Algoritmer og Datastrukturer 2

Gerth Stølting Brodal

Topologisk Sortering, Stærke Sammenhængskomponenter
[CLRS, kapitel 22.4-22.5]

AARHUS UNIVERSITET

Dybde Først Søgning (DFS)

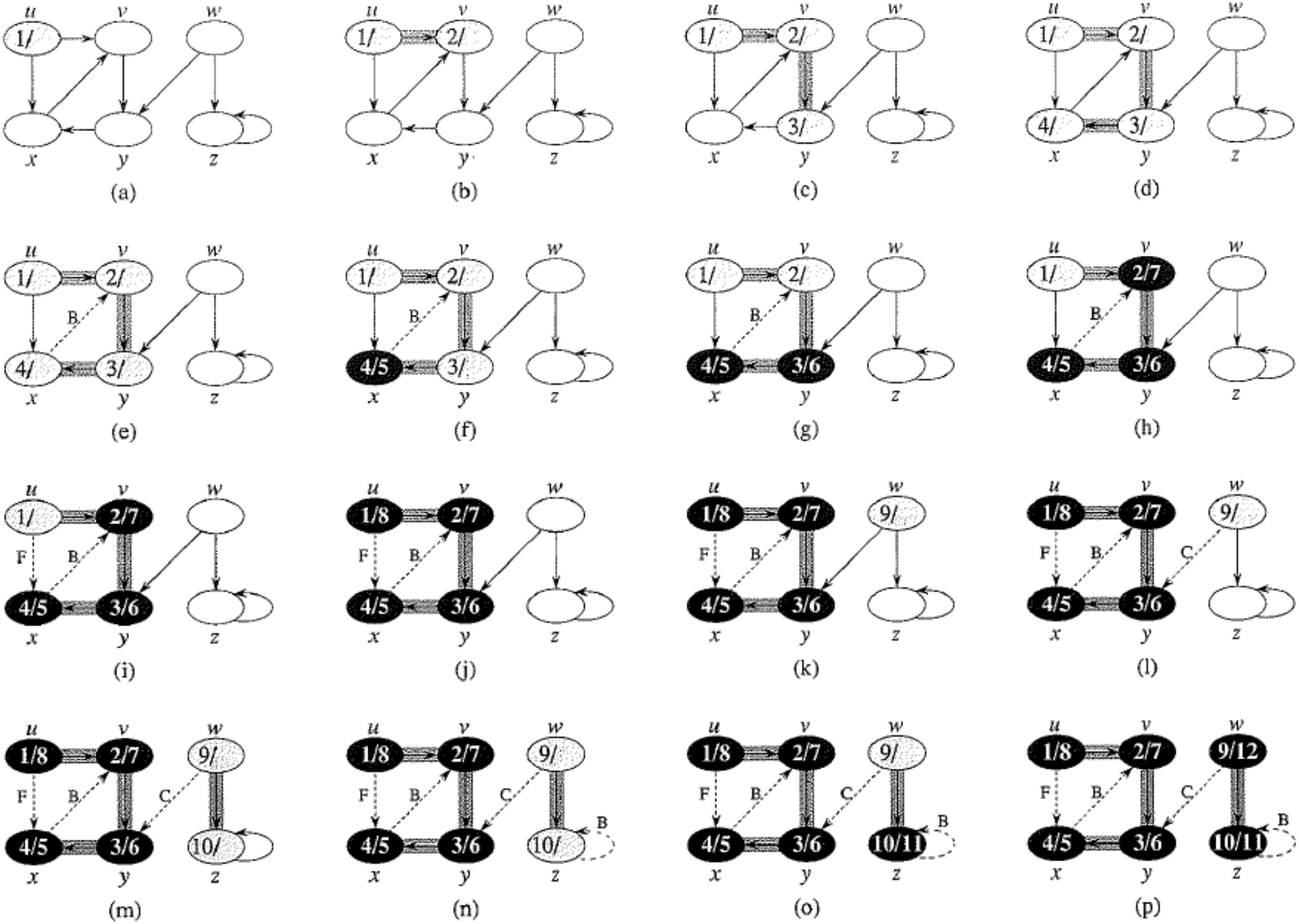
```
DFS(G)
1  for each vertex u ∈ G.V
2      u.color = WHITE
3      u.π = NIL
4  time = 0
5  for each vertex u ∈ G.V
6      if u.color == WHITE
7          DFS-VISIT(G, u)

DFS-VISIT(G, u)
 1  time = time + 1                  // white vertex u has just been discovered
 2  u.d = time
 3  u.color = GRAY
 4  for each v ∈ G.Adj[u]            // explore edge (u, v)
 5      if v.color == WHITE
 6          v.π = u
 7          DFS-VISIT(G, v)
 8  u.color = BLACK                  // blacken u; it is finished
 9  time = time + 1
10  u.f = time
```

u.color
WHITE = knuderne endnu ikke besøgt
GRAY = knuder på rekursionsstakken
BLACK = knuderne besøgt

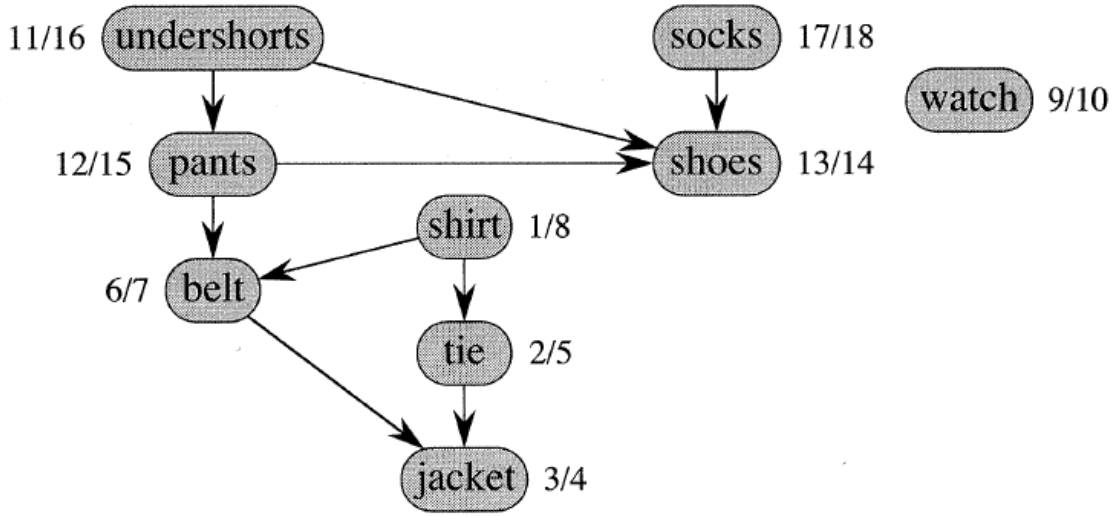
$u.\pi$ = faderen til u i DFS træet

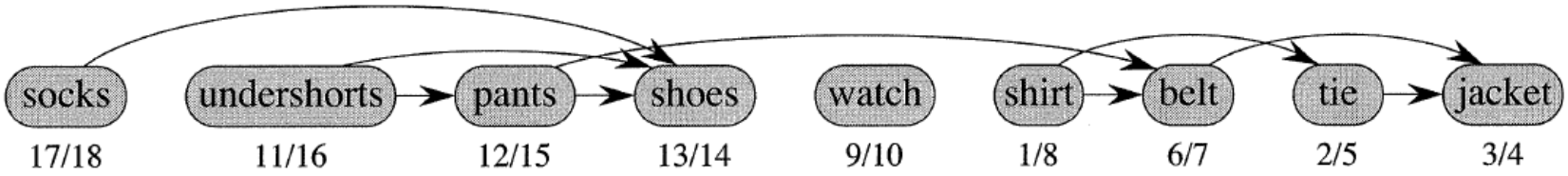
$u.d$ = "discover time" for u
$u.f$ = "finishing time" for u

Tid $O(n+m)$

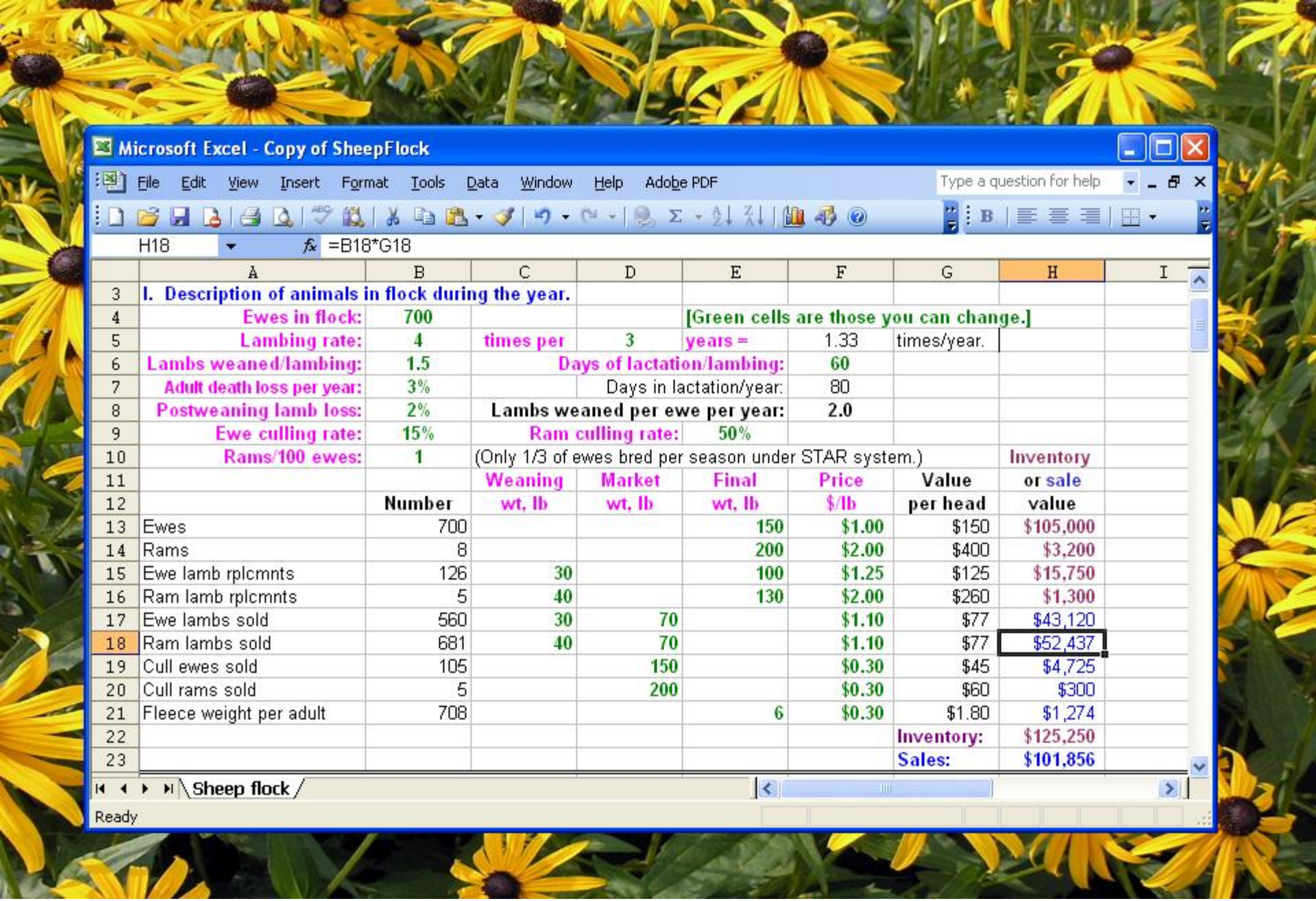

u v w
1/
x y z
(a)
1/ 2/
(b)
1/ 2/ 3/
(c)
1/ 2/ 4/ 3/
(d)
B
1/ 2/ 4/ 3/
(e)
1/ 2/ 4/5 3/
(f)
1/ 2/ 4/5 3/6
(g)
1/ 2/7 4/5 3/6
(h)
F B
1/ 2/7 4/5 3/6
(i)
1/8 2/7 4/5 3/6
(j)
9/
1/8 2/7 4/5 3/6
(k)
C
1/8 2/7 9/ 4/5 3/6
(l)
10/
1/8 2/7 9/ 4/5 3/6
(m)
1/8 2/7 9/ 4/5 3/6 10/
(n)
10/11
1/8 2/7 9/ 4/5 3/6 10/11
(o)
9/12
1/8 2/7 9/12 4/5 3/6 10/11
(p)

Acykliske Grafer: Topologisk Sortering





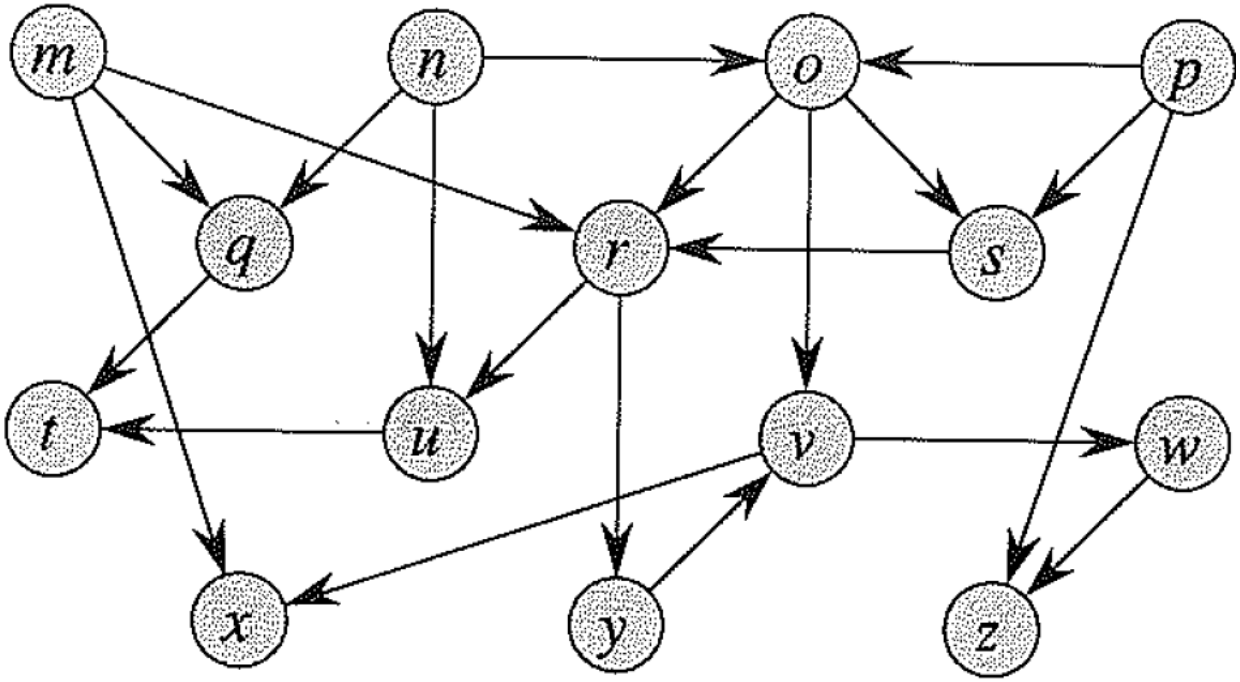
Alle kanter går fra venstre-mod-højre



Topologisk sortering = en rækkefølge hvor vi kan beregne cellernes indhold

Kahn 1962

Topologisk Sortering (I)

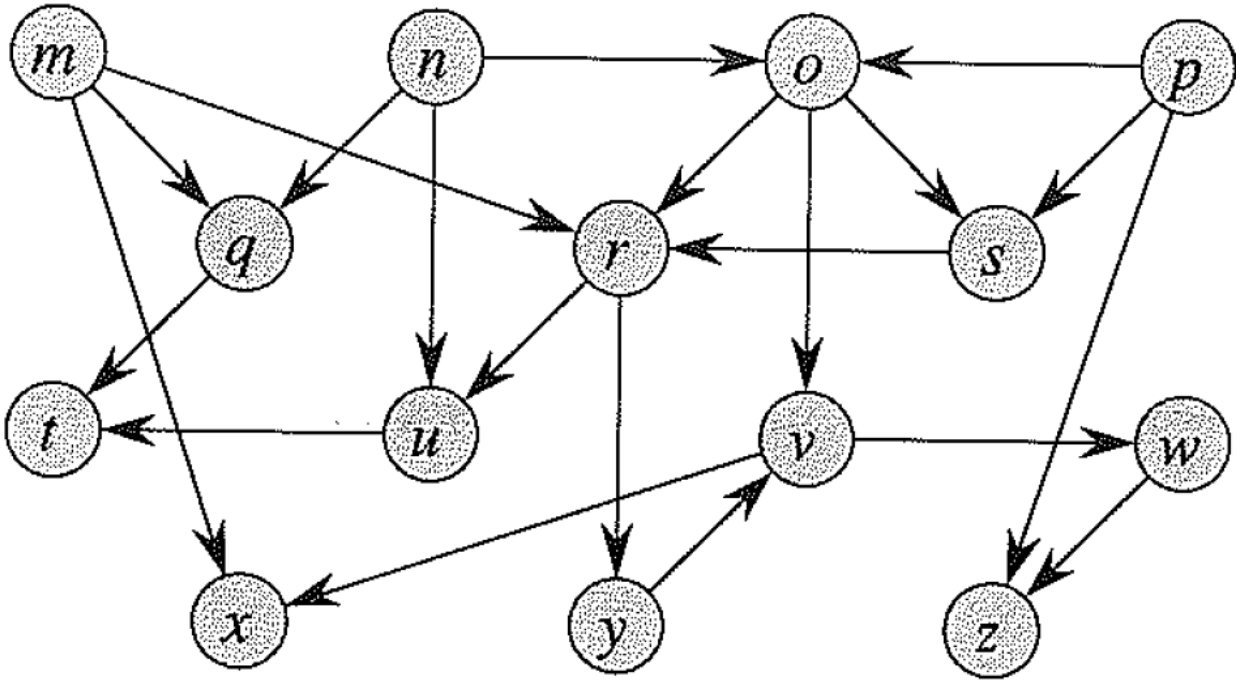


Algoritme: Grådigt slet en knude med indgrad 0 (og udgående kanter), og tilføj knuden sidst i den topologiske orden

Tid $O(m+n)$

Tarjan 1976

Topologisk Sortering (II)



TOPOLOGICAL-SORT(G)

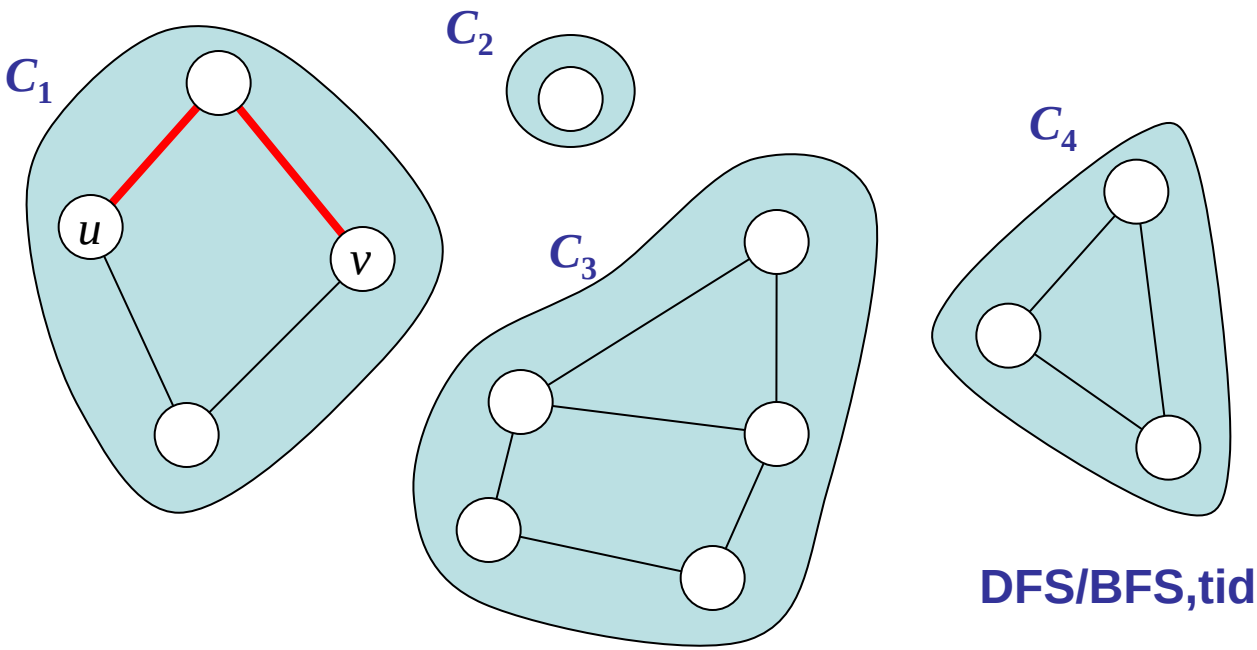
1 call DFS(G) to compute finishing times $v.f$ for each vertex v
2 as each vertex is finished, insert it onto the front of a linked list
3 **return** the linked list of vertices

Tid O(m+n)

Sammenhængskomponenter

Opdeling af knuderne i en uorienteret graf i **komponenter $C_1,\ldots,C_k$**, således at

u og v er i C_i hvis og kun hvis der er en sti mellem u og v



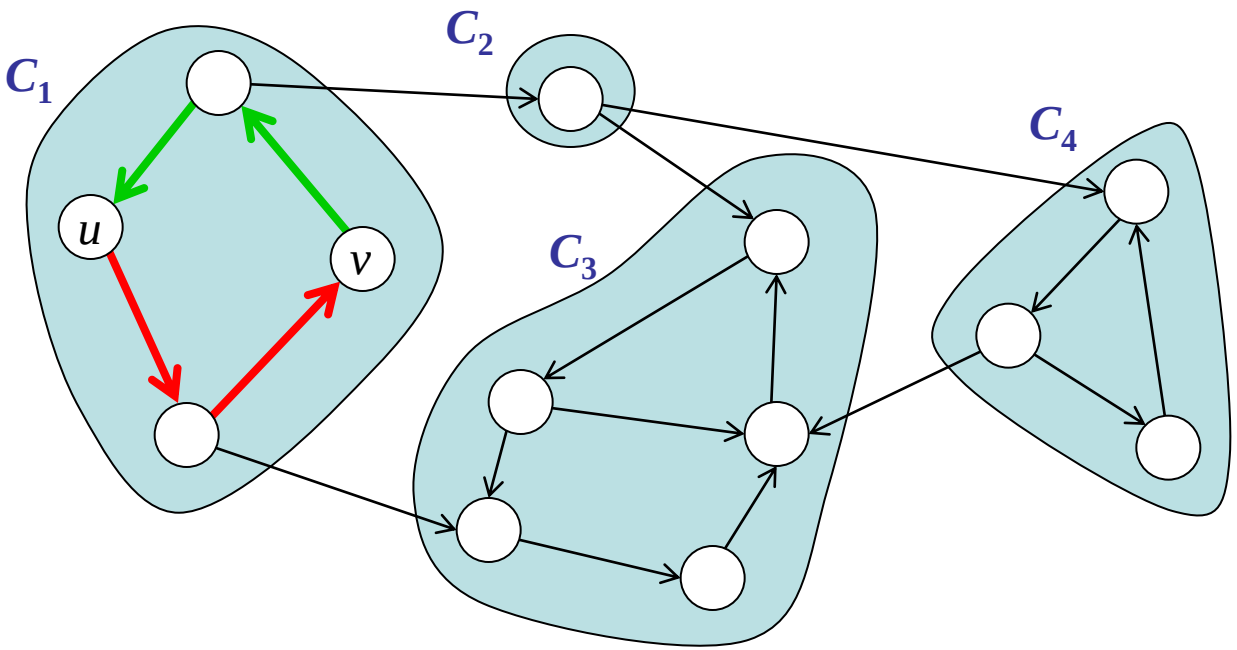
DFS/BFS,tid $O(m+n)$

Stærke Sammenhængskomponenter

Opdeling af knuderne i en orienteret graf i **komponenter $C_1,\ldots,C_k$**, således at

u og v er i C_i hvis og kun hvis der både er

- en sti fra u til v og
- en sti fra v til u



Tarjan 1972

Stærke Sammenhængskomponenter

STRONGLY-CONNECTED-COMPONENTS(G)

1 call DFS(G) to compute finishing times $u.f$ for each vertex u
2 compute G^T
3 call DFS(G^T), but in the main loop of DFS, consider the vertices in order of decreasing $u.f$ (as computed in line 1)
4 output the vertices of each tree in the depth-first forest formed in line 3 as separate strongly connected component

```
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5  for each vertex u ∈ G.V
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DFS-VISIT(G, u)
1  time = time + 1            // white vertex u has just been discovered
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8  u.color = BLACK            // blacken u; it is finished
9  time = time + 1
10 u.f = time
```
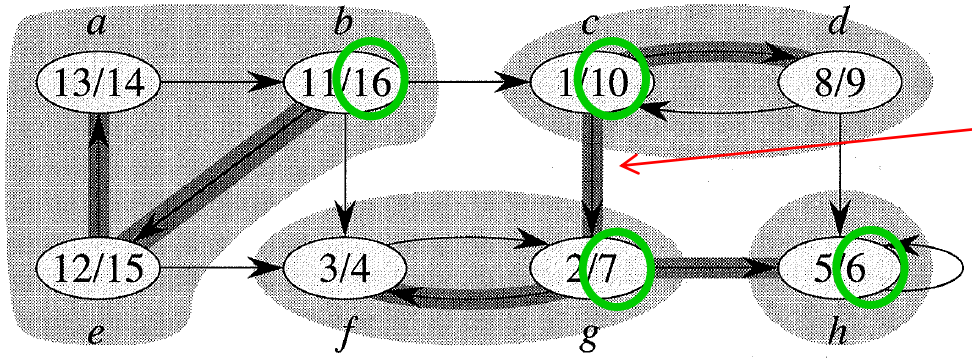
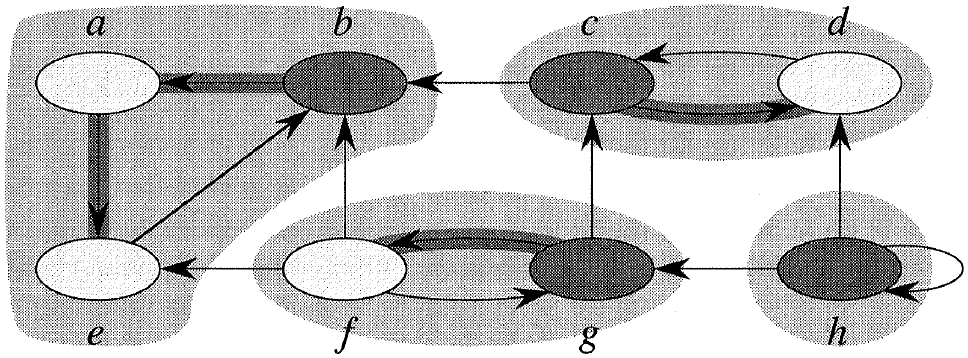

Tid $O(m+n)$

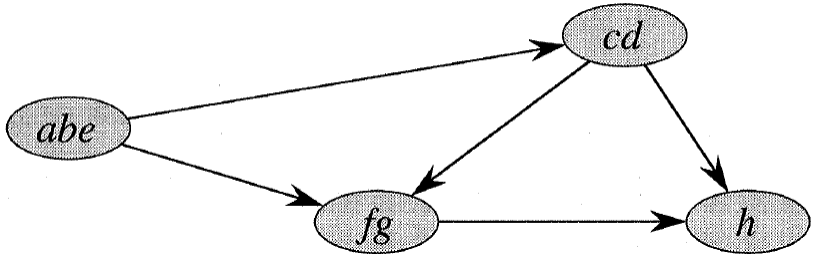
Stærke Sammenhængskomponenter



DFS trækanter mellem to stærke sammenhængs-komponenter



De største finising tider i hver komponent udgør en (omvendt) topologisk sortering af komponenterne



Tid $O(m+n)$