Edge Flips in Surface Meshes

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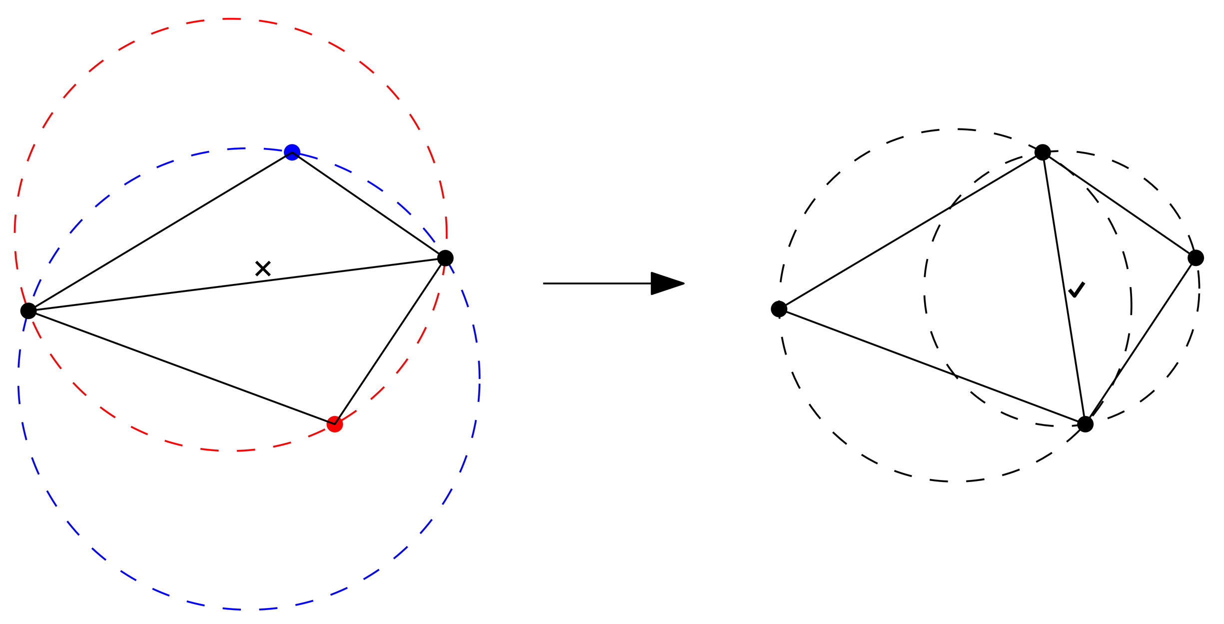
Problem

Edge flips in a planar triangulation:



- $O(n^2)$
- Delaunay

Can we generalize the idea to mesh surface?

- Definition of flippability?
- Termination? Complexity?
- Delaunay?

Definitions

Σ(surface) -

- 2-manifold

M(medial axis) -

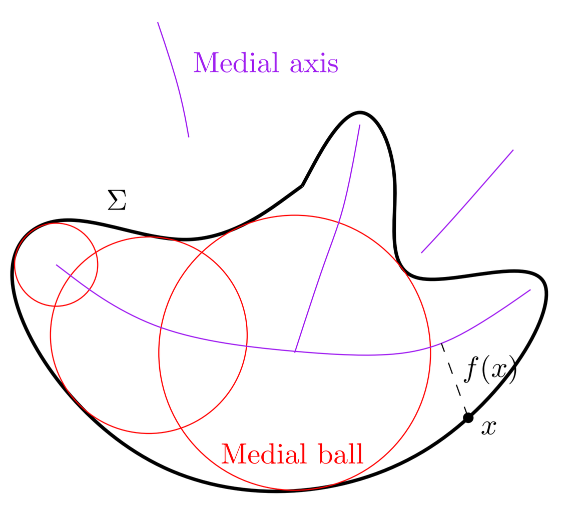
- set of maximum empty ball centers

$f(x)$(local feature size) -

- $d(x, M)$

ε-sample -

- $\forall x \in \Sigma$, $\exists$ a sample p s.t. $d(x,p) \leq \varepsilon f(x)$.



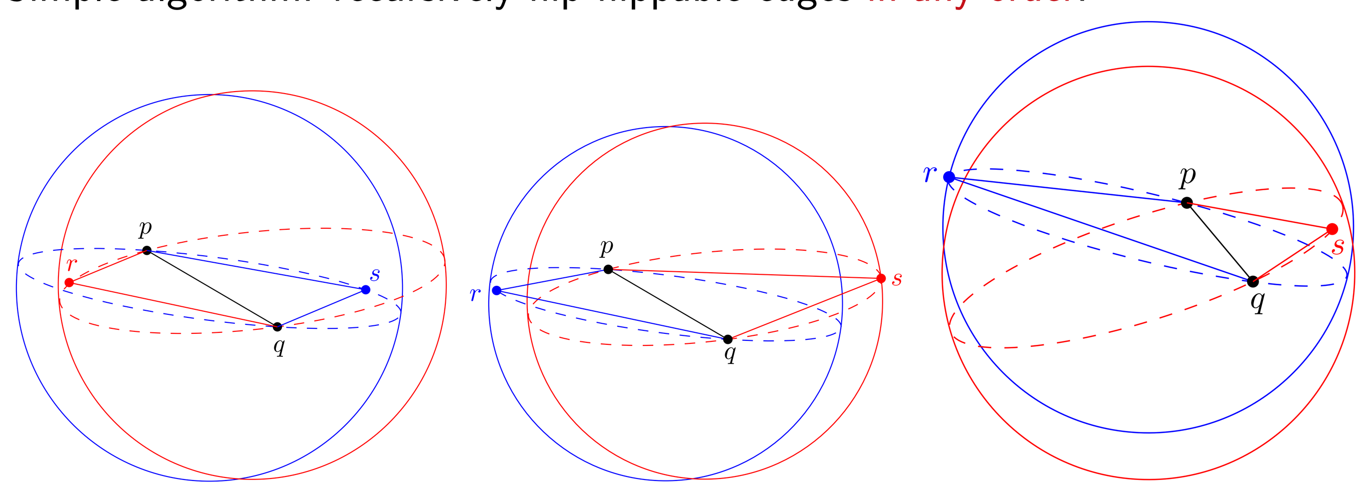
(ε, α)-mesh -

- vertex set forms an ε-sample
- all angles are at least α
- each triangle incident to p has circumradius $O(\varepsilon f(p))$
- nearest point map is a homeomorphism

(basically a fine surface mesh with a constant minimum angle)

Our Results

Simple algorithm: recursively flip flippable edges in any order.



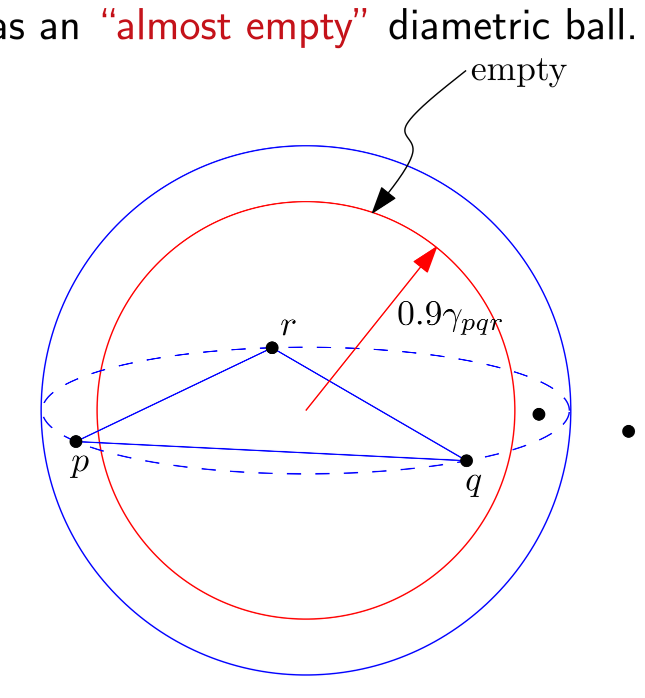
Flippable   Non-flippable   Non-flippable

Edge flips in a planer triangulation with constant minimum angle

- $O(n)$ time to make it Delaunay!

Edge flips in an (ε, α)-mesh:

- $O(n)$ time to make it non-flippable
- each triangle has an "almost empty" diametric ball.



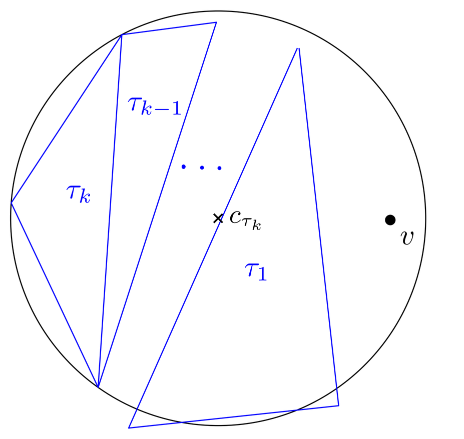
Proof Ideas

Linear flip complexity

- the minimum angle in the two new triangles after an edge flip is greater than that in the old triangles. (a well known fact for planer triangulations)
- (ε, α)-mesh remains an (ε, α)-mesh after each edge flip.
- $O(1)$ possible triangles incident to any vertex
- improvement of local angle vectors

Almost emptiness

- At the termination, all edges are non-flippable.
- For any vertex v and any triangle τ,
  - $d(v, c_\tau) \geq \omega\gamma_\tau$, where $\omega = 1 - O(\varepsilon)$.
    or
  - v *stabs* a closer triangle adjacent to τ.
- Suppose $d(v, c_\tau) < 0.9\gamma_\tau$.
  - a maximum sequence of triangles $\tau_1, \ldots, \tau_k (= \tau)$ stabbed by v
  - $d(v, c_{\tau_i}) \geq \omega^i \gamma_{\tau_i}$
  - $k \leq \kappa$(a packing argument) $\Rightarrow d(v, c_\tau) \geq \omega^\kappa \gamma_\tau \geq 0.9\gamma_\tau$
    contradiction!



Applications

Maintaining a deforming surface:

- Applications:
  cloth simulation, tracking fluid interface, surgery simulation, etc.
- [Cheng and Dey 08]
  - use edge-flip as a subroutine to improve the mesh quality
  - it requires a uniform mesh dense with respect to the reach(minimum local feature size) of the mesh
  - now we only require the sample to be dense with respect to local feature sizes(sparser in lower-curvature regions)