Smoothed Complexity of Visibility Maps for Realistic Terrains

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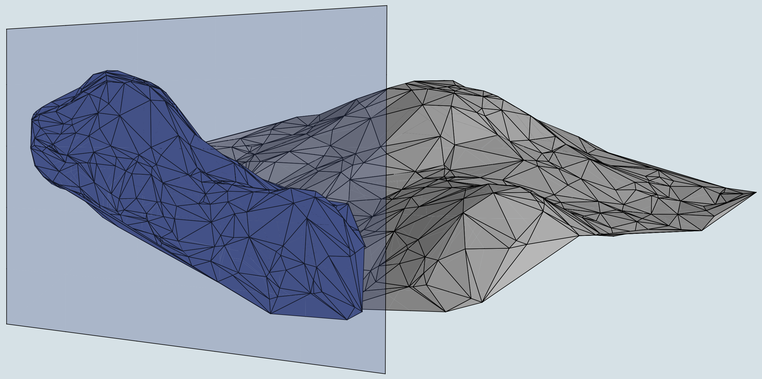
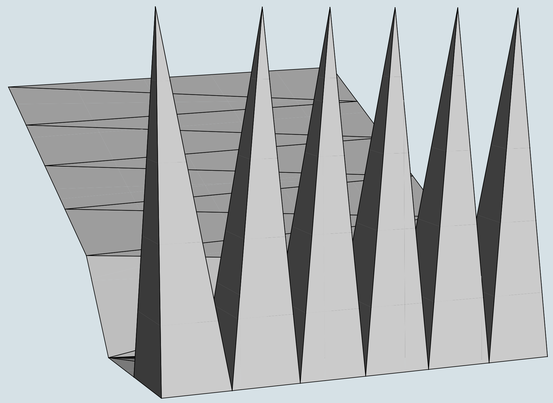
The Problem

The visibility map of a triangulated terrain $\mathcal{T}$ is the subdivision of the viewing plane into regions that correspond to the visible parts of triangles in $\mathcal{T}$ or to no triangles at all.



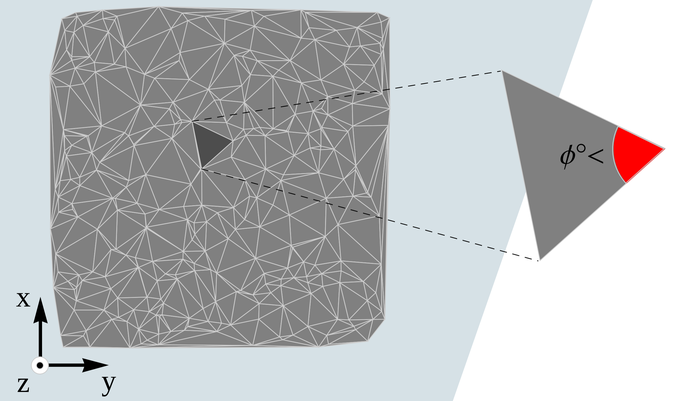
Unfortunately, there can be terrains that may induce visibility maps of complexity quadratic to the number of their vertices.



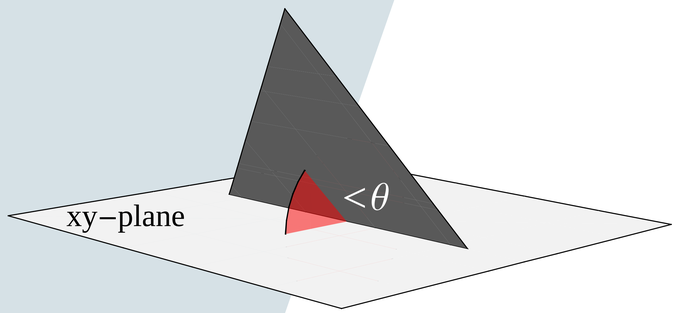
Input assumptions

Since such terrains do not appear in practice we impose specific assumptions upon our input. Terrains that fulfill these assumptions are called *realistic* [2]. The assumptions that we use in our approach are:

- The triangles in the xy-projection of the terrain are not arbitrarily skinny: each of their angles is not smaller than a constant ϕ



- The triangles in $3D$ space are not arbitrarily steep: Every terrain triangle forms a dihedral angle with the xy-plane which is not greater than a constant $\theta < 90°$.



Although there are still terrains under these assumptions that create a visibility map of quadratic complexity, these instances are *fragile* under noise. Therefore we add a small noise value to the z-coordinate of the terrain vertices. For every terrain vertex v let $\mathcal{E}_v$ be the set of the edges incident to v. Consider noise applied to v_z that is uniformly distributed in:

$$[-c \max_{e \in \mathcal{E}_v} |e|, c \max_{e \in \mathcal{E}_v} |e|]$$

where c is a small fraction of the unit.

For any input instance we do not consider the actual complexity of the visibility map but the expected complexity under the aforementioned noise scheme. The worst among the expected complexities of all the input instances is the *smoothed* complexity of the problem itself ([3],[1]).

Our results

We have proved $\Theta(n)$ smoothed complexity for the visibility map induced by realistic terrains with:

- perspective projection.
- orthographic projection of any viewing direction.

References

[1] Valentina Damerow and Christian Sohler. Extreme points under random noise. In *ESA*, pages 264–274, 2004.

[2] Esther Moet, Marc van Kreveld, and A. Frank van der Stappen. On realistic terrains. In *SCG '06: Proceedings of the twenty-second annual symposium on Computational geometry*, pages 177–186, New York, NY, USA, 2006. ACM.

[3] Daniel A. Spielman and Shang-Hua Teng. Smoothed analysis of algorithms: Why the simplex algorithm usually takes polynomial time. *J. ACM*, 51(3):385–463, 2004.