

Algoritmer og Datastrukturer 2

Gerth Stølting Brodal

Topologisk Sortering, Stærke Sammenhængskomponenter
[CLRS, kapitel 22.4-22.5]



Dybde Først Søgning (DFS)

DFS(G)

```
1 for each vertex  $u \in G.V$ 
2    $u.color = WHITE$ 
3    $u.\pi = NIL$ 
4    $time = 0$ 
5 for each vertex  $u \in G.V$ 
6   if  $u.color == WHITE$ 
7     DFS-VISIT( $G, u$ )
```

$u.color$

WHITE = knuderne endnu ikke besøgt
GRAY = knuder på rekursionsstakken
BLACK = knuderne besøgt

$u.\pi$ = faderen til u i DFS træet

$u.d$ = "discover time" for u

$u.f$ = "finishing time" for u

DFS-VISIT(G, u)

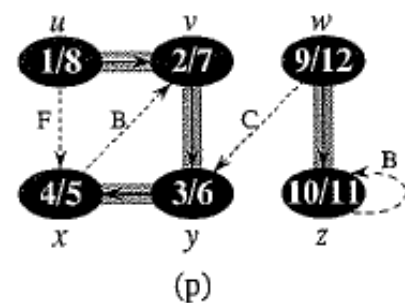
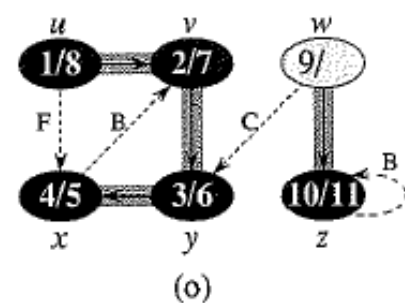
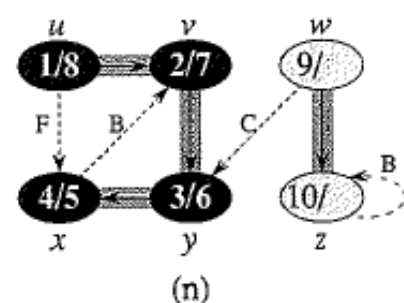
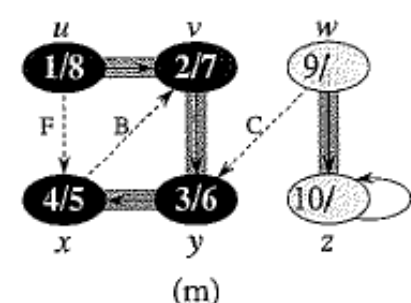
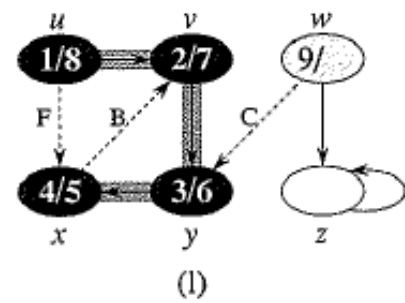
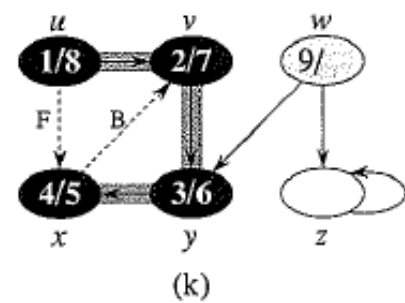
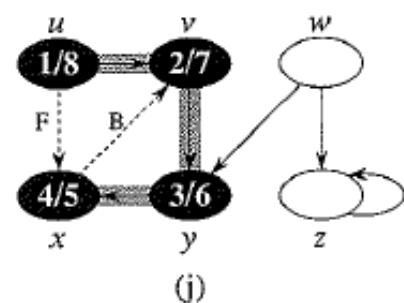
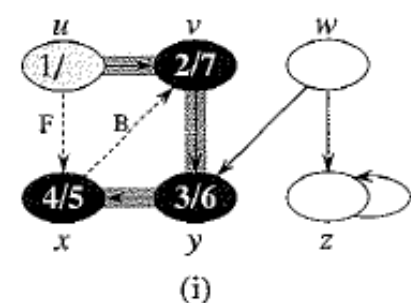
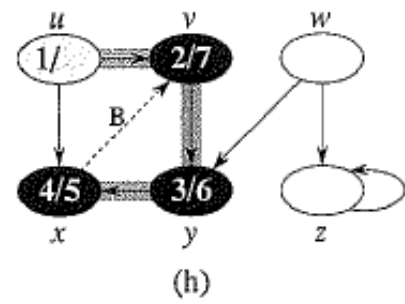
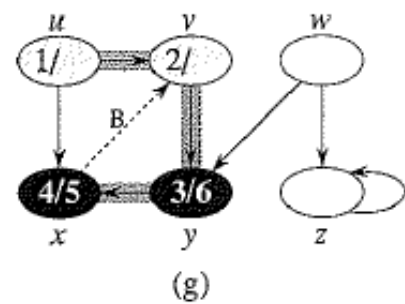
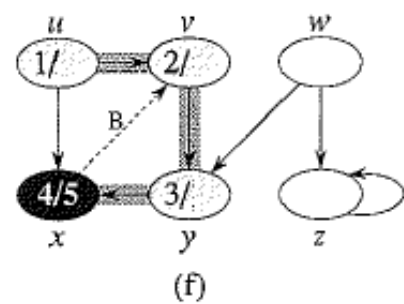
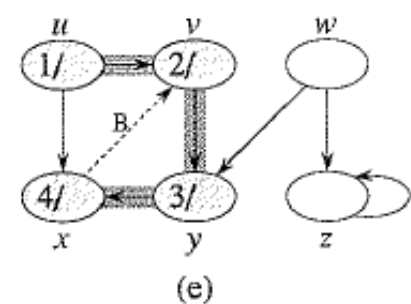
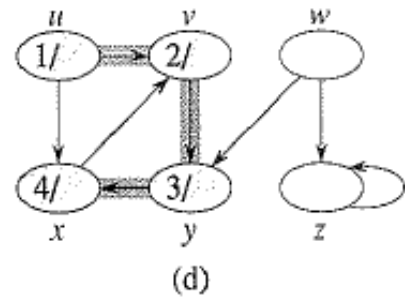
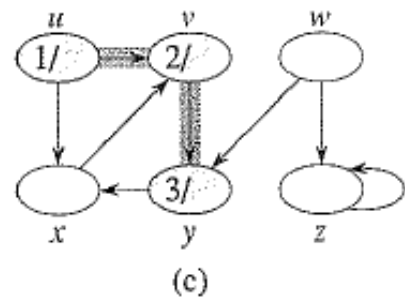
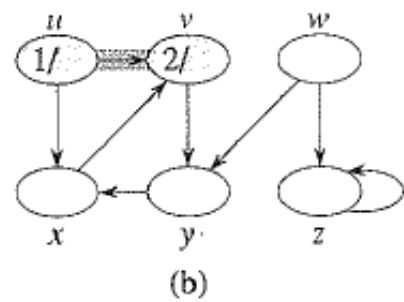
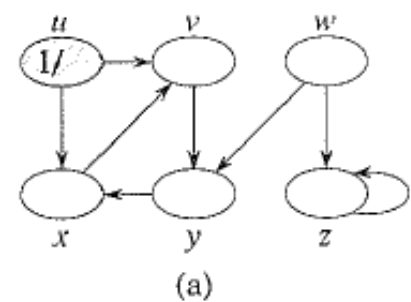
```
1  $time = time + 1$ 
2  $u.d = time$ 
3  $u.color = GRAY$ 
4 for each  $v \in G.Adj[u]$ 
5   if  $v.color == WHITE$ 
6      $v.\pi = u$ 
7     DFS-VISIT( $G, v$ )
8  $u.color = BLACK$ 
9  $time = time + 1$ 
10  $u.f = time$ 
```

// white vertex u has just been discovered

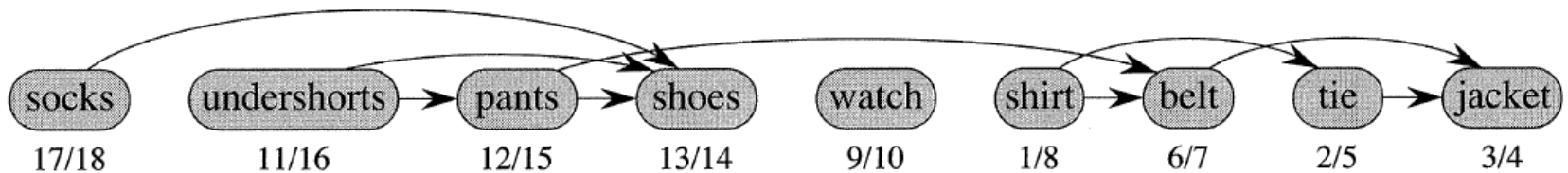
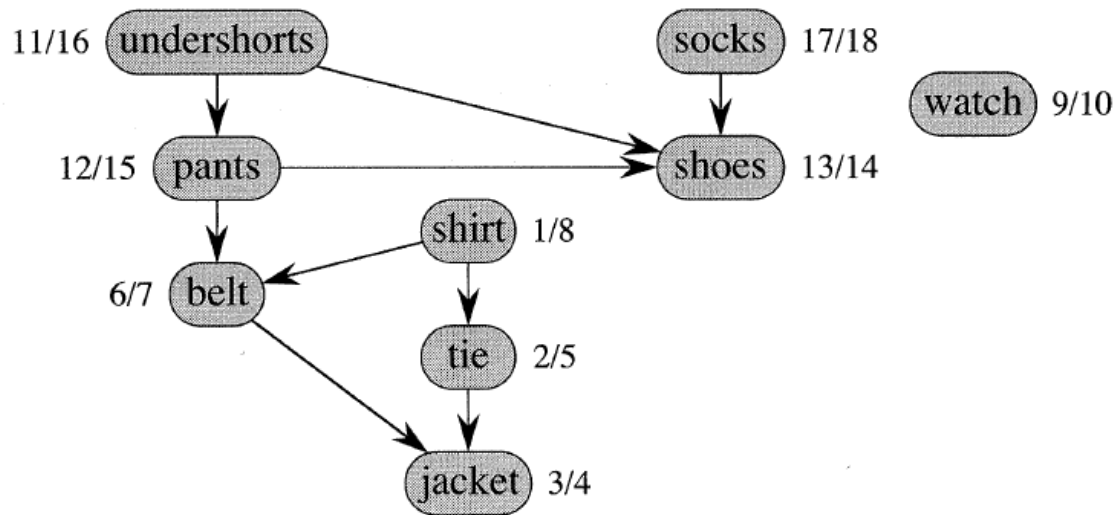
// explore edge (u, v)

// blacken u ; it is finished

Tid $O(n+m)$



Acykliske Grafer: Topologisk Sortering



Alle kanter går fra venstre-mod-højre

Microsoft Excel - Copy of SheepFlock

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

H18 fx =B18*G18

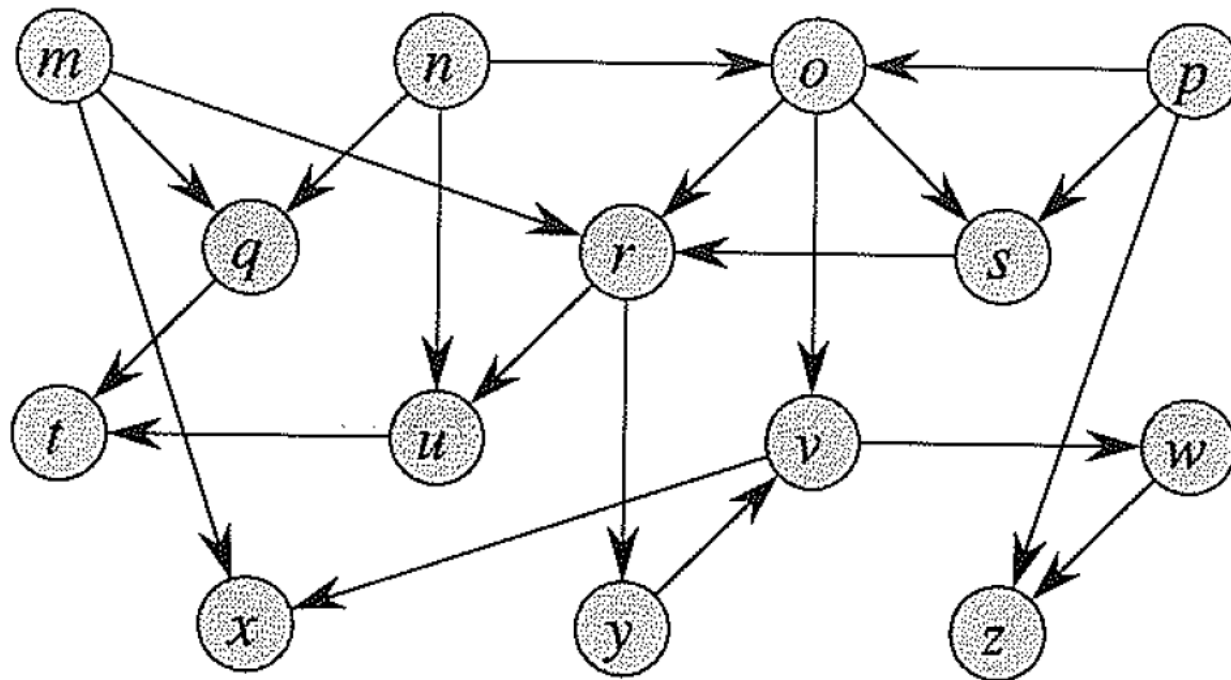
| | A | B | C | D | E | F | G | H | I |
|----|--|--------|---|--------|---|--------|-------------|-----------|-----------|
| 3 | I. Description of animals in flock during the year. | | | | | | | | |
| 4 | Ewes in flock: | 700 | | | [Green cells are those you can change.] | | | | |
| 5 | Lambing rate: | 4 | times per | 3 | years = | 1.33 | times/year. | | |
| 6 | Lambs weaned/lambing: | 1.5 | Days of lactation/lambing: | | 60 | | | | |
| 7 | Adult death loss per year: | 3% | Days in lactation/year: | | 80 | | | | |
| 8 | Postweaning lamb loss: | 2% | Lambs weaned per ewe per year: | | 2.0 | | | | |
| 9 | Ewe culling rate: | 15% | Ram culling rate: | | 50% | | | | |
| 10 | Rams/100 ewes: | 1 | (Only 1/3 of ewes bred per season under STAR system.) | | | | | | Inventory |
| 11 | | | Weaning | Market | Final | Price | Value | or sale | |
| 12 | | Number | wt, lb | wt, lb | wt, lb | \$/lb | per head | value | |
| 13 | Ewes | 700 | | | 150 | \$1.00 | \$150 | \$105,000 | |
| 14 | Rams | 8 | | | 200 | \$2.00 | \$400 | \$3,200 | |
| 15 | Ewe lamb replcmnts | 126 | 30 | | 100 | \$1.25 | \$125 | \$15,750 | |
| 16 | Ram lamb replcmnts | 5 | 40 | | 130 | \$2.00 | \$260 | \$1,300 | |
| 17 | Ewe lambs sold | 560 | 30 | 70 | | \$1.10 | \$77 | \$43,120 | |
| 18 | Ram lambs sold | 681 | 40 | 70 | | \$1.10 | \$77 | \$52,437 | |
| 19 | Cull ewes sold | 105 | | 150 | | \$0.30 | \$45 | \$4,725 | |
| 20 | Cull rams sold | 5 | | 200 | | \$0.30 | \$60 | \$300 | |
| 21 | Fleece weight per adult | 708 | | | 6 | \$0.30 | \$1.80 | \$1,274 | |
| 22 | | | | | | | Inventory: | \$125,250 | |
| 23 | | | | | | | Sales: | \$101,856 | |

Sheep flock /

Ready

Topologisk sortering = en rækkefølge hvor vi kan beregne cellernes indhold

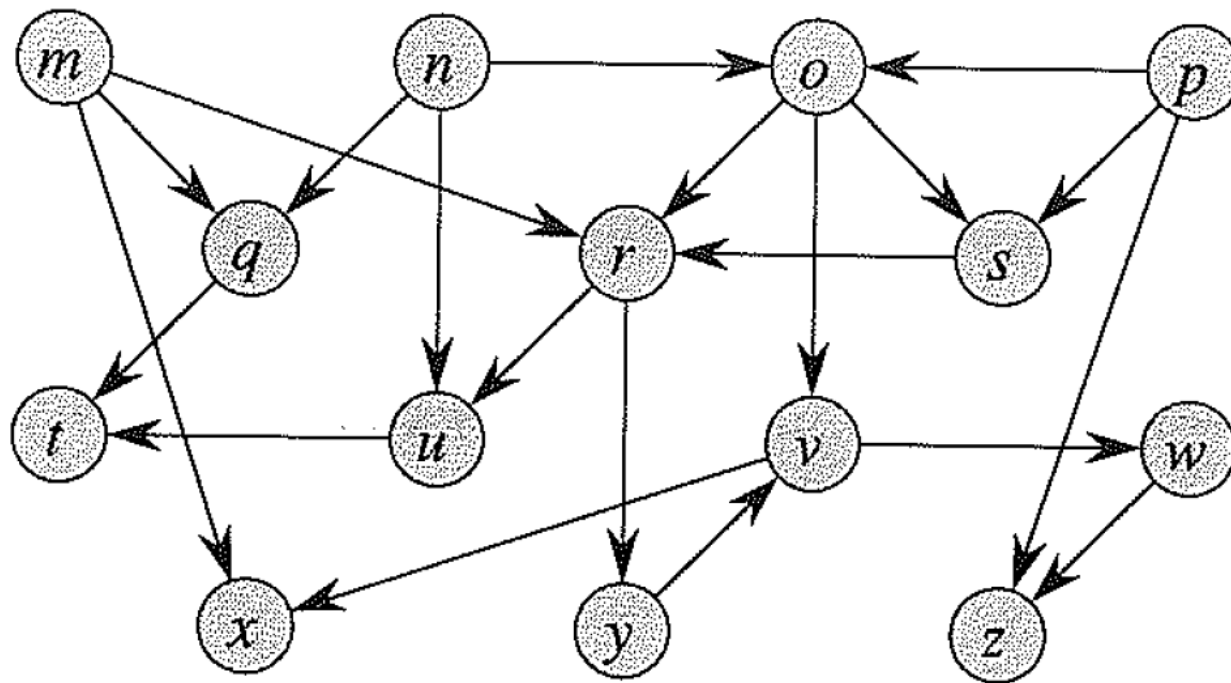
Topologisk Sortering (I)



Algoritme: Grådigt slet en knude med indgrad 0 (og udgående kanter), og tilføj knuden sidst i den topologiske orden

Tid $O(m+n)$

Topologisk Sortering (II)



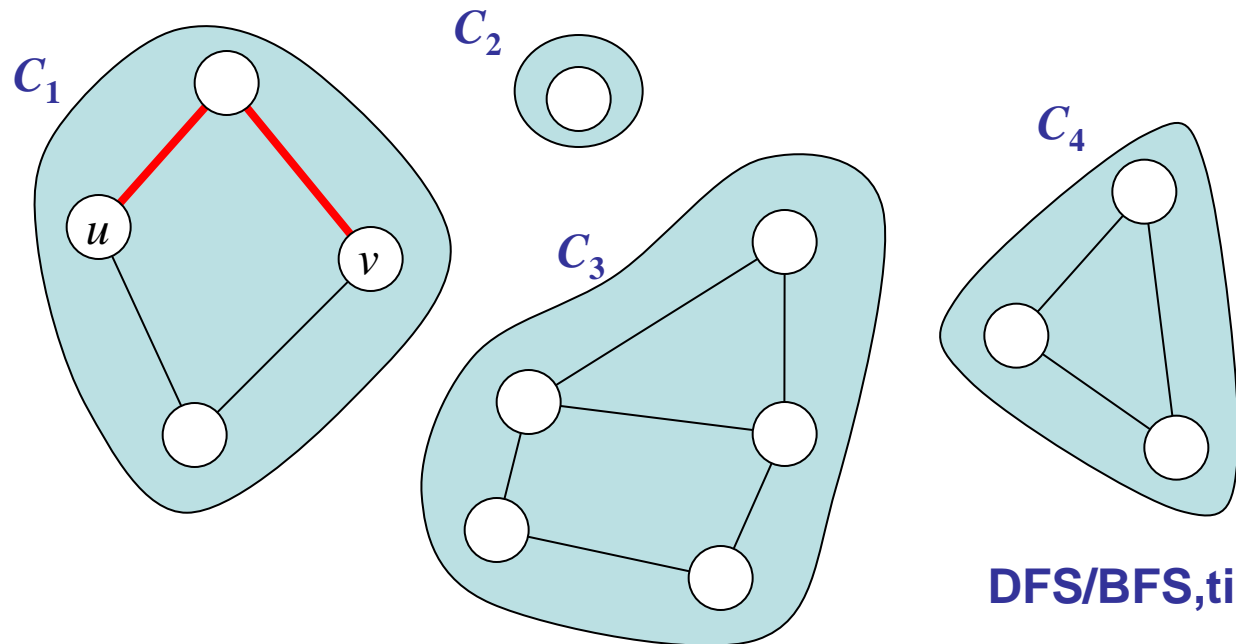
TOPOLOGICAL-SORT(G)

- 1 call DFS(G) to compute finishing times $v.f$ for each vertex v
- 2 as each vertex is finished, insert it onto the front of a linked list
- 3 **return** the linked list of vertices

Tid $O(m+n)$

Sammenhængskomponenter

Opdeling af knuderne i en **uorienteret** graf i **komponenter** C_1, \dots, C_k , således at u og v er i C_i hvis og kun hvis der er en **sti** mellem u og v

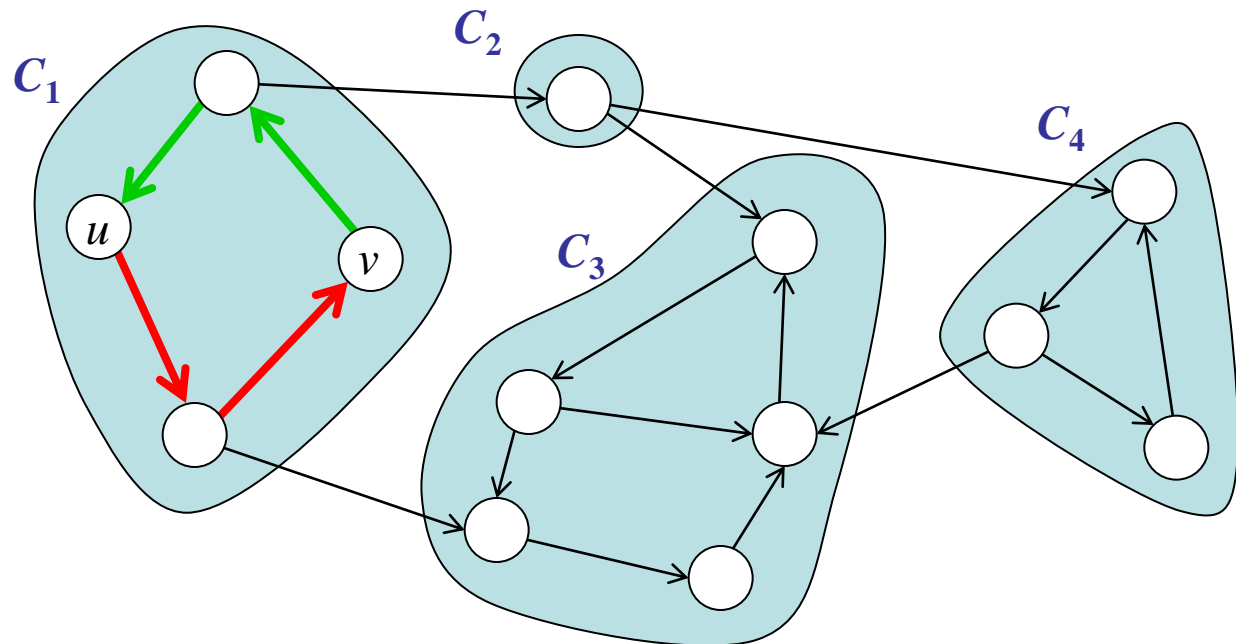


DFS/BFS, tid $O(m+n)$

Stærke Sammenhængskomponenter

Opdeling af knuderne i en **orienteret** graf i **komponenter** C_1, \dots, C_k , således at u og v er i C_i hvis og kun hvis der både er

- en **sti fra u til v** og
- en **sti fra v til u**



Stærke Sammenhængskomponenter

STRONGLY-CONNECTED-COMPONENTS (G)

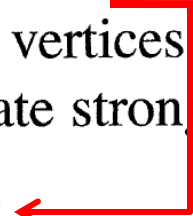
- 1 call DFS(G) to compute finishing times $u.f$ for each vertex u
- 2 compute G^T
- 3 call DFS(G^T), but in the main loop of DFS, consider the vertices in order of decreasing $u.f$ (as computed in line 1)
- 4 output the vertices of each tree in the depth-first forest formed in line 3 as separate strongly connected component

DFS(G)

```

1 for each vertex  $u \in G.V$ 
2    $u.color = WHITE$ 
3    $u.\pi = NIL$ 
4  $time = 0$ 
5 for each vertex  $u \in G.V$ 
6   if  $u.color == WHITE$ 
7     DFS-VISIT( $G, u$ )

```



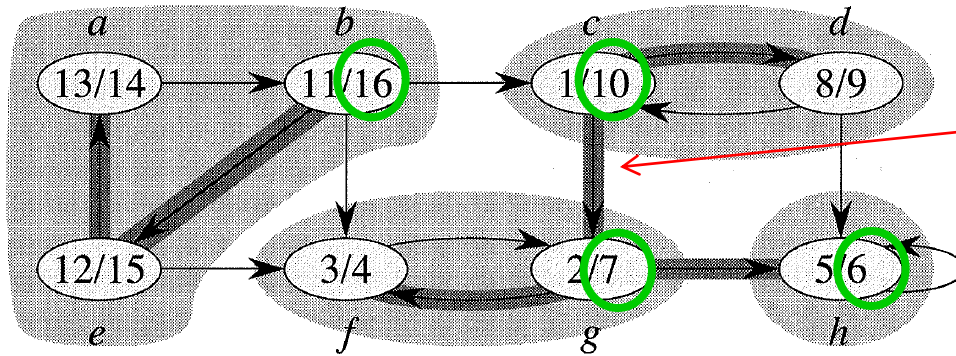
DFS-VISIT(G, u)

```

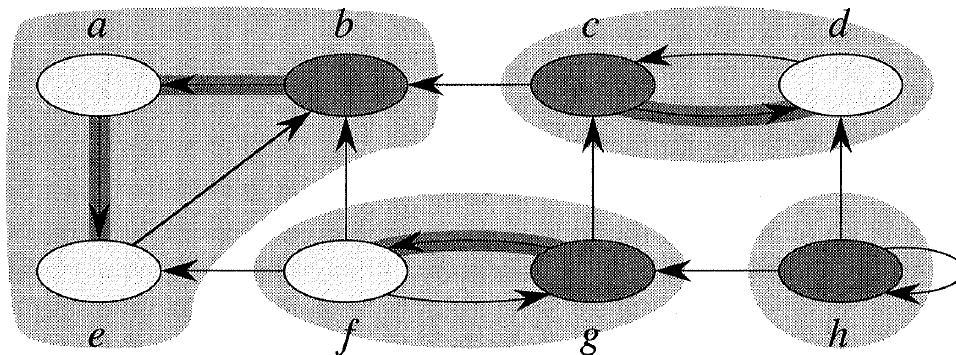
1  $time = time + 1$            // white vertex  $u$  has just been discovered
2  $u.d = time$ 
3  $u.color = GRAY$ 
4 for each  $v \in G.Adj[u]$      // explore edge  $(u, v)$ 
5   if  $v.color == WHITE$ 
6      $v.\pi = u$ 
7     DFS-VISIT( $G, v$ )
8  $u.color = BLACK$          // blacken  $u$ ; it is finished
9  $time = time + 1$ 
10  $u.f = time$ 

```

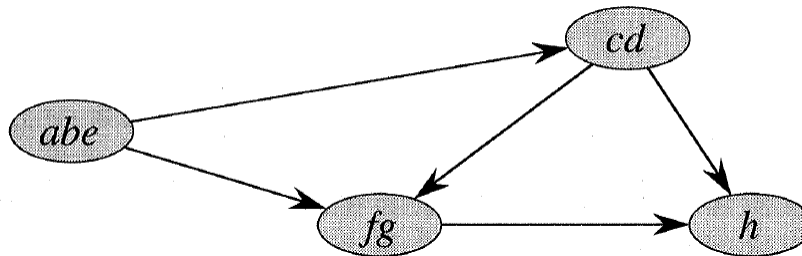
Stærke Sammenhængskomponenter



DFS trækker mellem to stærke sammenhængskomponenter



De største finishing tider i hver komponent udgør en (omvendt) topologisk sortering af komponenterne



Tid $O(m+n)$