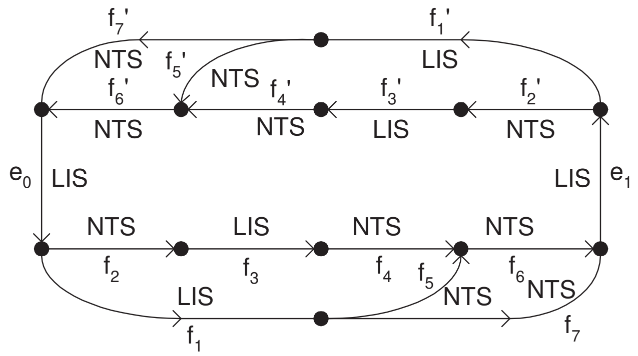

Figure 3: The network $\mathcal{G}_2$.

We will construct an adversary $\mathcal{A}$ such that the induction step will hold. Proving that the induction step holds, we ensure that the inductive hypothesis will hold at the beginning of phase $j+1$ for the symmetric edges with an increased value of s_j, $s_{j+1} > s_j$. From the inductive hypothesis, initially, there are s_j packets (called $S - flow$) in the queues $f_1^{'}, f_4^{'}, f_5^{'}, f_6^{'}$ requiring to traverse the edges e_0, f_2, f_3, f_4. In order to prove the induction step, it is assumed that there is a set S with a large enough number of $|S| = s_j$ packets in the initial system configuration.

During phase j, the adversary plays three rounds of injections. The sequence of injections is as follows:

- **Round 1:** This round lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round the edge f_2 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set X of $|X| = \rho C|T_1|$ packets in queue e_0 wanting to traverse the edges $e_0, f_1, f_5, f_6, e_1, f_2^{'}, f_3^{'}, f_4^{'}$. Also, the adversary injects a set S_1 of $|S_1| = \rho|T_1|$ packets in queue f_2 wanting to traverse the edge f_2.

  Evolution of the system configuration. S packets delay X packets in queue e_0 that uses LIS protocol because S packets are longer time in the system. At the same time, S packets are delayed in queue f_2 that uses NTS protocol due to S_1 packets which are nearest to their source (queue f_2) than S packets (queue f_4) and the unit capacity of the edge f_2. The remaining packets of the set S in f_2 at the end of this round are $|S_2| = |S| - (|T_1| - |S_1|)$. The rest packets of the set S that manage to traverse the edge f_2 traverse their remaining path and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|S_2|}{C}$ time steps.

  Adversary's behavior. During this round the edge f_1 has unit capacity, while all the other edges have capacity C. Also, the adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue f_3 requiring to traverse the edges $f_3, f_4, f_6, e_1, f_2^{'}, f_3^{'}, f_4^{'}$.

Evolution of the system configuration. S_2 packets delay Y packets in queue f_3 that uses LIS protocol because S_2 packets are longer time in the system. S_2 packets traverse the edge f_3 and they are absorbed. At the same time, the packet set X is delayed in queue f_1 due to the unit capacity of the edge f_1. Therefore, the remaining packets of X in queue f_1 is a set $|X^{'}|$ of $|X^{'}| = |X| - |T_2|$ packets.

- **Round 3:** It lasts $|T_3| = \frac{|X^{'}|}{C}$ time steps.

  Adversary's behavior. During this round the edges f_4, f_5, f_6 have unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Z of $|Z| = \rho C|T_3|$ packets in queue f_1 requiring to traverse the edges $f_1, f_7, e_1, f_2^{'}, f_3^{'}, f_4^{'}$. Furthermore, the adversary injects a set S_3 of $|S_3| = \rho|T_3|$ packets in queue f_4 wanting to traverse the edge f_4, a set S_4 of $|S_4| = \rho|T_3|$ packets in queue f_5 wanting to traverse the edge f_5 and a set S_5 of $|S_5| = \rho|T_3|$ packets in queue f_6 wanting to traverse the edge f_6.

  Evolution of the system configuration. $X^{'}$ packets delay Z packets in queue f_1 that uses LIS protocol because they are longer time in the system. At the same time $X^{'}$ packets are delayed in queue f_5 that uses NTS protocol due to the unit capacity of the edge f_5 during this round and the S_4 packets that are nearest to their source (queue f_5) than $X^{'}$ packets (queue $f_4^{'}$). Therefore, the remaining packets of $X^{'}$ in queue f_5 is a set $|X^{''}|$ of $|X^{''}| = |X^{'}| - (|T_3| - |S_4|)$ packets. Moreover Y packets are delayed in queue f_4 that uses NTS protocol due to the unit capacity of the edge f_4 during this round and the S_3 packets that are nearest to their source (queue f_4) than Y packets (queue $f_4^{'}$). Therefore, the remaining packets of Y in queue f_4 is a set $|Y^{'}|$ of $|Y^{'}| = |Y| - (|T_3| - S_3)$ packets.

  Note that during this round $|K| = 2|T_3| - |S_3| - |S_4|$ packets arrive in queue f_6 from queues f_4, f_5. However, the edge f_6 has unit capacity and uses NTS protocol that gives priority to the S_5 packets. Furthermore, the duration of this round is $|T_3|$ time steps. Consequently, the number of packets that remain in queue f_6 at the end of this round requiring to traverse the edges $f_6, e_1, f_2^{'}, f_3^{'}, f_4^{'}$ is $|L| = |K| + |S_5| - |T_3|$. Therefore, the number of packets in queues f_1, f_4, f_5, f_6 requiring to traverse the edges $e_1, f_2^{'}, f_3^{'}, f_4^{'}$ at the end of this round is $s_{j+1} = |X^{''}| + |Y^{'}| + |Z| + |L| = \rho s_j + \frac{\rho^2-\rho-1}{C} s_j + 2\frac{1-\rho}{C^2} s_j + \frac{-\rho^2+2\rho-1}{C^3} s_j + \rho s_j + \frac{2\rho^2-2\rho}{C} s_j + \frac{1-\rho}{C^2} s_j + \frac{-\rho^2+2\rho-1}{C^3} s_j + \rho^2 s_j - \frac{\rho}{C} s_j + \frac{\rho-\rho^2}{C^2} s_j + \frac{\rho-\rho^2}{C} s_j + 2\frac{\rho-1}{C^2} s_j + \frac{\rho^2-2\rho+1}{C^3} s_j$. In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^2[1 + \frac{2}{C} - \frac{1}{C^2} - \frac{1}{C^3}] + \rho[2 - \frac{3}{C} - \frac{1}{C^2} + \frac{2}{C^3}] + [-\frac{1}{C} + \frac{2}{C^2} - \frac{1}{C^3}] > 1$. Initially, note that the following inequalities hold:

  $$-1 + \frac{2}{C} - \frac{1}{C^2} - \frac{1}{C^3} < 1 + \frac{2}{C},$$

– $2 - \frac{3}{C} - \frac{1}{C^2} + \frac{2}{C^3} < 2$,

– $1 + \frac{1}{C} - \frac{2}{C^2} + \frac{1}{C^3} > 1 - \frac{2}{C^2}$.

Therefore, the inequality becomes $\rho^2[1+\frac{2}{C}] + 2\rho > 1 - \frac{2}{C^2}$. Thus, it suffices to be shown that $\rho^2[C^2+2C] + 2\rho C^2 > C^2 - 2$. This inequality holds for ρ larger than the largest root ρ_1 of the polynomial $\rho^2[C^2+2C] + 2\rho C^2 - (C^2-2)$. The largest root of the polynomial is $\rho_1 = \frac{-2C^2+\sqrt{4C^4+4(C^2-2)(C^2+2C)}}{2C(C+2)} = \frac{\sqrt{2C^2+2C-2-\frac{4}{C}}}{C+2} - \frac{C}{C+2}$. But, $\frac{\sqrt{2C^2+2C-2-\frac{4}{C}}}{C+2} < \sqrt{\frac{2C}{C+2}}$. Therefore, $\rho_1 < \sqrt{\frac{2C}{C+2}} - \frac{C}{C+2}$. Thus, it holds that $\rho > \sqrt{\frac{2C}{C+2}} - \frac{C}{C+2} > \rho_1$.

When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, the inequality $\rho^2[1+\frac{2}{C} - \frac{1}{C^2} - \frac{1}{C^3}] + \rho[2 - \frac{3}{C} - \frac{1}{C^2} + \frac{2}{C^3}] + [-\frac{1}{C} + \frac{2}{C^2} - \frac{1}{C^3}] > 1$ becomes $\rho^2 + 2\rho - 1 > 0$ which holds for $\rho > \sqrt{2} - 1$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > \sqrt{2} - 1$ the system is unstable. If we replace ρ' with $\sqrt{2} - 1 + \epsilon$ at the inequality $\rho' > \sqrt{\frac{2C}{C+2}} - \frac{C}{C+2}$, we estimate a $C(\rho') > 1$ such that $s_{j+1} > s_j$. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > \sqrt{2} - 1$. ∎

Then, we show an instability bound for the composition of protocols LIS and FTG on network $\mathcal{G}_3$ (Figure 4). The edges $e_0, e_1, f_1, f_1', f_3, f_3'$ of network $\mathcal{G}_3$ use LIS protocol, while the rest use FTG protocol.

Theorem 4.3 *Let $\rho' = \sqrt{2} - 1 + \epsilon$ with $\epsilon > 0$ and $C = C(\rho') > 1$. There is a network $\mathcal{G}_3$ and an adversary $\mathcal{A}_3$ of rate ρ that can change the link capacities of $\mathcal{G}_3$ between the two integer values 1 and $C(\rho')$ such that the system $\langle \mathcal{G}_3, \mathcal{A}_3, \mathsf{LIS}, \mathsf{FTG} \rangle$ is unstable for every $\rho > \rho'$. When $C(\rho') \to \infty$ the system $\langle \mathcal{G}_3, \mathcal{A}_3, \mathsf{LIS}, \mathsf{FTG} \rangle$ is unstable for $\rho > \sqrt{2} - 1$.*

Proof: We break the construction of the adversary $\mathcal{A}_3$ into phases.

Inductive Hypothesis: At the beginning of phase j (suppose j is even), there are s_j packets that are queued in the queues f_1', f_4', f_5', f_6' (in total) requiring to traverse the edges e_0, f_2, f_3, f_4.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets that are queued in the queues f_1, f_4, f_5, f_6 (in total) requiring to traverse the edges e_1, f_2', f_3', f_4'.

We will construct an adversary $\mathcal{A}_3$ such that the induction step holds. Proving that the induction step holds, we ensure that the inductive hypothesis will hold at the beginning of phase

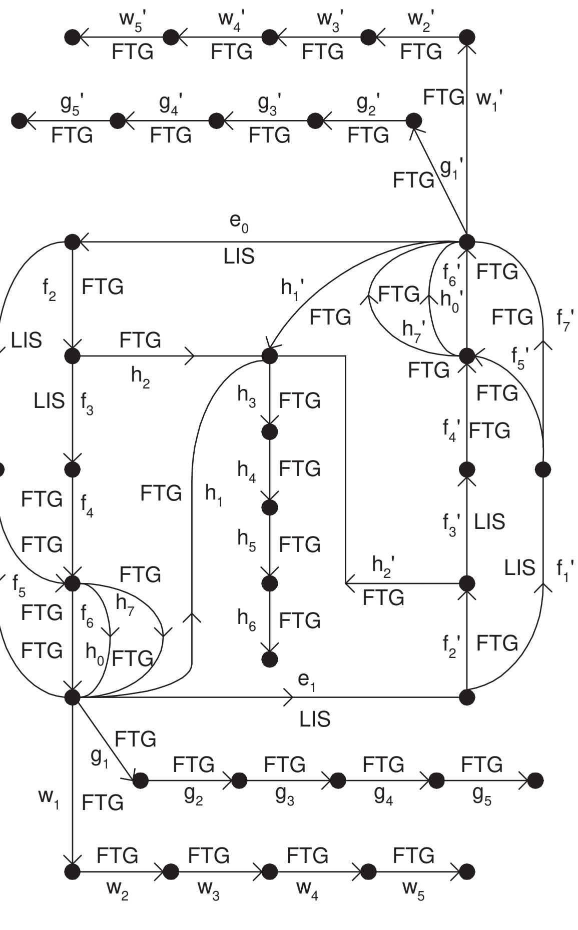

Figure 4: The network $\mathcal{G}_3$.

$j+1$ for the symmetric edges with an increased value of s_j, $s_{j+1} > s_j$. From the inductive hypothesis, initially, there are s_j packets (called $S-flow$) in the queues $f_1^{'}, f_4^{'}, f_5^{'}$, $f_6^{'}$ requiring to traverse the edges e_0, f_2, f_3, f_4. In order to prove the induction step, it is assumed that there is a set S with a large enough number of $|S| = s_j$ packets in the initial system configuration.

During phase j, the adversary plays three rounds of injections. The sequence of injections is as follows:

- **Round 1:** This round lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round the edge f_2 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set X of $|X| = \rho C|T_1|$ packets in queue e_0 wanting to traverse the edges $e_0, f_1,\ f_5, f_6, e_1, f_2^{'}, f_3^{'}, f_4^{'}$. Also, the adversary injects a set S_1 of $|S_1| = \rho|T_1|$ packets in queue f_2 wanting to traverse the edges f_2, h_2, h_3, h_4.

  Evolution of the system configuration. S packets delay X packets in queue e_0 that uses LIS protocol because S packets are longer time in the system. At the same time, S packets are delayed in queue f_2 that uses FTG protocol due to S_1 packets which have furthest to go (queue h_4) than S packets (queue f_4) and the unit capacity of the edge f_2. The remaining packets of the set S in f_2 at the end of this round are $|S_2| = |S| - (|T_1| - |S_1|)$. The rest packets of the set S that manage to traverse the edge f_2 traverse their remaining path and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|S_2|}{C}$ steps.

  Adversary's behavior. During this round the edge f_1 has unit capacity, while all the other edges have have capacity C. Also, the adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue f_3 requiring to traverse the edges $f_3, f_4, f_6, e_1, f_2^{'},\ f_3^{'}, f_4^{'}$.

  Evolution of the system configuration. S_2 packets delay Y packets in queue f_3 that uses LIS protocol because S_2 packets are longer time in the system. The packets of set S_2 traverse the edge f_3 and they are absorbed. At the same time, the packet set X is delayed in queue f_1 due to the unit capacity of the edge f_1. Therefore, the remaining packets of X in queue f_1 at the end of this round is a set $|X^{'}|$ of $|X^{'}| = |X| - |T_2|$ packets.

- **Round 3:** It lasts $|T_3| = \frac{|X^{'}|}{C}$ time steps.

  Adversary's behavior. During this round the edges f_4, f_5, f_6 have unit capacity, while all the other edges have capacity C. Also, the adversary injects a set Z of $|Z| = \rho C|T_3|$

packets in queue f_1 requiring to traverse the edges $f_1, f_7, e_1, f_2', f_3', f_4'$. Also, the adversary injects a set S_3 of $|S_3| = \rho|T_3|$ packets in queue f_4 wanting to traverse the edges $f_4, h_0, h_1, h_3, h_4, h_5, h_6$, a set S_4 of $|S_4| = \rho|T_3|$ packets in queue f_5 wanting to traverse the edges $f_5, h_7, g_1, g_2, g_3, g_4, g_5$ and a set S_5 of $|S_5| = \rho|T_3|$ packets in queue f_6 wanting to traverse the edges $f_6, w_1, w_2, w_3, w_4, w_5$.

Evolution of the system configuration. X' packets delay Z packets in queue f_1 that uses LIS protocol because they are longer time in the system. At the same time X' packets are delayed in queue f_5 that uses FTG protocol due to the unit capacity of the edge f_5 during this round and the S_4 packets that have furthest to go (queue g_5) than X' packets (queue f_4'). Therefore, the remaining packets of X' in queue f_5 is a set $|X''|$ of $|X''| = |X'| - (|T_3| - |S_4|)$ packets. Moreover Y packets are delayed in queue f_4 that uses FTG protocol due to the unit capacity of the edge f_4 during this round and the S_3 packets that have furthest to go (queue h_6) than Y packets (queue f_4'). Therefore, the remaining packets of Y in queue f_4 at the end of this round is a set $|Y'|$ of $|Y'| = |Y| - (|T_3| - |S_3|)$ packets.

Note that during this round $|K| = 2|T_3| - |S_3| - |S_4|$ packets arrive in queue f_6 from queues f_4, f_5. However, the edge f_6 has unit capacity and uses FTG protocol that gives priority to the S_5 packets. Furthermore, the duration of this round is $|T_3|$ time steps. Consequently, the number of packets that remain in queue f_6 at the end of this round requiring to traverse the edges $f_6, e_1, f_2', f_3', f_4'$ is $|L| = |K| + |S_5| - |T_3|$. Therefore, the number of packets in queues f_1, f_4, f_5, f_6 requiring to traverse the edges e_1, f_2', f_3', f_4' at the end of this round is $s_{j+1} = |X''| + |Y'| + |Z| + |L| = \rho s_j + \frac{\rho^2-\rho-1}{C} s_j + 2\frac{1-\rho}{C^2} s_j + \frac{-\rho^2+2\rho-1}{C^3} s_j + \rho s_j + \frac{2\rho^2-2\rho}{C} s_j + \frac{1-\rho}{C^2} s_j + \frac{-\rho^2+2\rho-1}{C^3} s_j + \rho^2 s_j - \frac{\rho}{C} s_j + \frac{\rho-\rho^2}{C^2} s_j + \frac{\rho-\rho^2}{C} s_j + 2\frac{\rho-1}{C^2} s_j + \frac{\rho^2-2\rho+1}{C^3} s_j$.

In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^2[1 + \frac{2}{C} - \frac{1}{C^2} - \frac{1}{C^3}] + \rho[2 - \frac{3}{C} - \frac{1}{C^2} + \frac{2}{C^3}] + [-\frac{1}{C} + \frac{2}{C^2} - \frac{1}{C^3}] > 1$. Initially, note that the following inequalities hold:

- $1 + \frac{2}{C} - \frac{1}{C^2} - \frac{1}{C^3} < 1 + \frac{2}{C}$,
- $2 - \frac{3}{C} - \frac{1}{C^2} + \frac{2}{C^3} < 2$,
- $1 + \frac{1}{C} - \frac{2}{C^2} + \frac{1}{C^3} > 1 - \frac{2}{C^2}$.

Therefore, the inequality becomes $\rho^2[1 + \frac{2}{C}] + 2\rho > 1 - \frac{2}{C^2}$. Thus, it suffices to be shown that $\rho^2[C^2 + 2C] + 2\rho C^2 > C^2 - 2$. This inequality holds for ρ larger than the largest root ρ_1 of the polynomial $\rho^2[C^2 + 2C] + 2\rho C^2 - (C^2 - 2)$. The largest root of the polynomial is $\rho_1 = \frac{-2C^2 + \sqrt{4C^4 + 4(C^2-2)(C^2+2C)}}{2C(C+2)} = \frac{\sqrt{2C^2+2C-2-\frac{4}{C}}}{C+2} - \frac{C}{C+2}$. But, $\frac{\sqrt{2C^2+2C-2-\frac{4}{C}}}{C+2} < \sqrt{\frac{2C}{C+2}}$. Therefore, $\rho_1 < \sqrt{\frac{2C}{C+2}} - \frac{C}{C+2}$. Thus, it holds that $\rho > \sqrt{\frac{2C}{C+2}} - \frac{C}{C+2} > \rho_1$.

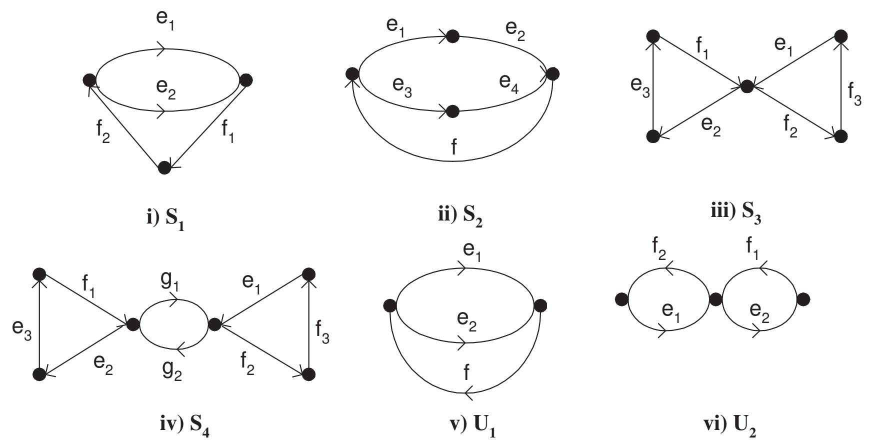

Figure 5: Pure simple-path networks $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4$, and not pure simple-path networks $\mathcal{U}_1, \mathcal{U}_2$.

When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, the inequality $\rho^2[1 + \frac{2}{C} - \frac{1}{C^2} - \frac{1}{C^3}] + \rho[2 - \frac{3}{C} - \frac{1}{C^2} + \frac{2}{C^3}] + [-\frac{1}{C} + \frac{2}{C^2} - \frac{1}{C^3}] > 1$ becomes $\rho^2 + 2\rho - 1 > 0$ which holds for $\rho > \sqrt{2} - 1$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C \to \infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C \to \infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > \sqrt{2} - 1$ the system is unstable. If we replace ρ' with $\sqrt{2} - 1 + \epsilon$ at the inequality $\rho' > \sqrt{\frac{2C}{C+2}} - \frac{C}{C+2}$, we estimate a $C(\rho') > 1$ such that $s_{j+1} > s_j$. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > \sqrt{2} - 1$. ■

5 Instability Bounds for Forbidden Subgraphs

In this section we show lower bounds on injection rate that guarantee instability for forbidden subgraphs. First, we consider the pure simple-path networks $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4$ (see Figure 5) that use NTG-U-LIS protocol.

Theorem 5.1 *Let $\rho \geq 0.82$. There is a network $\mathcal{S}_1$ and an adversary $\mathcal{A}_1$ of rate ρ that can change the link capacities of $\mathcal{S}_1$ between the two integer values 1 and $C > 1000$ such that the system $\langle \mathcal{S}_1, \mathcal{A}_1, \mathsf{NTG-U-LIS} \rangle$ is unstable. When $C \to \infty$ the system $\langle \mathcal{S}_1, \mathcal{A}_1, \mathsf{NTG-U-LIS} \rangle$ is unstable for $\rho > 0.8191$.*

Proof: We break the construction of the adversary $\mathcal{A}_1$ into phases.

Induction Hypothesis: At the beginning of phase j, there are s_j packets that are queued in the queues e_1, e_2 requiring to traverse the edge f_1.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets which will be queued in the queues e_1, e_2 requiring to traverse the edge f_1.

We will construct an adversary $\mathcal{A}_1$ such that the induction step holds. Proving that the induction step holds, we ensure that the induction hypothesis will hold at the beginning of phase $j+1$ with an increased value of s_j, $s_{j+1} > s_j$. From the induction hypothesis, initially, there are s_j packets (called S set of packets) in the queues e_1, e_2 requiring to traverse the edge f_1. In order to prove that the induction step works, we consider that there is a large enough number of packets s_j in the initial system configuration.

During phase j, the adversary plays three rounds of injections. The sequence of injections is as follows:

- **Round 1:** It lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects in queue f_1 a set X of $|X| = \rho C|T_1|$ packets wanting to traverse the edges f_1, f_2.

  Evolution of the system configuration. S packets delay X packets in queue f_1 because S packets are nearest to their destination (queue f_1) than X packets (queue f_2). S packets traverse the edge f_1 and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|X|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue f_2 requiring to traverse the edges f_2, e_1.

  Evolution of the system configuration. X packets delay Y packets in queue f_2 because they have nearest to go (queue f_2) than Y packets (queue e_1). X packets traverse the edge f_2 and they are absorbed.

- **Round 3:** It lasts $|T_3| = \frac{|Y|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects a set Z of $|Z| = \rho C|T_3|$ packets in queue f_2 requiring to traverse the edges f_2, e_2. Also, it injects a set Z_1 of $|Z_1| = \rho C|T_3|$ packets in queue e_1 requiring to traverse the edges e_1, f_1.

Evolution of the system configuration. Y packets delay Z packets in queue f_2 because although Y packets are in the same distance from their destination with the Z packets they are longer time in the system than Z packets. Moreover, Y packets delay Z_1 packets in queue e_1 because they have nearest to go (queue e_1) than Z_1 packets (queue f_1). Y packets traverse the edge e_1 and they are absorbed.

- **Round 4:** It lasts $|T_4| = \frac{|Z|}{C}$ time steps.

  Adversary's behavior. During this round the edge e_1 has unit capacity, while all the other edges have high capacity C. The adversary injects a set Z_2 of $|Z_2| = \rho C|T_4|$ packets in queue e_2 requiring to traverse the edges e_2, f_1.

  Evolution of the system configuration. Z packets delay Z_2 packets in queue e_2 because they have nearest to go (queue e_2) than Z_2 packets (queue f_1). Z packets traverse the edge e_2 and they are absorbed. Moreover Z_1 packets are delayed in queue e_1 due to the unit capacity of the edge e_1 during this round. Therefore, the remaining packets of Z_1 in queue e_1 at the end of this round is a set $|Z_1'|$ of $|Z_1'| = |Z_1| - |T_4|$ packets, while the rest Z_1 packets traverse their remaining path and they are absorbed. Therefore, the number of packets in queues e_1, e_2 requiring to traverse the edge f_1 at the end of this round is $s_{j+1} = |Z_2| + |Z_1'| = \rho^4 s_j + \rho^3 s_j - \frac{\rho^3}{C} s_j$.

  In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^4 s_j + \rho^3 s_j - \frac{\rho^3}{C} s_j > s_j$. Therefore, $\rho^4 C + \rho^3(C-1) > C$. Dividing by C the inequality, we take $\rho^4 + \rho^3(1 - \frac{1}{C}) > 1$. If we let $C = 1000$ and $\rho = 0.82$, the inequality holds. Thus, for $C > 1000$ and $\rho = 0.82$ the inequality holds, too.

  When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, our inequality becomes $\rho^4 + \rho^3 - 1 > 0$ which holds for $\rho > 0.8191$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > 0.8191$ the system is unstable. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > 0.8191$. ■

Theorem 5.2 *Let $\rho \geq 0.82$. There is a network $\mathcal{S}_2$ and an adversary $\mathcal{A}_2$ of rate ρ that can change the link capacities of $\mathcal{S}_2$ between the two integer values 1 and $C > 1000$ such that the system $\langle \mathcal{S}_2, \mathcal{A}_2, \mathsf{NTG-U-LIS} \rangle$ is unstable. When $C \to \infty$ the system $\langle \mathcal{S}_2, \mathcal{A}_2, \mathsf{NTG-U-LIS} \rangle$ is unstable for $\rho > 0.8191$.*

Proof: We break the construction of the adversary $\mathcal{A}_2$ into phases.

Induction Hypothesis: At the beginning of phase j, there are s_j packets that are queued in the queues e_2, e_4 requiring to traverse the edge f.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets which will be queued in the queues e_2, e_4 requiring to traverse the edge f.

We will construct an adversary $\mathcal{A}_2$ such that the induction step holds. Proving that the induction step holds, we ensure that the induction hypothesis will hold at the beginning of phase $j+1$ with an increased value of s_j, $s_{j+1} > s_j$. From the induction hypothesis, initially, there are s_j packets (called S set of packets) in the queues e_2, e_4 requiring to traverse the edge f. In order to prove that the induction step works, we consider that there is a large enough number of packets s_j in the initial system configuration.

During phase j, the adversary plays four rounds of injections. The sequence of injections is as follows:

- **Round 1:** It lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects in queue f a set X of $|X| = \rho C|T_1|$ packets wanting to traverse the edges f, e_3.

  Evolution of the system configuration. S packets delay X packets in queue f because S packets have nearest to go (queue f) than X packets (queue e_3). S packets traverse the edge f and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|X|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C. Also, the adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue e_3 requiring to traverse the edges e_3, e_4 and a set Z of $|Z| = \rho C|T_2|$ packets in queue f requiring to traverse the edges f, e_1.

  Evolution of the system configuration. X packets delay Z packets in queue f because although X packets are in the same distance from their destination with the Z packets they are longer time in the system than Z packets. Furthermore, X packets delay Y packets in queue e_3 because they have nearest to go (queue e_3) than Y packets (queue e_4). X packets traverse the edge e_3 and they are absorbed.

- **Round 3:** It lasts $|T_3| = \frac{|Y|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C. Also, the adversary injects a set Z_1 of $|Z_1| = \rho C|T_3|$ packets in queue e_4 requiring to traverse the edges e_4, f and a set Z_2 of $|Z_2| = \rho C|T_3|$ packets in e_1 requiring to traverse the edges e_1, e_2.

Evolution of the system configuration. Y packets delay Z_1 packets in e_4 because they have nearest to go (queue e_4) than Z_1 packets (queue f). Furthermore for the same reason Z packets delay Z_2 packets in queue e_1. Y and Z packets traverse the edges e_4 and e_1 correspondingly and they are absorbed.

- **Round 4:** It lasts $|T_4| = \frac{|Z_1|}{C}$ time steps.

  Adversary's behavior. During this round the edge e_4 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Z_3 of $|Z_3| = \rho C|T_4|$ packets in queue e_2 requiring to traverse the edges e_2, f.

  Evolution of the system configuration. Z_2 packets delay Z_3 packets in queue e_2 because they have nearest to go (queue e_2) than Z_3 packets (queue f). Moreover Z_1 packets are delayed in e_4 due to the unit capacity of the edge e_4 during this round. Therefore, the remaining packets of Z_1 in queue e_4 at the end of this round is a set Z_4 of $|Z_4| = |Z_1| - |T_4|$ packets, while the rest Z_1 packets traverse their remaining path and they are absorbed. Therefore, the number of packets in queues e_2, e_4 requiring to traverse the edge f at the end of this round is $s_{j+1} = |Z_3| + |Z_4| = \rho^4 s_j + \rho^3 s_j - \frac{\rho^3}{C} s_j$.

  In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^4 s_j + \rho^3 s_j - \frac{\rho^3}{C} s_j > s_j$. Therefore, $\rho^4 C + \rho^3 (C-1) > C$. Dividing by C the inequality, we take $\rho^4 + \rho^3(1 - \frac{1}{C}) > 1$. If we let $C = 1000$ and $\rho = 0.82$, the inequality holds. Thus, for $C > 1000$ and $\rho = 0.82$ the inequality holds, too.

  When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, our inequality becomes $\rho^4 + \rho^3 - 1 > 0$ which holds for $\rho > 0.8191$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > 0.8191$ the system is unstable. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > 0.8191$. ■

Theorem 5.3 *Let $\rho \geq 0.82$. There is a network $\mathcal{S}_3$ and an adversary $\mathcal{A}_3$ of rate ρ that can change the link capacities of $\mathcal{S}_3$ between the two integer values 1 and $C > 1000$ such that the system $\langle \mathcal{S}_3, \mathcal{A}_3, \mathsf{NTG-U-LIS} \rangle$ is unstable. When $C \to \infty$ the system $\langle \mathcal{S}_3, \mathcal{A}_3, \mathsf{NTG-U-LIS} \rangle$ is unstable for $\rho > 0.8191$.*

Proof: We break the construction of the adversary $\mathcal{A}_3$ into phases.

Induction Hypothesis: At the beginning of phase j, there are s_j packets that are queued in the queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, e_2 correspondingly.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets which will be queued in the queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, e_2 correspondingly.

We will construct an adversary $\mathcal{A}_3$, such that the induction step holds. Proving that the induction step holds, we ensure that the induction hypothesis will hold at the beginning of phase $j+1$ with an increased value of s_j, $s_{j+1} > s_j$. From the induction hypothesis, initially, there are s_j packets (called S set of packets) in the queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, e_2 correspondingly. In order to prove that the induction step works, we consider that there is a large enough number of packets s_j in the initial system configuration.

During phase j, the adversary plays four rounds of injections. The sequence of injections is as follows:

- **Round 1:** It lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects in queue e_2 a set X of $|X| = \rho C|T_1|$ packets wanting to traverse the edges e_2, e_3.

  Evolution of the system configuration. S packets delay X packets in queue e_2 because S packets have nearest to go (queue e_2) than X packets (queue e_3). S packets traverse the edge e_2 and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|X|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue e_3 requiring to traverse the edges e_3, f_1, f_2.

  Evolution of the system configuration. X packets delay Y packets in queue e_3 because they have nearest to go (queue e_3) than Y packets (queue f_2). X packets traverse the edge e_3 and they are absorbed.

- **Round 3:** It lasts $|T_3| = \frac{|Y|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C. Also, the adversary injects a set Z of $|Z| = \rho C|T_3|$ packets in queue f_1 requiring to traverse the edges f_1, e_2 and a set Z_1 of $|Z_1| = \rho C|T_3|$ packets in f_2 requiring to traverse the edges f_2, f_3.

Evolution of the system configuration. Y packets delay Z packets in f_1 because although Y packets are in the same distance from their destination with the Z packets, they are longer time in the system than Z packets. Furthermore, Y packets delay Z_1 packets in queue f_2 because they have nearest to go (queue f_2) than Z_1 packets (queue f_3). Y packets traverse the edge f_2 and they are absorbed.

- **Round 4:** It lasts $|T_4| = \frac{|Z|}{C}$ time steps.

  Adversary's behavior. During this round the edge f_1 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Z_2 of $|Z_2| = \rho C|T_4|$ packets in queue f_3 requiring to traverse the edges f_3, e_1, e_2.

  Evolution of the system configuration. Z_1 packets delay Z_2 packets in queue e_1 because they have nearest to go (queue f_3) than Z_2 packets (queue e_2). Z_1 packets traverse the edge f_3 and they are absorbed. Moreover Z packets are delayed in f_1 due to the unit capacity of the edge f_1 during this round. Therefore, the remaining packets of Z in queue f_1 at the end of this round is a set Z_3 of $|Z_3| = |Z| - |T_4|$ packets, while the rest Z packets traverse their remaining path and they are absorbed. Therefore, the number of packets in queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, e_2 correspondingly at the end of this round is $s_{j+1} = |Z_2| + |Z_3| = \rho^4 s_j + \rho^3 s_j - \frac{\rho^3}{C} s_j$.

  In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^4 s_j + \rho^3 s_j - \frac{\rho^3}{C} s_j > s_j$. Therefore, $\rho^4 C + \rho^3 (C-1) > C$. Dividing by C the inequality, we take $\rho^4 + \rho^3(1 - \frac{1}{C}) > 1$. If we let $C = 1000$ and $\rho = 0.82$, the inequality holds. Thus, for $C > 1000$ and $\rho = 0.82$ the inequality holds, too.

  When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, our inequality becomes $\rho^4 + \rho^3 - 1 > 0$ which holds for $\rho > 0.8191$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > 0.8191$ the system is unstable. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > 0.8191$. ∎

Theorem 5.4 *Let $\rho \geq 0.82$. There is a network $\mathcal{S}_4$ and an adversary $\mathcal{A}_4$ of rate ρ that can change the link capacities of $\mathcal{S}_4$ between the two integer values 1 and $C > 1000$ such that the system $\langle \mathcal{S}_4, \mathcal{A}_4, \mathsf{NTG-U-LIS} \rangle$ is unstable. When $C \to \infty$ the system $\langle \mathcal{S}_4, \mathcal{A}_4, \mathsf{NTG-U-LIS} \rangle$ is unstable for $\rho > 0.8191$.*

Proof: We break the construction of the adversary $\mathcal{A}_4$ into phases.

Induction Hypothesis: At the beginning of phase j, there are s_j packets that are queued in the queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, g_2, e_2 correspondingly.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets which will be queued in the queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, g_2, e_2 correspondingly.

We will construct an adversary $\mathcal{A}_4$, such that the induction step holds. Proving that the induction step holds, we ensure that the induction hypothesis will hold at the beginning of phase $j+1$ with an increased value of s_j, $s_{j+1} > s_j$. From the induction hypothesis, initially, there are s_j packets (called S set of packets) in the queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, g_2, e_2 correspondingly. In order to prove that the induction step works, we consider that there is a large enough number of packets s_j in the initial system configuration.

During phase j, the adversary plays four rounds of injections. The sequence of injections is as follows:

- **Round 1:** It lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects in queue e_2 a set X of $|X| = \rho C|T_1|$ packets wanting to traverse the edges e_2, e_3.

  Evolution of the system configuration. S packets delay X packets in queue e_2 because S packets have nearest to go (queue e_2) than X packets (queue e_3). S packets traverse the edge e_2 and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|X|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects a set Y of $|Y| = \rho C|T_2|$ packets in queue e_3 requiring to traverse the edges e_3, f_1, g_1.

  Evolution of the system configuration. X packets delay Y packets in queue e_3 because they have nearest to go (queue e_3) than Y packets (queue g_1). X packets traverse the edge e_3 and they are absorbed.

- **Round 3:** It lasts $|T_3| = \frac{|Y|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C. Also, the adversary injects a set Z_1 of $|Z_1| = \rho C|T_3|$ packets in queue f_1 requiring to traverse the edges f_1, e_2 and a set Z_2 of $|Z_2| = \rho C|T_3|$ packets in g_1 requiring to traverse the edges g_1, f_2, f_3.

Evolution of the system configuration. Y packets delay Z_1 packets in f_1 because although Y packets are in the same distance from their destination with the Z_1 packets, Y packets are longer time in the system than Z_1 packets. Furthermore, Y packets delay Z_2 packets in queue g_1 because they have nearest to go (queue g_1) than Z_2 packets (queue f_3). Y packets traverse the edge g_1 and they are absorbed.

- **Round 4:** It lasts $|T_4| = \frac{|Z_1|}{C}$ time steps.

  Adversary's behavior. During this round the edge f_1 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Z_3 of $|Z_3| = \rho C|T_4|$ packets in queue f_3 requiring to traverse the edges f_3, e_1, g_2, e_2.

  Evolution of the system configuration. Z_2 packets delay Z_3 packets in queue f_3 because they have nearest to go (queue f_3) than Z_3 packets (queue e_2). Moreover Z_1 packets are delayed in f_1 due to the unit capacity of the edge f_1 during this round. Therefore, the remaining packets of Z_1 in queue f_1 at the end of this round is a set Z_4 of $|Z_4| = |Z_1| - |T_4|$ packets, while the rest Z_1 packets traverse their remaining path and they are absorbed. Therefore, the number of packets in queues f_1, f_3 requiring to traverse the edges f_1, e_2 and f_3, e_1, g_2, e_2 correspondingly at the end of this round is $s_{j+1} = |Z_3| + |Z_4| = \rho^4 s_j + \rho^3 s_j - \frac{\rho^3 s_j}{C}$.

  In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^4 s_j + \rho^3 s_j - \frac{\rho^3}{C} s_j > s_j$. Therefore, $\rho^4 C + \rho^3 (C-1) > C$. Dividing by C the inequality, we take $\rho^4 + \rho^3 (1 - \frac{1}{C}) > 1$. If we let $C = 1000$ and $\rho = 0.82$, the inequality holds. Thus, for $C > 1000$ and $\rho = 0.82$ the inequality holds, too.

  When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, our inequality becomes $\rho^4 + \rho^3 - 1 > 0$ which holds for $\rho > 0.8191$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > 0.8191$ the system is unstable. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > 0.8191$. ■

Now, we consider the not pure simple-path networks $\mathcal{U}_1, \mathcal{U}_2$ (see Figure 5) that use NTG-U-LIS protocol.

Theorem 5.5 *Let $\rho = 0.8$. There is a network $\mathcal{U}_1$ and an adversary $\mathcal{A}$ of rate ρ that can change the link capacities of $\mathcal{U}_1$ between the two integer values 1 and $C > 1000$ such that the* *system* $\langle \mathcal{U}_1, \mathcal{A}, \mathsf{NTG-U-LIS} \rangle$ *is unstable. When* $C \to \infty$ *the system* $\langle \mathcal{U}_1, \mathcal{A}, \mathsf{NTG-U-LIS} \rangle$ *is unstable for* $\rho > \sqrt[3]{0.5}$.

Proof: We break the construction of the adversary $\mathcal{A}$ into phases.

Induction Hypothesis: At the beginning of phase j, there are s_j packets that are queued in the queues e_1, e_2 requiring to traverse the edge f.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets which will be queued in the queues e_1, e_2 requiring to traverse the edge f.

We will construct an adversary $\mathcal{A}$ such that the induction step holds. Proving that the induction step holds, we ensure that the induction hypothesis will hold at the beginning of phase $j+1$ with an increased value of s_j, $s_{j+1} > s_j$. From the induction hypothesis, initially, there are s_j packets (called S set of packets) in the queues e_1, e_2 requiring to traverse the edge f. In order to prove that the induction step works, we consider that there is a large enough number of packets s_j in the initial system configuration.

During phase j the adversary plays four rounds of injections. The sequence of injections is as follows:

- **Round 1:** It lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects in queue f a set X of $|X| = \rho C |T_1|$ packets wanting to traverse the edges f, e_1.

  Evolution of the system configuration. S packets delay X packets in queue f because S packets have nearest to go (queue f) than X packets (queue e_1). S packets traverse the edge f and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|X|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C. Also, the adversary injects a set Y of $|Y| = \rho C |T_2|$ packets in queue f requiring to traverse the edges f, e_2 and a set Z of $|Z| = \rho C |T_2|$ packets in queue e_1 requiring to traverse the edge e_1.

  Evolution of the system configuration. X packets delay Y packets in queue f because although X packets are in the same distance from their destination with the Y packets, X packets are longer time in the system than Y packets. Furthermore for the same reason X packets delay Z packets in queue e_1. X packets traverse the edge e_1 and they are absorbed.

- **Round 3:** It lasts $|T_3| = \frac{|Y|}{C}$ time steps.

  Adversary's behavior. During this round the edge e_2 has unit capacity, while all the network edges have high capacity C. Also, the adversary injects a set Z_1 of $|Z_1| = \rho C|T_3|$ packets in queue e_1 requiring to traverse the edges e_1, f.

  Evolution of the system configuration. Z packets delay Z_1 packets in queue e_1 because they have nearest to go (queue e_1) than Z_1 packets (queue f). Furthermore, Y packets are delayed in e_2 due to the unit capacity of e_2 during this round. Therefore, the remaining packets of Y in queue e_2 at the end of this round is a set Y' of $|Y'| = |Y| - |T_3|$ packets, while the rest Y packets traverse their remaining path and they are absorbed.

- **Round 4:** It lasts $|T_4| = \frac{|Y'|}{C}$ time steps.

  Adversary's behavior. During this round the edge e_1 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Z_2 of $|Z_2| = \rho C|T_4|$ packets in queue e_2 requiring to traverse the edges e_2, f.

  Evolution of the system configuration. Y' packets delay Z_2 packets in queue e_2 because they have nearest to go (queue e_2) than Z_2 packets (queue f). Y' packets traverse the edge e_2 and they are absorbed. Moreover, Z_1 packets are delayed in queue e_1 due to the unit capacity of the edge e_1 during this round. Therefore, the remaining packets of Z_1 in queue e_1 at the end of this round is a set Z_1' of $|Z_1'| = |Z_1| - |T_4|$ packets, while the rest Z_1 packets traverse their remaining path and they are absorbed. Therefore, the number of packets in queues e_1, e_2 requiring to traverse the edges f at the end of this round is $s_{j+1} = |Z_2| + |Z_1'| = \rho^3 s_j - \frac{\rho^3}{C}s_j + \rho^3 s_j - \frac{\rho^2}{C}s_j + \frac{\rho^2}{C^2}s_j$.

  In order to have instability, we must have $s_{j+1} > s_j$, that is $2\rho^3 s_j - \frac{\rho^3}{C}s_j - \frac{\rho^2}{C}s_j + \frac{\rho^2}{C^2}s_j > s_j$. Therefore, $2\rho^3C^2 - \rho^3C - \rho^2C + \rho^2 > C^2$. Dividing by C^2 the inequality, we take $\rho^3(2 - \frac{1}{C}) - \rho^2(\frac{1}{C} - \frac{1}{C^2}) > 1$. If we let $C = 1000$ and $\rho = 0.8$, the inequality holds. Thus, for $C > 1000$ and $\rho = 0.8$ the inequality holds, too.

  When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, our inequality becomes $2\rho^3 - 1 > 0$ which holds for $\rho > \sqrt[3]{0.5}$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty}\rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > \sqrt[3]{0.5}$ the system is unstable. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > \sqrt[3]{0.5}$. ∎

Theorem 5.6 *Let $\rho = 0.76$. There is a network $\mathcal{U}_2$ and an adversary $\mathcal{A}'$ of rate ρ that can* *change the link capacities of $\mathcal{U}_2$ between the two integer values 1 and $C > 1000$ such that the system $\langle \mathcal{U}_2, \mathcal{A}', \mathsf{NTG-U-LIS} \rangle$ is unstable. When $C \to \infty$ the system $\langle \mathcal{U}_2, \mathcal{A}', \mathsf{NTG-U-LIS} \rangle$ is unstable for $\rho > 0.754$.*

Proof: We break the construction of the adversary $\mathcal{A}'$ into phases.

Induction Hypothesis: At the beginning of phase j, there are s_j packets that are queued in the queues e_1, f_1 requiring to traverse the edge f_2.

Induction Step: At the beginning of phase $j+1$ there will be $s_{j+1} > s_j$ packets which will be queued in the queues e_1, f_1 requiring to traverse the edge f_2.

We will construct an adversary $\mathcal{A}'$ such that the induction step holds. Proving that the induction step holds, we ensure that the induction hypothesis will hold at the beginning of phase $j+1$ with an increased value of s_j, $s_{j+1} > s_j$. From the induction hypothesis, initially, there are s_j packets (called S set of packets) in the queues e_1, f_1 requiring to traverse the edge f_2. In order to prove that the induction step works, we consider that there is a large enough number of packets s_j in the initial system configuration.

During phase j the adversary plays three rounds of injections. The sequence of injections is as follows:

- **Round 1:** It lasts $|T_1| = \frac{s_j}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects in queue f_2 a set X of $|X| = \rho C |T_1|$ packets wanting to traverse the edges f_2, e_1, e_2.

  Evolution of the system configuration. S packets delay X packets in queue f_2 because S packets have nearest to go (queue f_2) than X packets (queue e_2). S packets traverse the edge f_2 and they are absorbed.

- **Round 2:** It lasts $|T_2| = \frac{|X|}{C}$ time steps.

  Adversary's behavior. During this round all the network edges have high capacity C and the adversary injects a set Y of $|Y| = \rho C |T_2|$ packets in queue e_1 requiring to traverse the edges e_1, f_2 and a set Z of $|Z| = \rho C |T_2|$ packets in queue e_2 requiring to traverse the edges e_2, f_1.

  Evolution of the system configuration. X packets delay Y packets in queue e_1 because although X packets are in the same distance from their destination with the Y packets, X packets are longer time in the system than Y packets. Furthermore, X packets delay

Z packets in queue e_2 because they have nearest to go (queue e_2) than Z packets (queue f_1). X packets traverse the edge e_2 and they are absorbed.

- **Round 3:** It lasts $|T_3| = \frac{|Y|}{C}$ time steps.

  Adversary's behavior. During this round the edge e_1 has unit capacity, while all the other edges have high capacity C. Also, the adversary injects a set Z_1 of $|Z_1| = \rho C|T_3|$ packets in queue f_1 requiring to traverse the edges f_1, f_2.

  Evolution of the system configuration. Z packets delay Z_1 packets in queue f_1 because they have nearest to go (queue f_1) than Z_1 packets (queue f_2). Moreover Y packets are delayed in e_1 due to the unit capacity of e_1 during this round. Therefore, the remaining packets of Y in queue e_1 at the end of this round is a set $|Y'|$ of $|Y'| = |Y| - |T_3|$ packets, while the rest Y packets traverse their remaining path and they are absorbed. Therefore, the number of packets in queues e_1, f_1 requiring to traverse the edge f_2 at the end of this round is $s_{j+1} = |Z_1| + |Y'| = \rho^3 s_j + \rho^2 s_j - \frac{\rho^2 s_j}{C}$.

  In order to have instability, we must have $s_{j+1} > s_j$, that is $\rho^3 s_j + \rho^2 s_j - \frac{\rho^2 s_j}{C} > s_j$. Therefore, $\rho^3 C + \rho^2(C-1) > C$. Dividing by C the inequality, we take $\rho^3 + \rho^2(1 - \frac{1}{C}) > 1$. If we let $C = 1000$ and $\rho = 0.76$, the inequality holds. Thus, for $C > 1000$ and $\rho = 0.76$ the inequality holds, too.

  When $C \to \infty$, it holds that $\frac{1}{C^k} \to 0$ for all $k \geq 1$. Then, our inequality becomes $\rho^3 + \rho^2 - 1 > 0$ which holds for $\rho > 0.754$. Note that if we have a sequence of equations $f_C(\rho)$ and there exists the limit $\lim_{C\to\infty} f_C(\rho) = f_\infty(\rho)$, then it holds fundamentally by the theory of function limits that if $\rho(C)$ is the root of $f_C(\rho) = 0$, then $\lim_{C\to\infty} \rho(C)$ is the root of $f_\infty(\rho)$. Therefore, for $\rho > 0.754$ the system is unstable. This argument can be repeated for an infinite number of phases showing that the number of packets in the system increases forever for $\rho > 0.755$. ■

6 Discussion and Directions for Further Research

In this work we have studied how the dynamical change of network link capacities affects the instability properties of greedy contention-resolution protocols. In particular, we propose an extension of the adversarial model that was first initiated by Borodin *et al.* in [5], the Adversarial, Quasi-Static Queueing Theory Model. According to this model each link capacity takes on values in the two-valued set of integers $\{1, C\}$ for $C > 1$ such that each capacity value remains fixed for a continuous period of time, at least proportional to the number of packets in the system at the time when the capacity was last set.

In the framework of Adversarial, Quasi-Static Queueing Theory Model we have studied the instability properties of LIS showing an improved lower bound on injection rate that guarantees instability than the one in the model of dynamic capacities [5]. This result represents the current state-of-the-art for the instability bound of LIS protocol. Also, we have shown that the lower bound on injection rate that guarantees instability for specific compositions of universally stable protocols drops lower than $\frac{1}{2}$ when the link capacities can change by an adversary. This result improves the corresponding one in the classical adversarial model [10] and also, it is the first attempt to study the stability behavior of compositions of protocols in dynamic adversarial systems. Finally, we have studied the instability behavior which is induced by all the known forbidden subgraphs in the classical adversarial queueing model on networks running NTG-U-LIS protocol presenting adversarial constructions that improve the instability bound when the link capacities can change dynamically.

In this work we have presented some examples of the impact the dynamical change of link capacities has on the instability behavior of greedy protocols and networks. However, a lot of problems remain open. Our results in this work suggest that, for every unstable network, its instability bound in the model of quasi-static capacities may be lower than for the classical adversarial queueing model. Proving (or disproving) this remains an open problem. Another important problem is to study the impact of dynamically changing link capacities on other greedy protocols and networks. Another problem is whether the lower bound on injection rate that guarantees instability for compositions of protocols can be dropped further. Furthermore, an interesting problem is whether there are upper bounds on injection rate that guarantee stability for forbidden subgraphs when the link capacities can change by an adversary. Finally, it is interesting to study the stability problems that can be obtained if we allow the adversary to control the movement or not of the network nodes and the contention-resolution protocol that is used in each network queue at each time step.

References

[1] M. Andrews, B. Awerbuch, A. Fernandez, J. Kleinberg, T. Leighton, and Z. Liu. Universal Stability Results for Greedy Contention-Resolution Protocols. *Journal of the ACM*, Vol. 48, No. 1, pp. 39–69, January 2001.

[2] C. Alvarez, M. Blesa, and M. Serna. Universal stability of undirected graphs in the adversarial queueing model. *Proceedings of the 14th Annual ACM Symposium on Parallel Algorithms and Architectures*, pp. 183–197, August 2002.

[3] W. Aiello, E. Kushilevitz, R. Ostrovsky, and A. Rosén. Adaptive Packet Routing for Bursty Adversarial Traffic. *Proceedings of the 30th Annual ACM Symposium on Theory of Computing*, pp. 359–368, May 1998.

[4] A. Borodin, J. Kleinberg, P. Raghavan, M. Sudan, and D. Williamson. Adversarial Queueing Theory. *Journal of the ACM*, Vol. 48, No. 1, pp. 13–38, January 2001.

[5] A. Borodin, R. Ostrovsky, and Y. Rabani. Stability Preserving Transformations: Packet Routing Networks with Edge Capacities and Speeds. *Proceedings of the 12th Annual ACM-SIAM Symposium on Discrete Algorithms*, pp. 601–610 January 2001.

[6] H. Chen, D. D. Yao. *Fundamentals of Queueing Networks*, Springer, 2000.

[7] J. Diaz, D. Koukopoulos, S. Nikoletseas, M. Serna, P. Spirakis, and D. Thilikos. Stability and Non-Stability of the FIFO Protocol. *Proceedings of the 13th Annual ACM Symposium on Parallel Algorithms and Architectures*, pp. 48–52, July 2001.

[8] A. Goel. Stability of Networks and Protocols in the Adversarial Queueing Model for Packet Routing. *Networks*, Vol. 37, No. 4, pp. 219–224, 2001.

[9] D. Koukopoulos, M. Mavronicolas, S. Nikoletseas, and P. Spirakis. On the Stability of Compositions of Universally Stable, Greedy, Contention-Resolution Protocols. *Proceedings of the 16th International Symposium on DIStributed Computing,* Dahlia Mahkli ed., pp. 88–102, LNCS 2508, Springer-Verlag, October 2002.

[10] D. Koukopoulos, M. Mavronicolas, S. Nikoletseas, and P. Spirakis. The Impact of Network Structure on the Stability of Greedy Protocols. *Proceedings of the 5th Italian Conference on Algorithms and Complexity,* R. Petreschi, G. Persiano, and R. Silvestri eds., pp. 251–263, LNCS 2653, Springer-Verlag, May 2003.

[11] D. Koukopoulos, S. Nikoletseas, and P. Spirakis. Stability Issues in Heterogeneous and FIFO Networks under the Adversarial Queueing Model. Invited Keynote Address, *Proceedings of the 8th International Conference on High Performance Computing*, B. Monien, V.K. Prasanna, and S. Vajapeyam eds., pp. 3–14, LNCS 2228, Springer-Verlag, December 2001.

[12] Z. Lotker, B. Patt-Shamir, and A. Rosén. New Stability Results for Adversarial Queueing. *Proceedings of the 15th Annual ACM Symposium on Parallel Algorithms and Architectures*, pp. 192–199, August 2002.

[13] P. Tsaparas. Stability in Adversarial Queueing Theory. *M.Sc. Thesis*, Computer Science Department, University of Toronto, 1997.