Algoritmer og Datastrukturer 2

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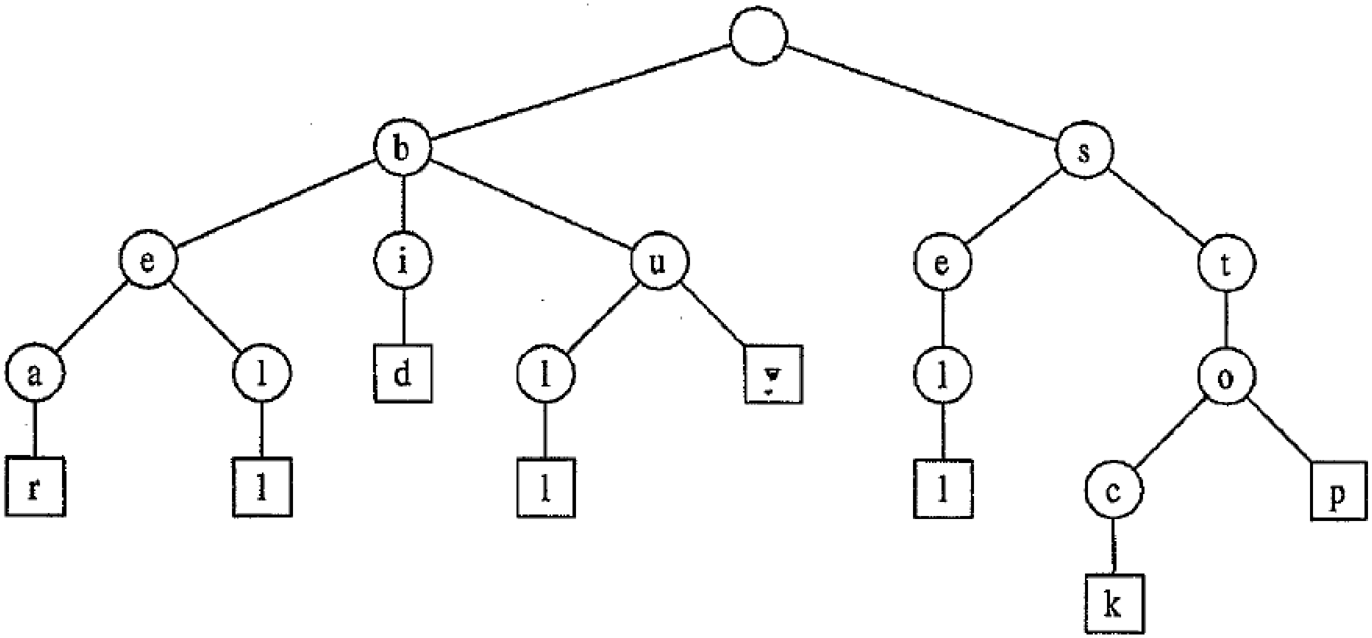
Suffiks træer [GT, kapitel 9.2], Suffiks arrays [Smyth, kapitel 5.3.2]

AARHUS UNIVERSITET

Trier

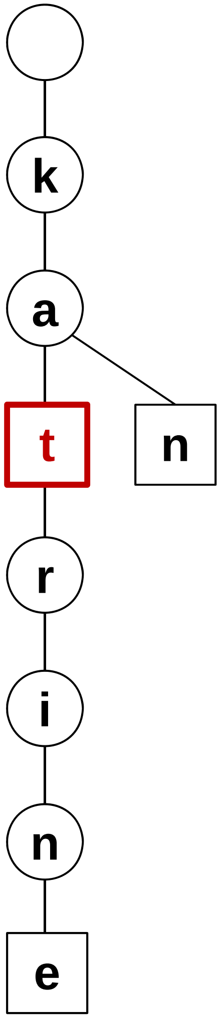
(text re**trie**val)

bear
bell
bid
bull
buy
sell
stock
stop

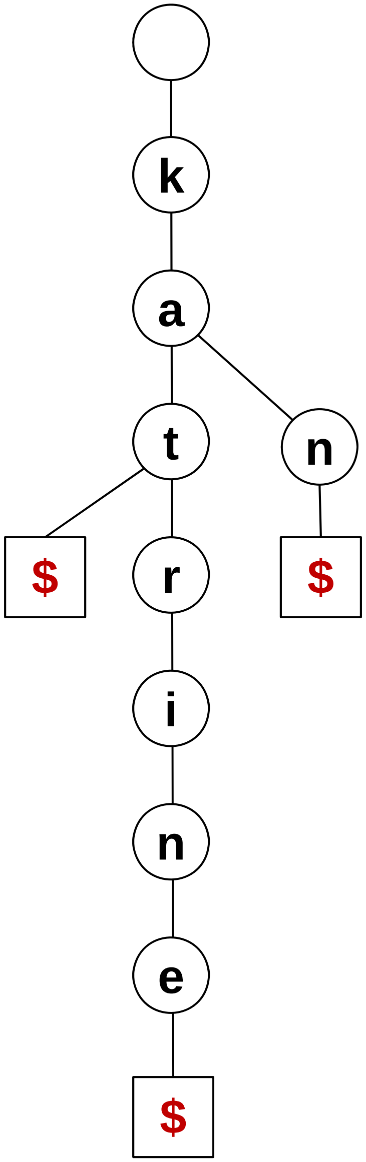


Præfiks-fri ?

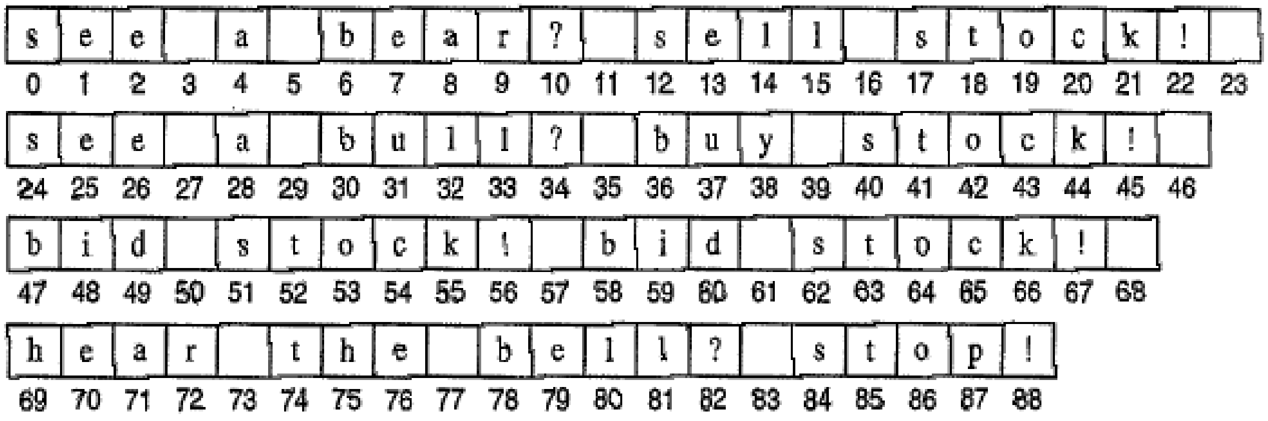
kan
kat
katrine



kan$
kat$
katrine$



Søgning i Streng



s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!	
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

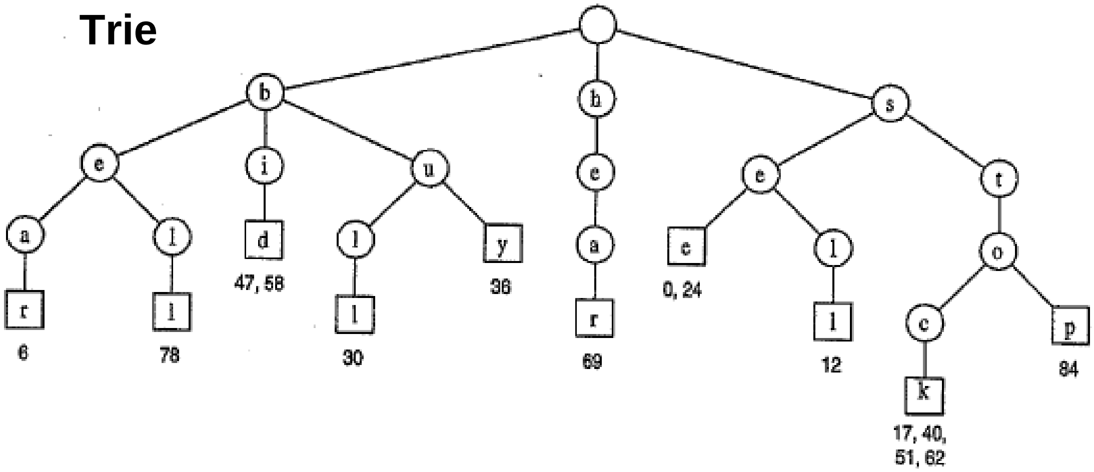
b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!	
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68

h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88

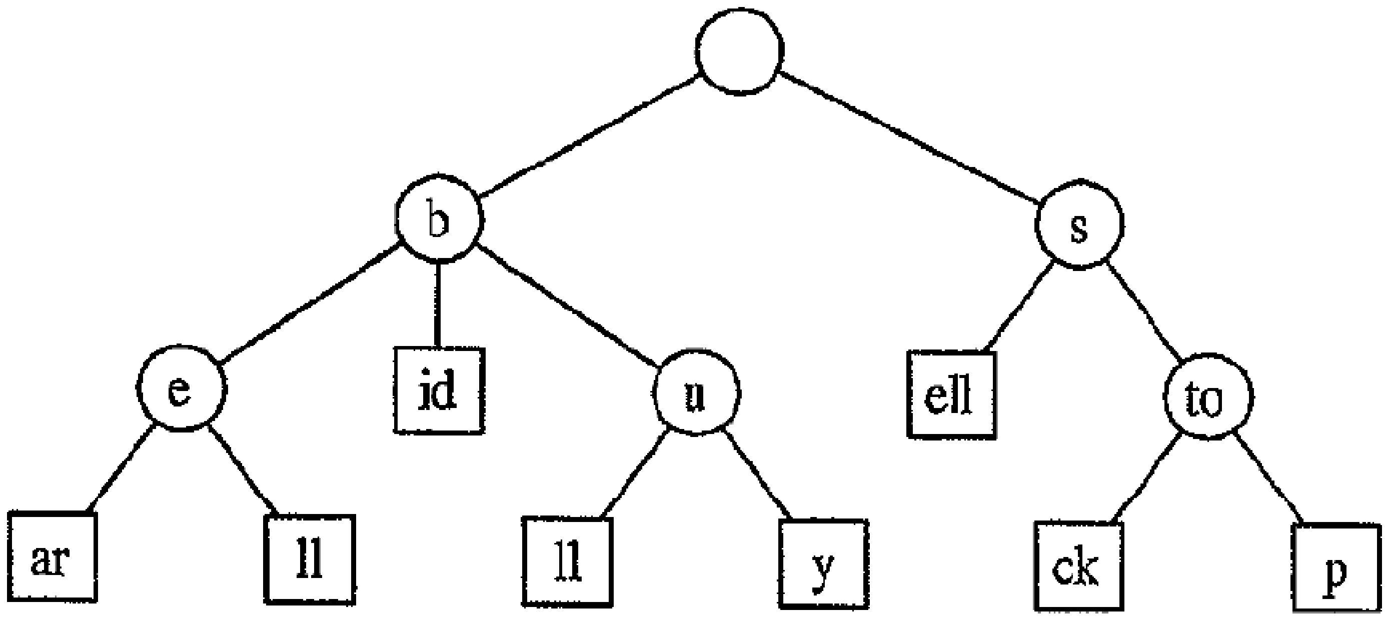
Inverteret fil

Ord	Position(er)
a	4,28
bear	6
bell	78
bid	47,58
bull	30
buy	36
hear	69
see	0,24
sell	12
stock	17,40,51,62
stop	84
the	74

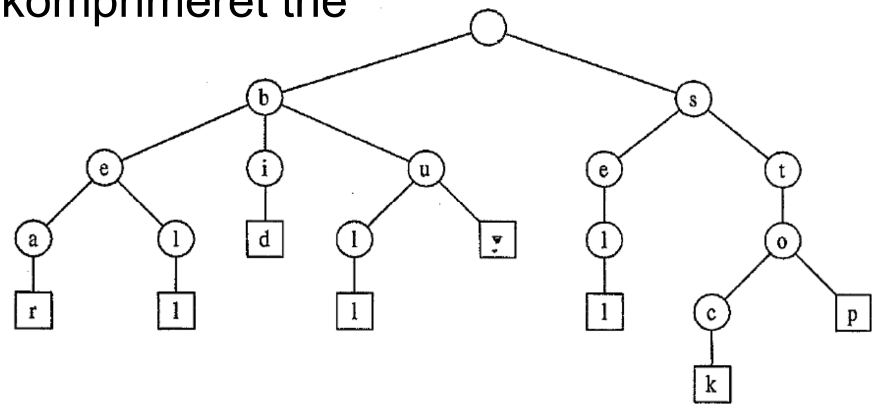
Trie



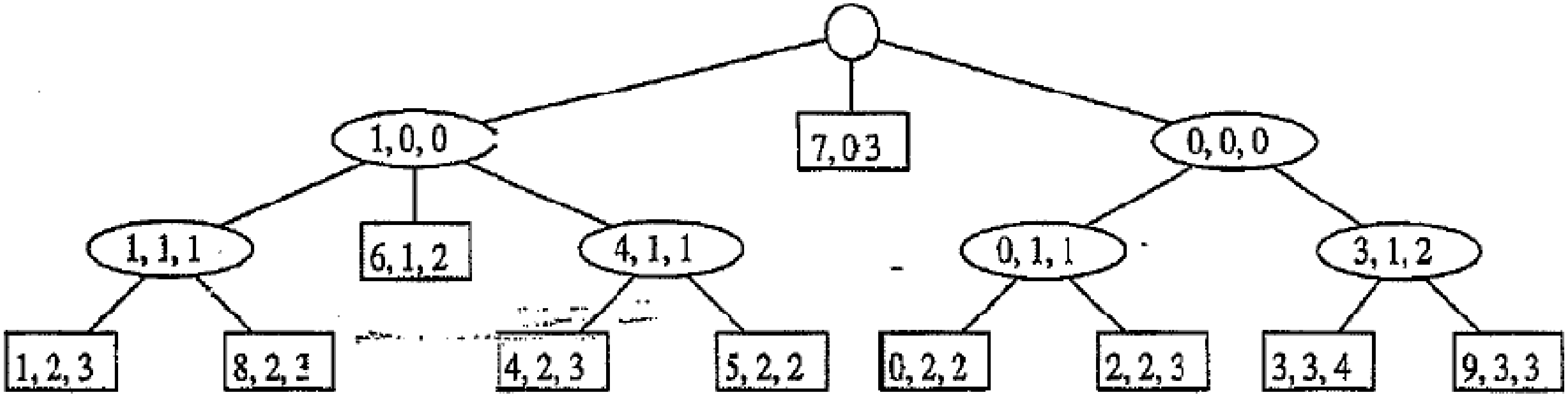
Komprimeret Trie



Ukomprimeret trie



Komprimeret Trie over Ordbog



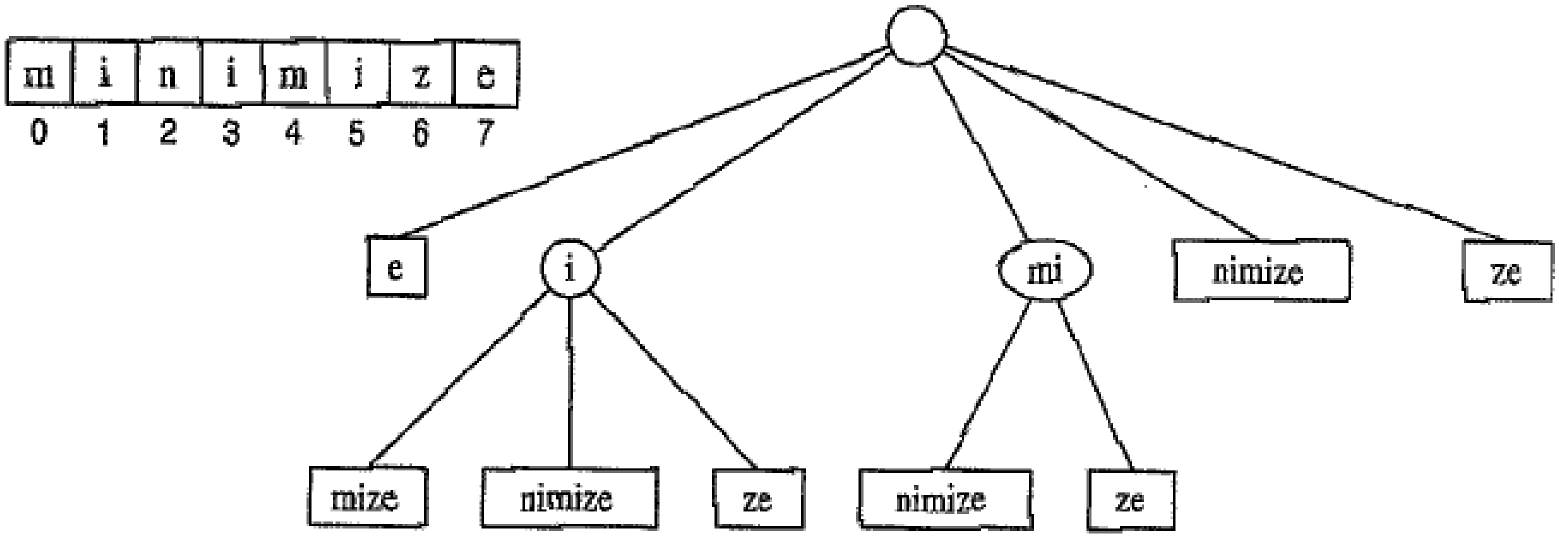
	0	1	2	3	4
$S[0]$ =	s	e	e		
$S[1]$ =	b	e	a	r	
$S[2]$ =	s	e	l	l	
$S[3]$ =	s	t	o	c	k

	0	1	2	3
$S[4]$ =	b	u	l	l
$S[5]$ =	b	u	y	
$S[6]$ =	b	i	d	

	0	1	2	3
$S[7]$ =	h	e	a	r
$S[8]$ =	b	e	l	l
$S[9]$ =	s	t	o	p

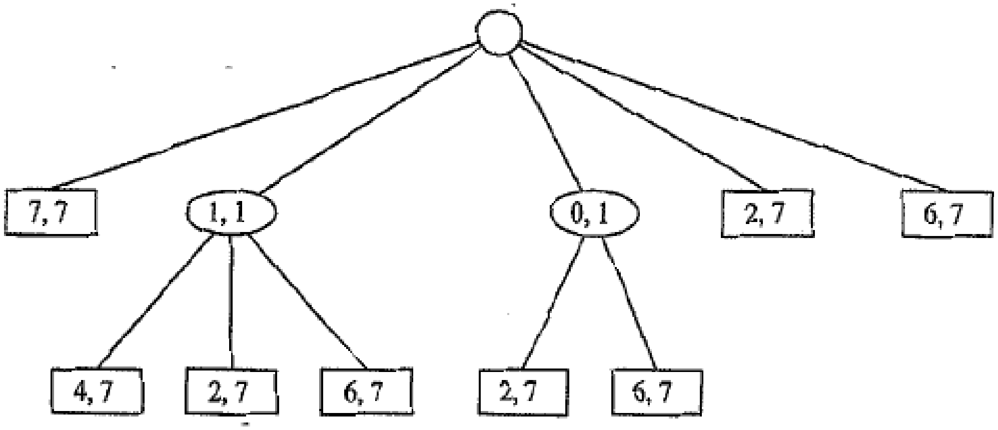
(i,j,k) er delstrengen $S[i][j..k]$

Suffiks Træer



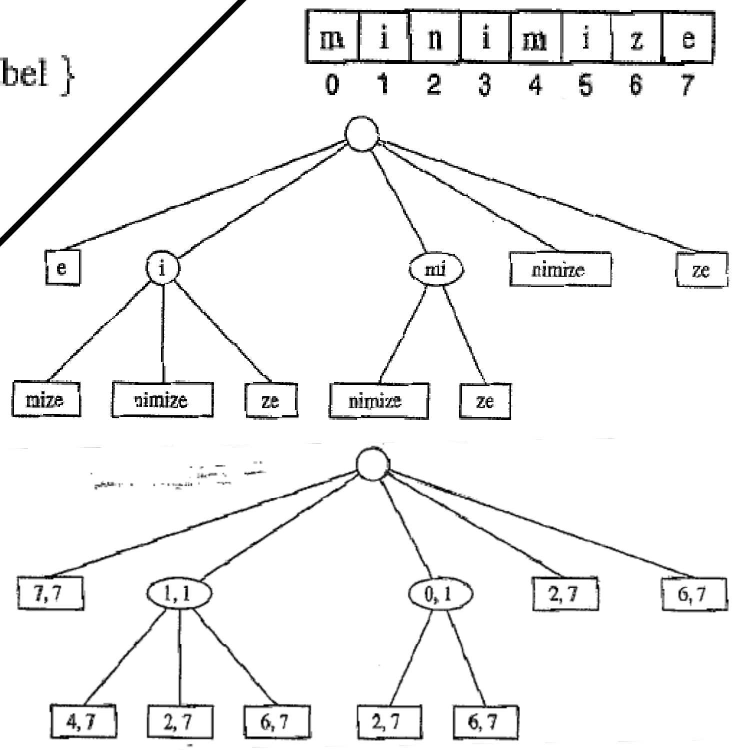
Suffix træ =
komprimeret trie
over suffixer

Plads O(*n*)



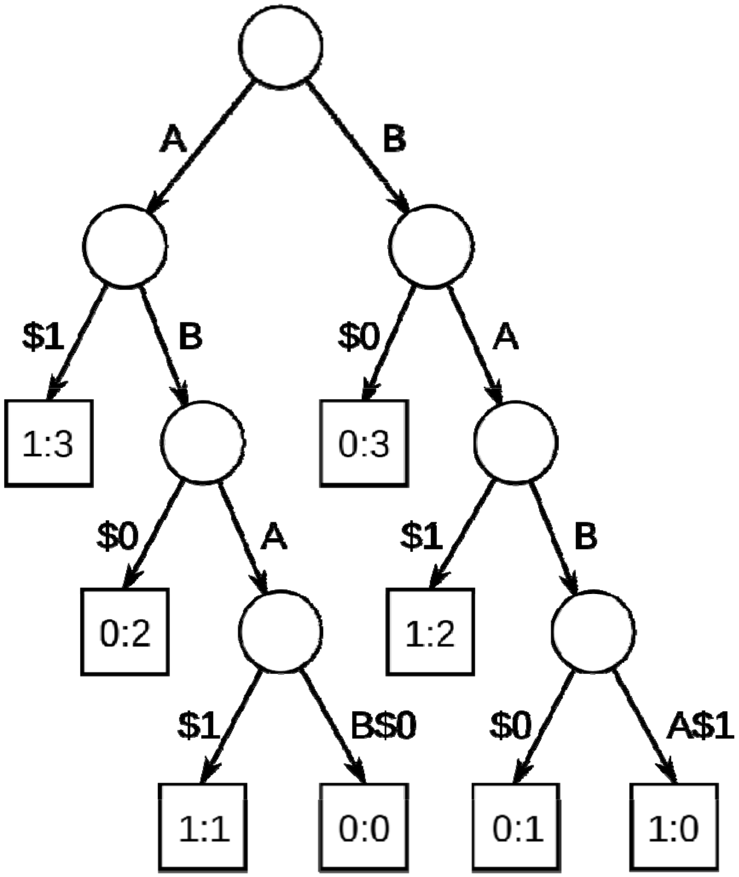
Kan konstrueres i O(*n*) tid (for konstant størrelse alfabet)

```
Algorithm suffixTrieMatch($T$,$P$):
  Input: Compact suffix trie $T$ for a text $X$ and pattern $P$
  Output: Starting index of a substring of $X$ matching $P$ or an indication that $P$
    is not a substring of $X$
  $p \leftarrow P$.length() { length of suffix of the pattern to be matched }
  $j \leftarrow 0$ { start of suffix of the pattern to be matched }
  $v \leftarrow T$.root()
  repeat
    $f \leftarrow$ true { flag indicating that no child was successfully processed }
    for each child $w$ of $v$ do
      $i \leftarrow$ start($w$)
      if $P[j] = X[i]$ then
        { process child $w$ }
        $x \leftarrow$ end($w$) $- i + 1$
        if $p \leq x$ then
          { suffix is shorter than or of the same length of the node label }
          if $P[j..j+p-1] = X[i..i+p-1]$ then
            return $i - j$ { match }
          else
            return "$P$ is not a substring of $X$"
        else
          { suffix is longer than the node label }
          if $P[j..j+x-1] = X[i..i+x-1]$ then
            $p \leftarrow p - x$ { update suffix length }
            $j \leftarrow j + x$ { update suffix start index }
            $v \leftarrow w$
            $f \leftarrow$ false
            break out of the for loop
  until $f$ or $T$.isExternal($v$)
  return "$P$ is not a substring of $X$"
```



Suffiks Træ over to Strenge

ABAB$\$_0$BABA$\$_1$



wikipedia.org

Suffiks Array

1 2 3 4 5 6 7 8
tekst = *a b a a b a a b*

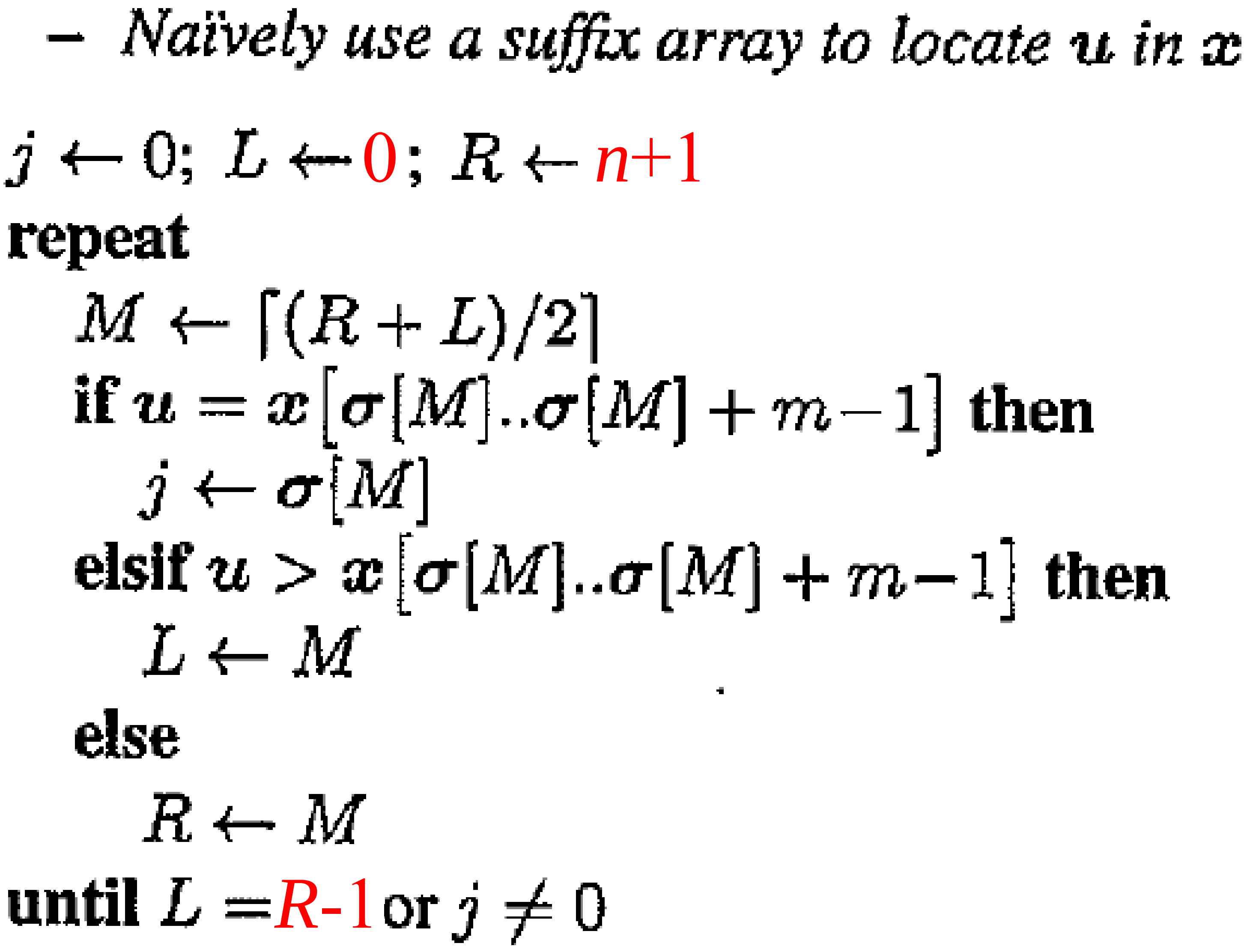
Suffikser	
1	*a b a a b a a b*
2	*b a a b a a b*
3	*a a b a a b*
4	*a b a a b*
5	*b a a b*
6	*a a b*
7	*a b*
8	*b*

Sorterede suffikser	
6	*a a b*
3	*a a b a a b*
7	*a b*
4	*a b a a b*
1	*a b a a b a a b*
8	*b*
5	*b a a b*
2	*b a a b a a b*

Suffix array

σ	6	3	7	4	1	8	5	2

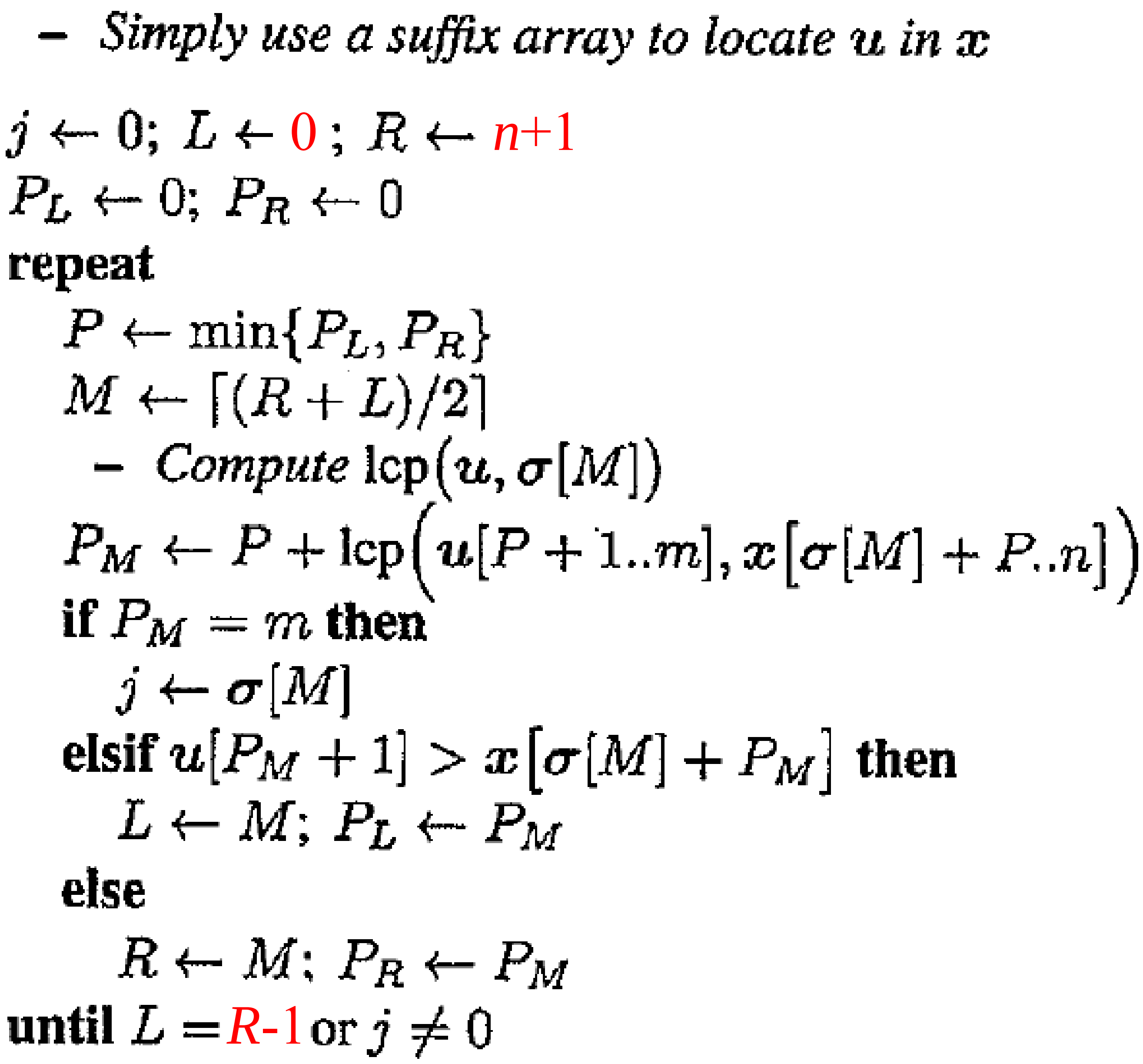
Algorithm SANaïve [Smyth, s.151]



– *Naïvely use a suffix array to locate u in x*

```
$j \leftarrow 0$; $L \leftarrow 0$; $R \leftarrow n+1$
repeat
    $M \leftarrow \lceil (R+L)/2 \rceil$
    if $u = x[\sigma[M]..\sigma[M]+m-1]$ then
        $j \leftarrow \sigma[M]$
    elsif $u > x[\sigma[M]..\sigma[M]+m-1]$ then
        $L \leftarrow M$
    else
        $R \leftarrow M$
until $L = R-1$ or $j \neq 0$
```

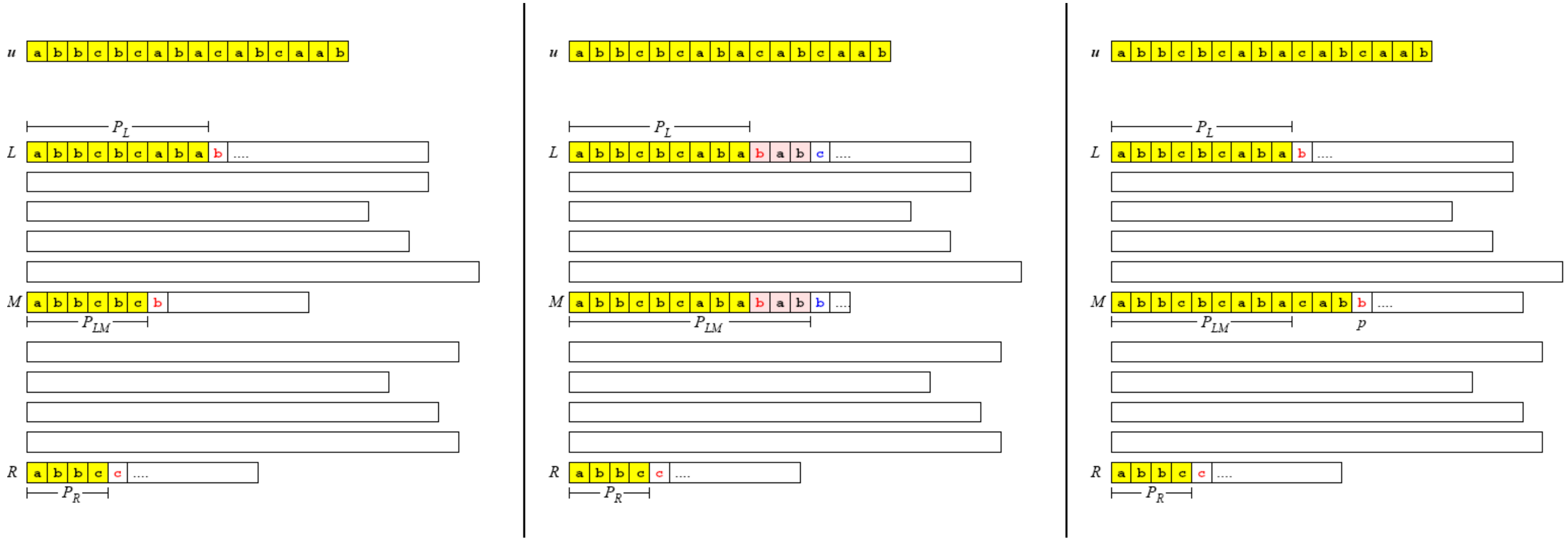
Algorithm SASimple [Smyth, s.151]



– *Simply use a suffix array to locate u in x*

```
$j \leftarrow 0$; $L \leftarrow 0$ ; $R \leftarrow n+1$
$P_L \leftarrow 0$; $P_R \leftarrow 0$
repeat
    $P \leftarrow \min\{P_L, P_R\}$
    $M \leftarrow \lceil (R+L)/2 \rceil$
     – Compute $\mathrm{lcp}(u, \sigma[M])$
    $P_M \leftarrow P + \mathrm{lcp}\Big(u[P+1..m], x\big[\sigma[M]+P..n\big]\Big)$
    if $P_M = m$ then
        $j \leftarrow \sigma[M]$
    elsif $u[P_M+1] > x\big[\sigma[M]+P_M\big]$ then
        $L \leftarrow M$; $P_L \leftarrow P_M$
    else
        $R \leftarrow M$; $P_R \leftarrow P_M$
until $L = R-1$ or $j \neq 0$
```

SAComplex ($P_L \geq P_R$)


u a b b c b c a b a c a b c a a b
P_L
L a b b c b c a b a b ....
M a b b c b c b
P_{LM}
R a b b c c ....
P_R
u a b b c b c a b a c a b c a a b
P_L
L a b b c b c a b a b a b c ....
M a b b c b c a b a b a b b ....
P_{LM}
R a b b c c ....
P_R
u a b b c b c a b a c a b c a a b
P_L
L a b b c b c a b a b ....
M a b b c b c a b a c a b b ....
P_{LM}
p
R a b b c c ....
P_R

$P_{LM} < P_L$: $R \leftarrow M$, $P_R \leftarrow P_{LM}$

$P_{LM} > P_L$: $L \leftarrow M$

$P_{LM} = P_L$: **Start sammenligning på position** $P_{LM}+1$

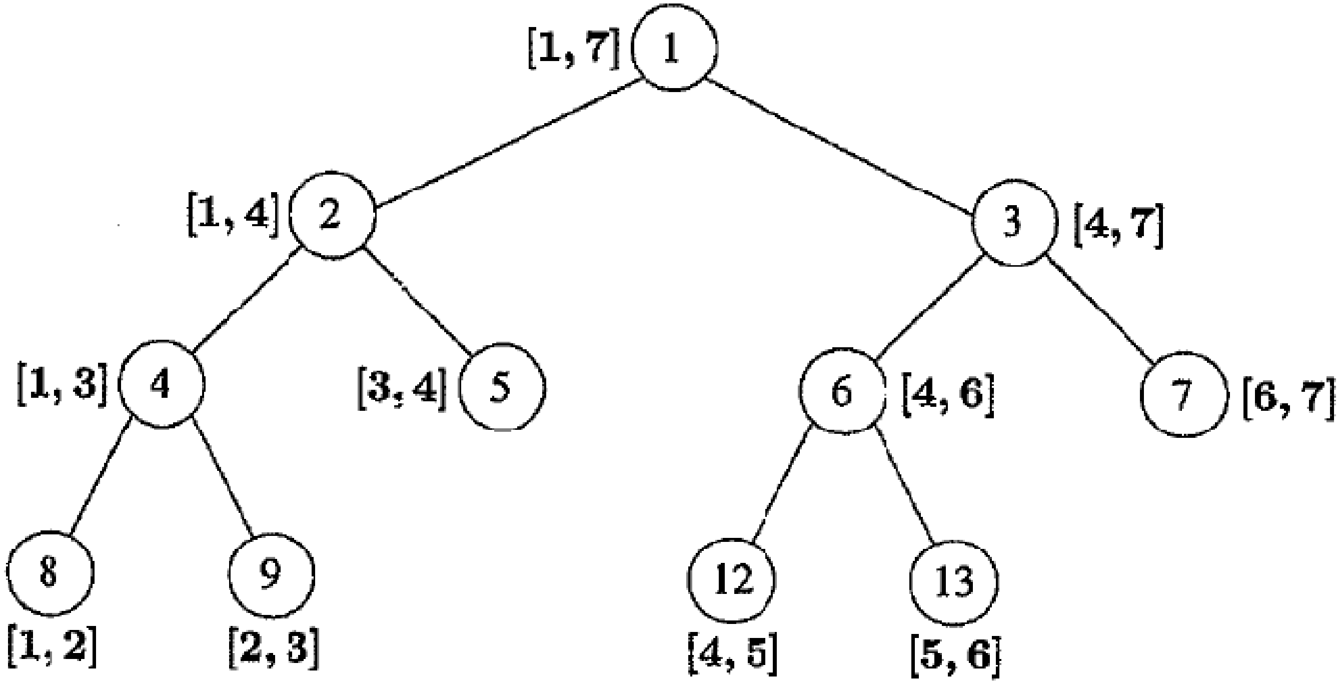
Lad p **være første forskellige position:**

$u[p] < \sigma[M][p]$: $R \leftarrow M$, $P_R \leftarrow p\text{-}1$

$u[p] > \sigma[M][p]$: $L \leftarrow M$, $P_L \leftarrow p\text{-}1$

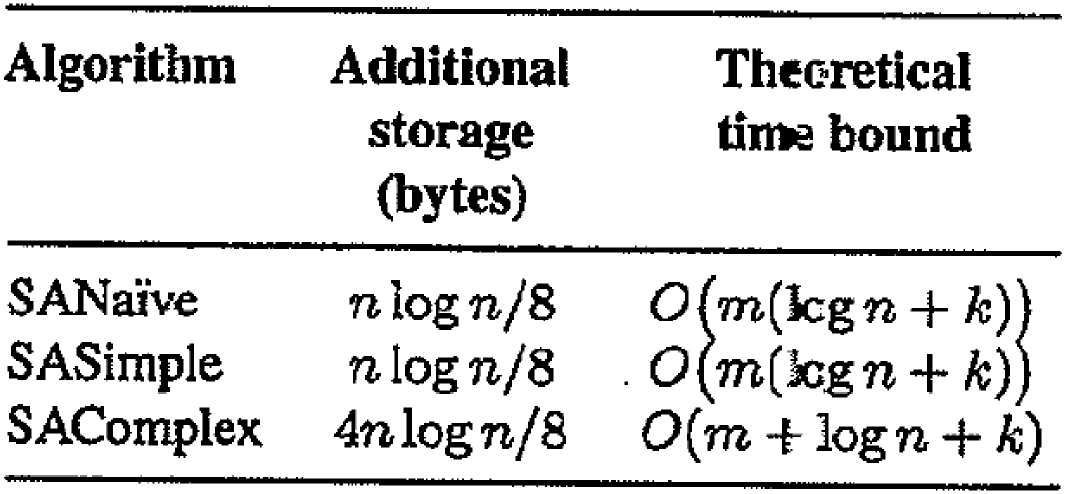
præberegnet

Binært træ over intervaller



Mihai Pătraşcu

Søgninger i et Suffiks Array



Algorithm	Additional storage (bytes)	Theoretical time bound
SANaïve	$n \log n/8$	$O(m(\log n + k))$
SASimple	$n \log n/8$	$O(m(\log n + k))$
SAComplex	$4n \log n/8$	$O(m + \log n + k)$

n = tekst længde, m = mønster længde, k = antal forekomster