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# Competitive Analysis of On–line Randomized Call Control in Cellular Networks*

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## Abstract

*In this paper we address an important communication issue arising in cellular (mobile) networks that utilize Frequency Division Multiplexing (FDM) technology. In such networks, many users within the same geographical region can communicate simultaneously with other users of the network using distinct frequencies. The spectrum of the available frequencies is limited; thus, efficient solutions to the call control problem are essential. The objective of the call control problem is, given a spectrum of available frequencies and users that wish to communicate, to maximize the number of users that communicate without signal interference.*

*Using competitive analysis, we study the performance of algorithm $p$–*RANDOM*; an intuitive on-line randomized call control algorithm proposed in [4] for cellular networks. We give upper and lower bounds of its competitive ratio against oblivious adversaries as a function of the parameter $p$. Optimizing the upper bound function, we prove that there exists a $2.651$–competitive randomized call control algorithm. In this way, we significantly improve the best known upper bound on the competitiveness of on–line randomized call control which was $2.934$.*

## 1. Introduction

In the area of mobile communications, which combines wireless and high speed networking technologies, rapid technological progress has been made. It is expected that in the near future, mobile users have access to a wide variety of services available over mobile communication networks.

An architectural approach widely common for mobile networks is the following. A geographical area in which communication takes place is divided into regions. Each region is the calling area of a base station. Base stations are connected via a high speed network. In this work, the topology of the high speed network is not of interest. When a mobile user A wishes to communicate with some other user B, a path must be established between the base stations of the regions in which the users A and B are located. Then communication is performed in three steps: (a) wireless communication between A and its base station, (b) communication between the base stations, and (c) wireless communication between B and its base station. Thus, the transmission of a message from A to B first takes place between A and its base station, the base station of A sends the message to the base station of B which will transmit it to B. At least one base station is involved in the communication even if both mobile users are located in the same region.

Many users of the same region can communicate simultaneously with other users of the network. This can be achieved via frequency division multiplexing (FDM). The base station is responsible for allocating distinct frequencies from the available spectrum to users so that signal interference is avoided both within the same region and adjacent regions. Since the spectrum of available frequencies is limited, important engineering problems related to the efficient use of the frequency spectrum arise.

The network topology usually adopted is a finite portion of the infinite triangular lattice. This results from the uniform distribution of base stations within the network, as well as from the fact that the calling area of a base station is a circle which, for simplicity reasons, is idealized as a regular hexagon. Associated with the network is an *interference graph* $G$ that reflects possible signal interference. Vertices correspond to cells and an edge $(u, v)$ exists in the graph if and only if the cells corresponding to $u$ and $v$ are adjacent. Due to geometry, we call this graph a hexagon graph (see Figure 1). If the above assumptions (uniform distribution of base stations and equivalence of transmitters) do not hold, arbitrary interference graphs can be used to model the underlying network. At the rest of this paper, we use the term cellular network especially for networks with hexagon

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interference graph, like the one depicted in Figure 1.

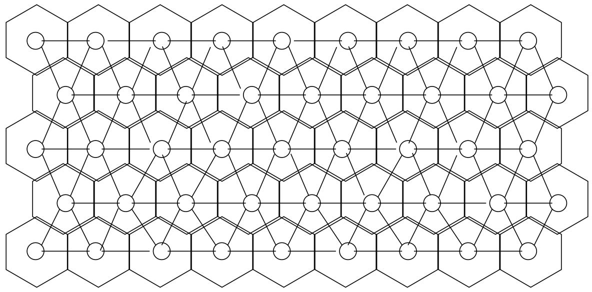

**Figure 1. A cellular network and the corresponding interference (hexagon) graph**

In this paper we study the *call control* (or call admission) problem which is defined as follows:

> Given users that wish to communicate, the *call control* problem on a network that supports a spectrum of $w$ available frequencies is to assign frequencies to users so that signal interference is avoided and the number of users served is maximized.

We assume that calls corresponding to users that wish to communicate appear in the cells of the network in an on–line manner. When a call arrives, a call–control algorithm decides either to accept the call (assigning a frequency to it), or to reject it. Once a call is accepted, it cannot be rejected (preempted). Furthermore, the frequency assigned to the call cannot be changed in the future. We assume that all calls have infinite duration; this assumption is equivalent to considering calls of the same duration.

Competitive analysis [10] has been used for evaluating the performance of on–line algorithms for various problems. In our setting, given a sequence of calls, the performance of an on–line algorithm $A$ is compared to the performance of the optimal algorithm $OPT$.

Let $B_A(\sigma)$ be the benefit of the on–line algorithm $A$ on the sequence of calls $\sigma$, i.e. the number of calls of $\sigma$ accepted by $A$ and $B_{OPT}(\sigma)$ the benefit of the optimal algorithm $OPT$. If $A$ is a deterministic algorithm, we define its competitive ratio $\rho$ as

$$\rho = \max_{\sigma} \frac{B_{OPT}(\sigma)}{B_A(\sigma)},$$

where the maximum is taken over all possible sequences of calls. If $A$ is a randomized algorithm, we define its competitive ratio $\rho$ as

$$\rho = \max_{\sigma} \frac{B_{OPT}(\sigma)}{\mathcal{E}[B_A(\sigma)]},$$

where $\mathcal{E}[B_A(\sigma)]$ is the expectation of the number of calls accepted by $A$, and the maximum is taken over all possible sequences of calls.

Usually, we compare the performance of deterministic algorithms against *off–line adversaries*, i.e. adversaries that have knowledge of the behaviour of the deterministic algorithm in advance. In the case of randomized algorithms, we consider *oblivious adversaries* whose knowledge is limited to the probability distribution of the random choices of the randomized algorithm.

The static version of the call control problem is very similar to the famous maximum independent set problem. The on–line version of the problem is studied in [1, 2, 3, 4, 7, 9]. [1], [2], and [7] study the call control problem in the context of optical networks. Pantziou et al. [9] present upper bounds for planar and arbitrary mobile networks. Applying the CLASSIFY AND RANDOMLY SELECT paradigm [2, 9] on cellular networks, we obtain a 3–competitive randomized call control algorithm. Usually, competitive analysis of call control focuses on networks supporting one frequency. Awerbuch et al. [1] present a simple way to transform algorithms designed for one frequency to algorithms for arbitrarily many frequencies with a small sacrifice in competitiveness. Lower bounds for call control in arbitrary networks are presented in [3].

The authors in [4] describe algorithm $p$–RANDOM, an intuitive on–line randomized call control algorithm for networks that support one frequency. Using simple arguments they prove an upper bound of $2.97$ on its competitive ratio against oblivious adversaries. In this way they beat the barrier of $3$ which is a lower bound on the competitiveness of deterministic algorithms. Motivated by algorithm $p$–RANDOM, they also design another randomized algorithm (algorithm $p_\delta$–RANDOM), for which they prove a competitiveness of $2.934$. Also, they present a lower bound of $1.857$ on the competitive ratio of any randomized call control algorithm in cellular networks.

In this paper, using more involved competitive analysis, we give stronger bounds on the performance of algorithm $p$–RANDOM. We present upper and lower bounds of its competitive ratio against oblivious adversaries as a function of the parameter $p$. Optimizing the upper bound function, we prove that there exists a $2.651$–competitive randomized call control algorithm. In this way, we significantly improve the best known upper bound on the competitiveness of on–line randomized call control in cellular networks.

The rest of the paper is structured as follows. In Section 2 we describe in detail algorithm $p$–RANDOM. We prove the upper bound on its competitive ratio in Section 3 and the lower bound in Section 4. We discuss some extensions and conclude with open problems revealed by our work in Section 5.

## 2. The randomized call control algorithm

Consider a cellular network that supports one frequency. Algorithm $p$–RANDOM receives as input a sequence of calls in an on–line manner, and works as follows.

1.Initially, all cells are unmarked.
2.for any new call $c$ in a cell $v$
3. if $v$ is marked then reject $c$.
4. if $v$ has an accepted call or is adjacent to a cell with an accepted call, then reject $c$
5. else
6. with probability $p$ accept $c$.
7. with probability $1-p$ reject $c$ and mark $v$.

The algorithm uses a parameter $p \in [1/3, 1]$. It is clear that if $p < 1/3$, the competitive ratio is larger than 3. The algorithm is simple and can be easily implemented with small communication overhead (exchange of messages) between the base stations of the network.

Marking cells on rejection guarantees that algorithm $p$–RANDOM does not simulate the greedy deterministic one. Assume otherwise, that marking is not used. Then, consider an adversary that presents $t$ calls in a cell $v$ and one call in 3 (mutually not adjacent) cells adjacent to $v$. The probability that the randomized algorithm does not accept a call in cell $v$ drops exponentially as $t$ increases, and the benefit approaches 1, while the optimal benefit is 3.

Note that algorithm $p$–RANDOM may accept at most one call in each cell but this is also the case for any algorithm running in networks that support one frequency (including the optimal one). Thus, for the competitive analysis of algorithm $p$–RANDOM, we will only consider sequences of calls with at most one call per cell.

## 3. The upper bound

In this section we prove the upper bound on the competitive ratio of algorithm $p$–RANDOM as a function of $p$. Our main statement is the following.

**Theorem 1** *For $p \in [1/3, 1]$, algorithm $p$–RANDOM has competitive ratio at most*

$$\frac{3}{5p - 7p^2 + 3p^3}$$

*against oblivious adversaries.*

**Proof:** Let $\sigma$ be a sequence of calls. We assume that $\sigma$ has been fixed in advance and will be revealed to the algorithm in an on–line manner. We make this assumption because we are interested in the competitiveness of the algorithm against oblivious adversaries whose knowledge is limited to the probability distribution of the random choices of the algorithm (i.e., the parameter $p$).

Consider the execution of algorithm $p$–RANDOM on $\sigma$. For any call $c \in \sigma$, we denote by $X(c)$ the random variable that indicates whether the algorithm accepted $c$. Obviously,

$$B(\sigma) = \sum_{c \in \sigma} X(c).$$

Let $A(\sigma)$ be the set of calls in $\sigma$ accepted by the optimal algorithm. For each call $c \in A(\sigma)$, we define the amortized benefit $\bar{b}(c)$ as

$$\bar{b}(c) = X(c) + \sum_{c' \in \gamma(c)} \frac{X(c')}{d(c')},$$

where $\gamma(c)$ denotes the set of calls of the sequence in cells adjacent to $c$. For each call $c' \notin A(\sigma)$, $d(c')$ is the number of calls in $A(\sigma)$ that are in cells adjacent to the cell of $c$. By the two equalities above, it is clear that

$$B(\sigma) = \sum_{c \in A(\sigma)} \bar{b}(c).$$

Furthermore, note that for any call $c' \notin A(\sigma)$, $d(c') \leq 3$. We obtain that

$$\bar{b}(c) \geq X(c) + \frac{\sum_{c' \in \gamma(c)} X(c')}{3}$$

and, by linearity of expectation,

$$\mathcal{E}[B(\sigma)] \geq \sum_{c \in A(\sigma)} \left( \mathcal{E}\left[ X(c) + \frac{\sum_{c' \in \gamma(c)} X(c')}{3} \right] \right) \quad (1)$$

Let $\gamma'(c)$ be the set of calls in cells adjacent to the cell of $c$ which appear prior to $c$ in the sequence $\sigma$. Clearly, $\gamma'(c) \subseteq \gamma(c)$, which implies that

$$\sum_{c' \in \gamma(c)} X(c') \geq \sum_{c' \in \gamma'(c)} X(c').$$

Thus, (1) yields

$$\mathcal{E}[B(\sigma)] \geq \sum_{c \in A(\sigma)} \left( \mathcal{E}\left[ X(c) + \frac{\sum_{c' \in \gamma'(c)} X(c')}{3} \right] \right) \quad (2)$$

In what follows we will try to bound from below the expectation of the random variable

$$Y(c) = X(c) + \frac{\sum_{c' \in \gamma'(c)} X(c')}{3},$$

for each call $c \in A(\sigma)$.

We concentrate on a call $c \in A(\sigma)$. Let $\Omega = 2^{\gamma'(c)}$ be the set which contains all possible subsets of $\gamma'(c)$. We define the *effective neighborhood* of $c$, denoted by $\Gamma(c)$, to be the subset of $\gamma'(c)$ that contains the calls of $\gamma'(c)$ which, when they appear, they are unconstrained by calls of $\sigma$ at distance 2 from $c$. Clearly, $\Gamma(c)$ is a random variable taking its values from the sample space $\Omega$. Intuitively, whether an optimal call $c$ is accepted by the algorithm depends on its effective neighborhood $\Gamma(c)$. We have

$$\begin{aligned}
\mathcal{E}[Y(c)] &= \sum_{\gamma\in\Omega}\left(\mathcal{E}\left[X(c)+\frac{\sum_{c'\in\gamma}X(c')}{3}|\Gamma(c)=\gamma\right]\right. \\
&\qquad \left.\cdot \Pr[\Gamma(c)=\gamma]\right) \\
&\geq \min_{\gamma\in\Omega}\left\{\mathcal{E}\left[X(c)+\frac{\sum_{c'\in\gamma}X(c')}{3}|\Gamma(c)=\gamma\right]\right\} \\
&= \min_{\gamma\in\Omega}\left\{\mathcal{E}[X(c)|\Gamma(c)=\gamma]\right. \\
&\qquad \left. + \frac{\mathcal{E}[\sum_{c'\in\gamma}X(c')|\Gamma(c)=\gamma]}{3}\right\} \qquad (3)
\end{aligned}$$

To compute $\mathcal{E}[X(c)|\Gamma(c)=\gamma]$, we observe that algorithm $p$–RANDOM may accept $c$ only if it has rejected all calls of in its effective neighborhood $\gamma$. The probability that all calls of $\gamma$ are rejected given that $\Gamma(c)=\gamma$ is $(1-p)^{|\gamma|}$, while then $c$ is accepted with probability $p$. Thus,

$$\mathcal{E}[X(c)|\Gamma(c)=\gamma] = p(1-p)^{|\gamma|}. \qquad (4)$$

We now bound from below $\mathcal{E}[\sum_{c'\in\gamma}X(c')|\Gamma(c)=\gamma]$ by distinguishing between cases on the size of the effective neighborhood $|\gamma|$.

**Claim 2** *For any* $p\in[1/3,1]$,

$$\mathcal{E}\left[\sum_{c'\in\gamma}X(c')|\Gamma(c)=\gamma\right] \geq \begin{cases} 0 & if|\gamma|=0 \\ p & if|\gamma|=1 \\ 2p-p^2 & if|\gamma|=2 \\ 3p-2p^2 & if|\gamma|=3 \\ 4p-3p^2+p^3 & if|\gamma|=4 \\ 5p-4p^2+p^3 & if|\gamma|=5 \\ 6p-5p^2+p^3 & if|\gamma|=6 \end{cases}$$

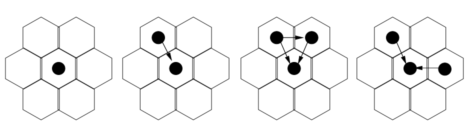

**Figure 2. The cases $|\gamma|=0,1,2$.**

**Proof:** In Figures 2, 3, 4, 5, and 6 we give all possible cases for the effective neighborhood of an optimal call $c$ in a sequence of calls $\sigma$. In each figure the optimal call is denoted by the black circle in the middle cell while black

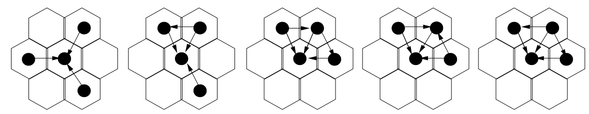

**Figure 3. The case $|\gamma|=3$.**

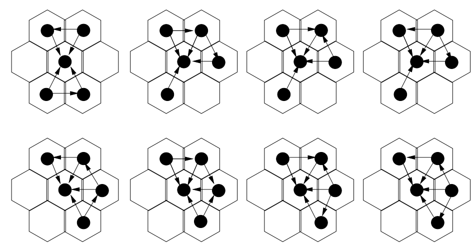

**Figure 4. The case $|\gamma|=4$.**

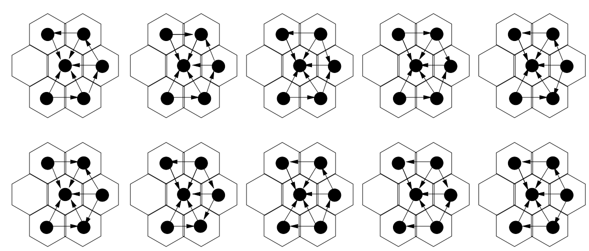

**Figure 5. The case $|\gamma|=5$.**

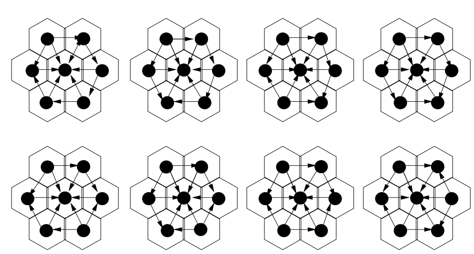

**Figure 6. The case $|\gamma|=6$.**

circles in the outer cells denote calls in the effective neighborhood $\gamma$ of $c$. An arrow from a call $c_1$ to another call $c_2$ indicates that $c_1$ appears in $\sigma$ prior to $c_2$. In the figures, we have eliminated the symmetric cases.

The proof is trivial for the cases $|\gamma| = 0$ and $|\gamma| = 1$ (the two leftmost cases in Figure 2). In the third case of Figure 2 (where $|\gamma| = 2$), we observe that the algorithm accepts the first call in $\gamma$ with probability $p$ and the second one with probability $p(1-p)$. In total, the expectation of the number of accepted calls in $\gamma$ is $2p - p^2$. In the rightmost case of Figure 2, the expectation of the number of accepted calls in $\gamma$ is $2p < 2p - p^2$.

Similarly, we can compute the desired lower bounds on $\mathcal{E}[\sum_{c' \in \gamma} X(c')|\Gamma(c) = \gamma]$ for the cases $|\gamma| = 3, 4, 5, 6$. ∎

By making calculations with (3), (4), and Claim 2, we obtain that

$$\begin{aligned}
\mathcal{E}[Y(c)] &\geq \min_{\gamma \in \Omega} \left\{ \mathcal{E}[X(c)|\Gamma(c) = \gamma] \right. \\
&\qquad \left. + \frac{\mathcal{E}[\sum_{c' \in \gamma} X(c')|\Gamma(c) = \gamma]}{3} \right\} \\
&\geq \min_{\gamma \in \Omega : |\gamma| = 2} \left\{ \mathcal{E}[X(c)|\Gamma(c) = \gamma] \right. \\
&\qquad \left. + \frac{\mathcal{E}[\sum_{c' \in \gamma} X(c')|\Gamma(c) = \gamma]}{3} \right\} \\
&\geq p(1-p)^2 + \frac{2p - p^2}{3} \\
&= \frac{5p - 7p^2 + 3p^3}{3}
\end{aligned}$$

Now, using (2) we obtain that

$$\begin{aligned}
\mathcal{E}[B(\sigma)] &\geq \sum_{c \in A(\sigma)} \frac{5p - 7p^2 + 3p^3}{3} \\
&= \frac{5p - 7p^2 + 3p^3}{3} \cdot B_{OPT}(\sigma)
\end{aligned}$$

This completes the proof of Theorem 1. ∎

The expression in Theorem 1 is minimized to $729/265 = 2.651$ for $p = 5/9$. Thus, we obtain the following result.

**Corollary 3** *There exists an on–line randomized call control algorithm for cellular networks with one frequency which is at most $2.651$–competitive against oblivious adversaries.*

## 4. The lower bound

In this section we show that our analysis is not far from being tight. In particular, we prove the following.

**Theorem 4** *For $p \in [1/3, 1]$, the competitive ratio of algorithm $p$–*RANDOM *against oblivious adversaries is at least*

$$\max\left\{ \frac{3}{4p - 3p^2}, \frac{3}{5p - 7p^2 + 4p^3 - p^4} \right\}.$$

**Proof:** We will prove the lower bound by constructing two sequences $\sigma_1$ and $\sigma_2$ of calls for which the competitive ratio of algorithm $p$–RANDOM is $\frac{3}{4p-3p^2}$ and $\frac{3}{5p-7p^2+4p^3-p^4}$, respectively.

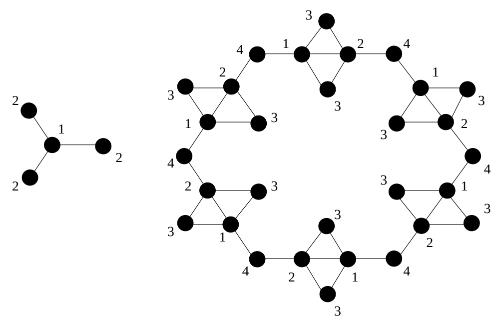

**Figure 7. The lower bound on the performance of algorithm $p$–RANDOM.**

Sequence $\sigma_1$ is depicted in the left part of Figure 7. In round 1, a call appears at some cell $v$, and in round 2 one call appears in each one of the three mutually not adjacent cells in the neighborhood of $v$. Clearly, the benefit of the optimal algorithm is 3. To compute the expectation of the benefit of algorithm $p$–RANDOM, we observe that

– with probability $p$, the call presented in round 1 is accepted, and

– with probability $1 - p$, the call presented in round 1 is rejected and each of the three calls presented in round 2 is accepted with probability $p$.

Thus, the expectation of the benefit of the algorithm on sequence $\sigma_1$ is $p + (1-p)3p = 4p - 3p^2$.

Sequence $\sigma_2$ is depicted in the right part of Figure 7. Calls appear in four rounds. The labels on the calls denote the round in which they appear. Clearly, the benefit of the optimal algorithm is $18$ since the optimal algorithm could accept the calls which appear in rounds 3 and 4. To compute the expectation of the benefit of algorithm $p$–RANDOM on sequence $\sigma_2$, we first compute the probability that each call is accepted.

– A call which appears in round 1 is accepted with probability $p$.

– A call which appears in round 2 can be accepted if its adjacent call which appeared in round 1 has been rejected; thus, the probability that a call which appears in round 2 is accepted is $p(1-p)$.

– A call which appears in round 3 can be accepted if both its adjacent calls which appeared in rounds 1 and 2 have been rejected; thus, the probability that a call which appears in round 3 is accepted is $p(1-p)^2$.

– A call which appears in round 4 can be accepted if its adjacent calls which appeared in rounds 1 and 2 have been rejected. The probability that a call which appeared in round 1 is rejected is $1-p$ while the probability that a call which appeared in round 2 is rejected is $1-p(1-p)$. Thus, the probability that a call which appears in round 4 is accepted is $p(1-p)\,(1-p(1-p))$.

Note that the number of calls that appear in rounds 1, 2, 3, and 4 is 6, 6, 12, and 6, respectively. Thus, we obtain that the expectation of the benefit of the algorithm is

$$6p+6p(1-p)+12p(1-p)^2+6p(1-p)(1-p(1-p)) =$$

$$30p-42p^2+24p^3-6p^4.$$

This yields a competitive ratio of algorithm $p$–RANDOM on sequence $\sigma_2$ of

$$\frac{18}{30p-42p^2+24p^3-6p^4} = \frac{3}{5p-7p^2+4p^3-p^4}.$$

This completes the proof of the theorem. ∎

The expression in Theorem 4 is minimized to $2.469$ for $p \approx 0.6145$. Thus, we obtain the following corollary.

**Corollary 5** *For any $p \in (1/3,1)$, algorithm $p$–RANDOM is at least $2.469$–competitive against oblivious adversaries.*

Let $p^*$ be the value in $[1/3,1]$ for which algorithm $p^*$–RANDOM has better competitive ratio than any algorithm $p$–RANDOM, for $p \in [1/3,1]$. By Theorem 4 and Corollary 3, solving the inequality

$$\max\left\{\frac{3}{4p-3p^2}, \frac{3}{5p-7p^2+4p^3-p^4}\right\} \leq 2.651$$

we obtain the following.

**Corollary 6** $p^* \in [0.421, 0.843]$.

## 5. Open problems

We have computed upper and lower bounds on the competitiveness of algorithm $p$–RANDOM on cellular networks that support one frequency as a function of $p$. A graphical representation of both functions is depicted in Figure 8.

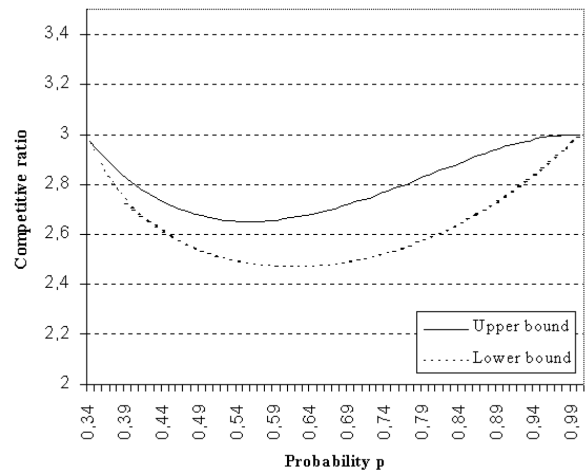


**Figure 8. A graphical representation of the the upper and the lower bound on the competitiveness of algorithm $p$–RANDOM for $p \in [1/3,1]$.**

Note that there is still a small gap between the upper and the lower bound which is up to $0,25$ for some values of $p$. Closing this gap is an interesting open problem. We conjecture that the competitive ratio of algorithm $p$–RANDOM is closer to the lower bound function.

There is also a large gap between the competitive ratio of algorithm $p$–RANDOM and the lower bound of $1.857$ on the competitive ratio of any randomized call control algorithm [4]. Closing this gap is the most interesting open problem to solve.

In this work we have addressed the case of one frequency. Using a technique of Awerbuch et al. [1], for any integer $w \geq 2$, we can design an algorithm for $w$ frequencies which is based on algorithm $p$–RANDOM and has competitive ratio

$$\frac{1}{1-\left(1-\frac{1}{w\rho(p)}\right)^w}$$

where $\rho(p)$ is the competitive ratio of algorithm $p$–RANDOM. The competitive ratio we achieve in this way for two frequencies is $2.927$. Unfortunately, for larger numbers of frequencies, the competitive ratio we obtain using the same technique is larger than $3$. When the number of frequencies $w$ is not a multiple of $3$, we can design another randomized algorithm (which is based on algorithm $p$–RANDOM and a simple deterministic one) which has competitive ratio $3-\Omega(1/w)$. However, it is still an interesting open problem to beat the barrier of $3$ on the competitiveness of randomized call control in cellular networks with arbitrarily many frequencies.

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