Homework Exercises for Lecture 4

4-1 Let $t \geqslant 1$ be a real number. Prove that there exists a set S of n real numbers such that every t-spanner for S, whose diameter is 2, contains $\Omega(n \log n)$ edges.

4-2 Consider n disks centering at $c_1, \cdots, c_n$. Let G be the disk graph of given disks; more precisely, the vertices of G are $c_1, \cdots, c_n$ and there is an edge between c_i and c_j if and only if the corresponding disks overlap. Show that Θ-graph approach does not necessarily produce a t-spanner of G for any constant θ.

4-3 Let P be a simple polygon with n vertices. Let G be the complete graph over vertices of P such that the weight assigned to (p, q) is the length of the shortest path from p to q inside P where p and q are vertices of P. Prove that using Θ-graph approach, we can produce a t-spanner of $O(n)$ size.

4-4 Prove that there is a set S of n points such that any WSPD $\{A_i, B_i\}$ for S with separation ratio s holds $\sum |A_i| + |B_i| = \Omega(n^2)$. Moreover, prove that if WSPD is obtained by a quad-tree, then $\sum \min(|A_i|, |B_i|) = O(n \log n)$

4-5 Let I be all intervals of form $[2^i, 2^{i+1}]$ where i is an integer number. Let S be a set of n points in a plane. An interval $[2^i, 2^{i+1}] \in I$ is called a marked interval if there is a pair $p, q \in S$ such that $|pq|$ (the Euclidian distance) lies in $[2^i, 2^{i+1}]$. Shows that the number of marked intervals is $O(n)$. Is it possible to generalize the above problem to any metric space D defined over n points.