

Algoritmer og Datastrukturer 2

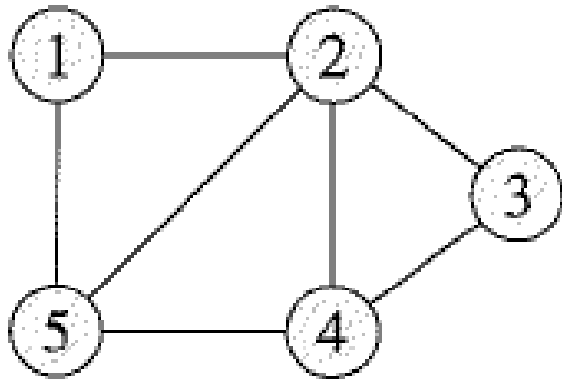
Gerth Stølting Brodal

Graf repræsentationer, BFS og DFS
[CLRS, kapitel 22.1-22.3]

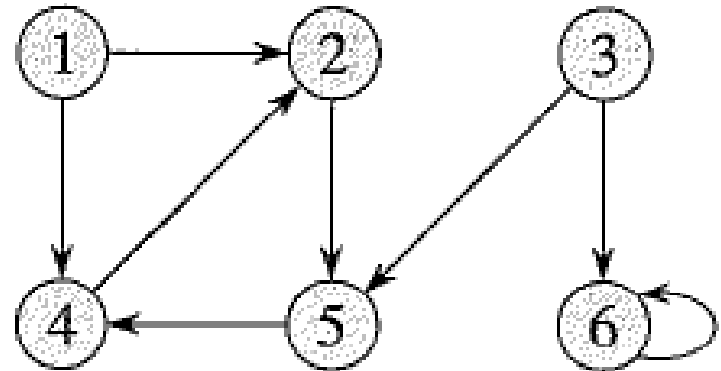


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Grafer



Uorienterede grafer



Orienterede grafer

$G = (V, E)$ graf med knuder V og kanter E

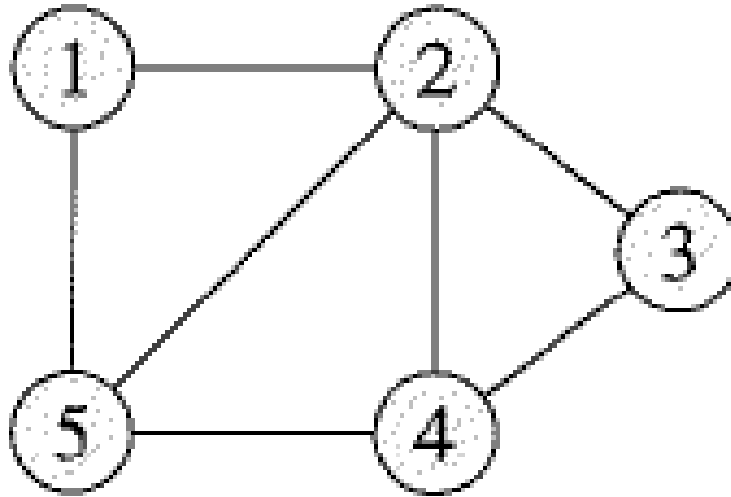
$E : \{u, v\}$ kant mellem u og v i en uorienteret graf og

(u, v) en orienteret kant fra u til v .

$n = |V| =$ antal knuder

$m = |E| =$ antal kanter (forbindelser mellem knuder)

Planar Grafer - Eulers formel



$$V = 5$$

$$E = 7$$

$$\# \text{ flader} = 4$$

For en sammenhengende planar graf gælder:

Eulers formel: $|V| - |E| + \# \text{ flader} = 2$

Korollar:

$$|E| \leq 3|V| - 6$$

(for $|V| \geq 3$, ingen selvløkker, ingen parallelle kanter)

Hvilken løsning finder den grådige algoritme?

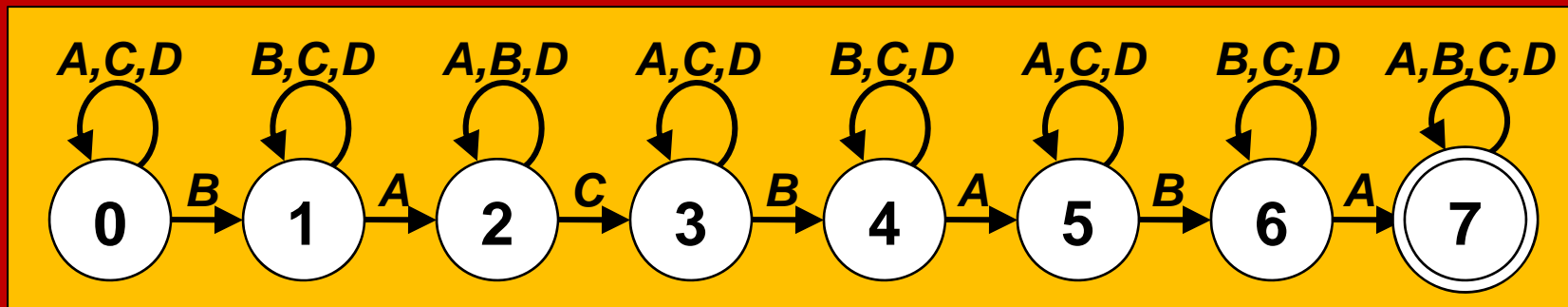
a) ABABGACBABAD



b) ABABGACBABAD

c) ABABGACBABAD

d) Ved ikke



Microsoft Excel - Copy of SheepFlock

File Edit View Insert Format Tools Data Window Help Adobe PDF Type a question for help

H18 =B18*G18

	A	B	C	D	E	F	G	H	I
3	I. Description of animals in flock during the year.								
4	Ewes in flock:	700			[Green cells are those you can change.]				
5	Lambing rate:	4	times per	3	years =	1.33	times/year.		
6	Lambs weaned/lambing:	1.5	Days of lactation/lambing:		60				
7	Adult death loss per year:	3%	Days in lactation/year:		80				
8	Postweaning lamb loss:	2%	Lambs weaned per ewe per year:		2.0				
9	Ewe culling rate:	15%	Ram culling rate:		50%				
10	Rams/100 ewes:	1	(Only 1/3 of ewes bred per season under STAR system.)						Inventory
11			Weaning	Market	Final	Price	Value	or sale	
12		Number	wt, lb	wt, lb	wt, lb	\$/lb	per head	value	
13	Ewes	700			150	\$1.00	\$150	\$105,000	
14	Rams	8			200	\$2.00	\$400	\$3,200	
15	Ewe lamb rplcmnts	126	30		100	\$1.25	\$125	\$15,750	
16	Ram lamb rplcmnts	5	40		130	\$2.00	\$260	\$1,300	
17	Ewe lambs sold	560	30	70		\$1.10	\$77	\$43,120	
18	Ram lambs sold	681	40	70		\$1.10	\$77	\$52,437	
19	Cull ewes sold	105		150		\$0.30	\$45	\$4,725	
20	Cull rams sold	5		200		\$0.30	\$60	\$300	
21	Fleece weight per adult	708			6	\$0.30	\$1.80	\$1,274	
22							Inventory:	\$125,250	
23							Sales:	\$101,856	

Sheep flock /

Ready

Hvilken beregningsrækkefølge ?

	A	B	C
1	10	20	=A1+B1
2	50	30	=A2+B2
3	=(A1+A2)/C3	=(B1+B2)/C3	=C1+C2

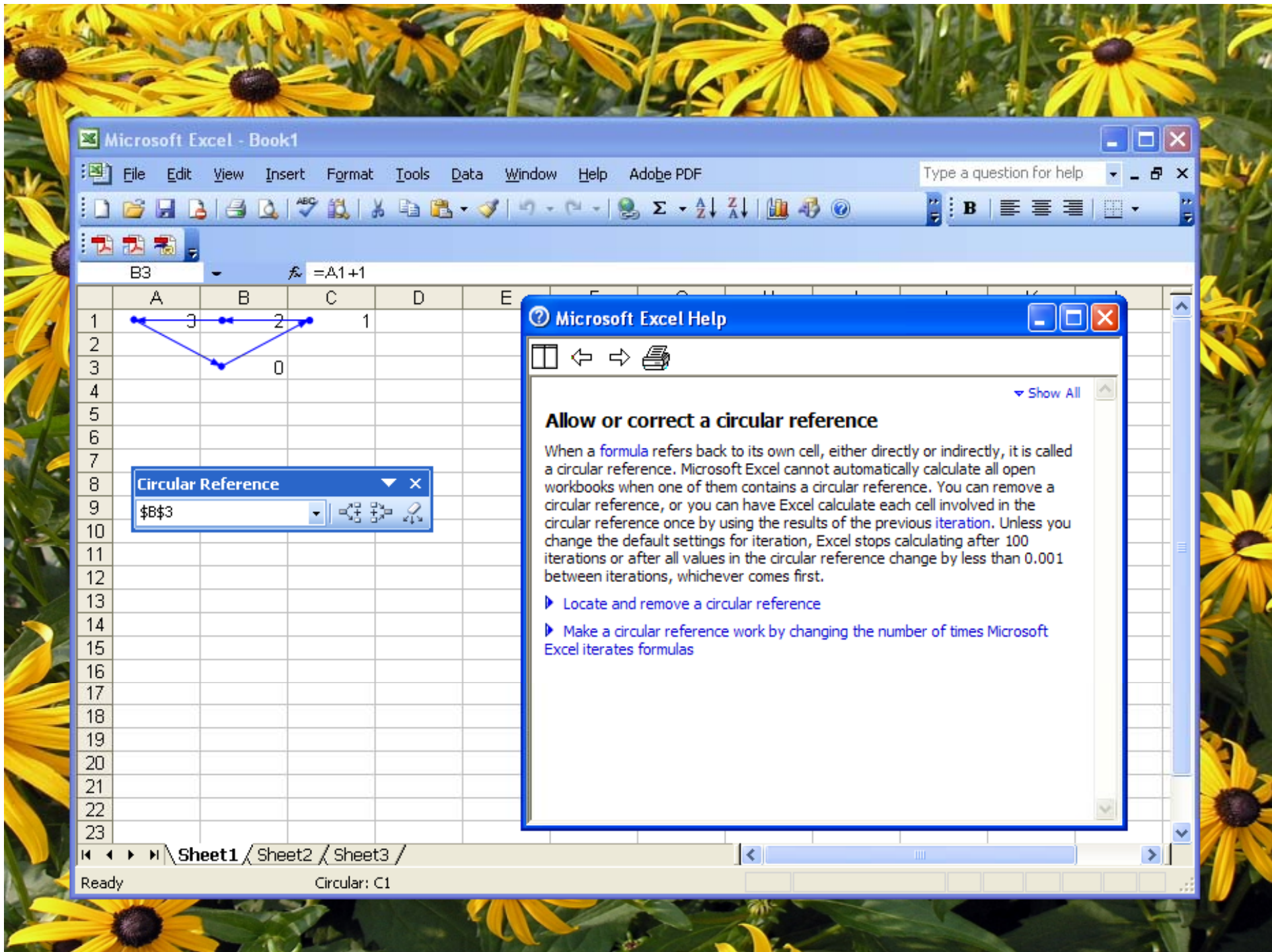
- a) C1 C2 A3 B3 C3
- b) A3 B3 C2 C1 C3
- c) C2 C1 C3 B3 A3
- d) Ved ikke

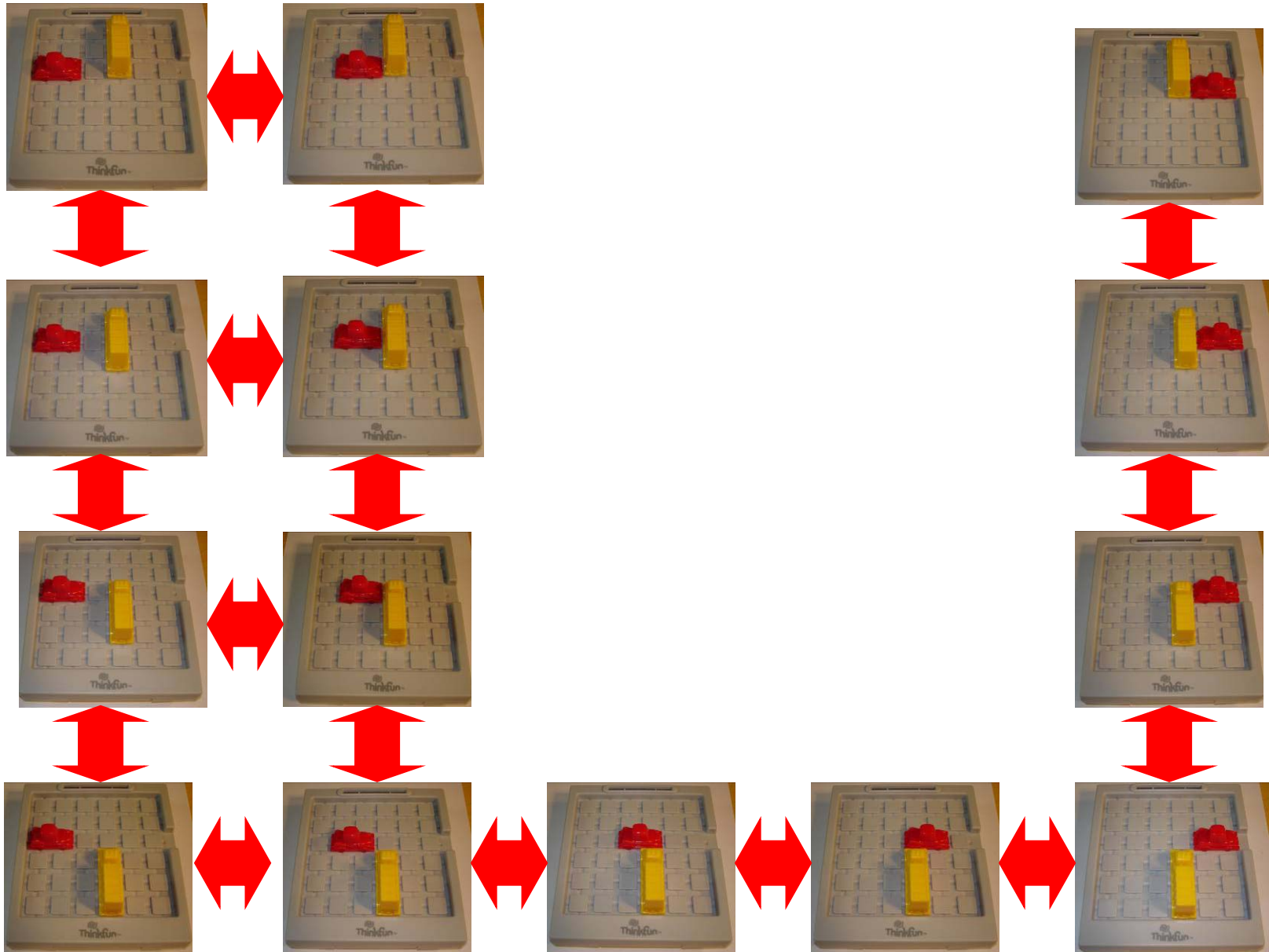
Hvilken beregningsrækkefølge ?

	A	B	C
1	10	20	=A1+B1
2	50	30	=A2+B2
3	=(A1+A2)/C3	=(B1+B2)/C3	=C1+C2



- a) C1 C2 A3 B3 C3
- b) A3 B3 C2 C1 C3
- c) C2 C1 C3 B3 A3
- d) Ved ikke





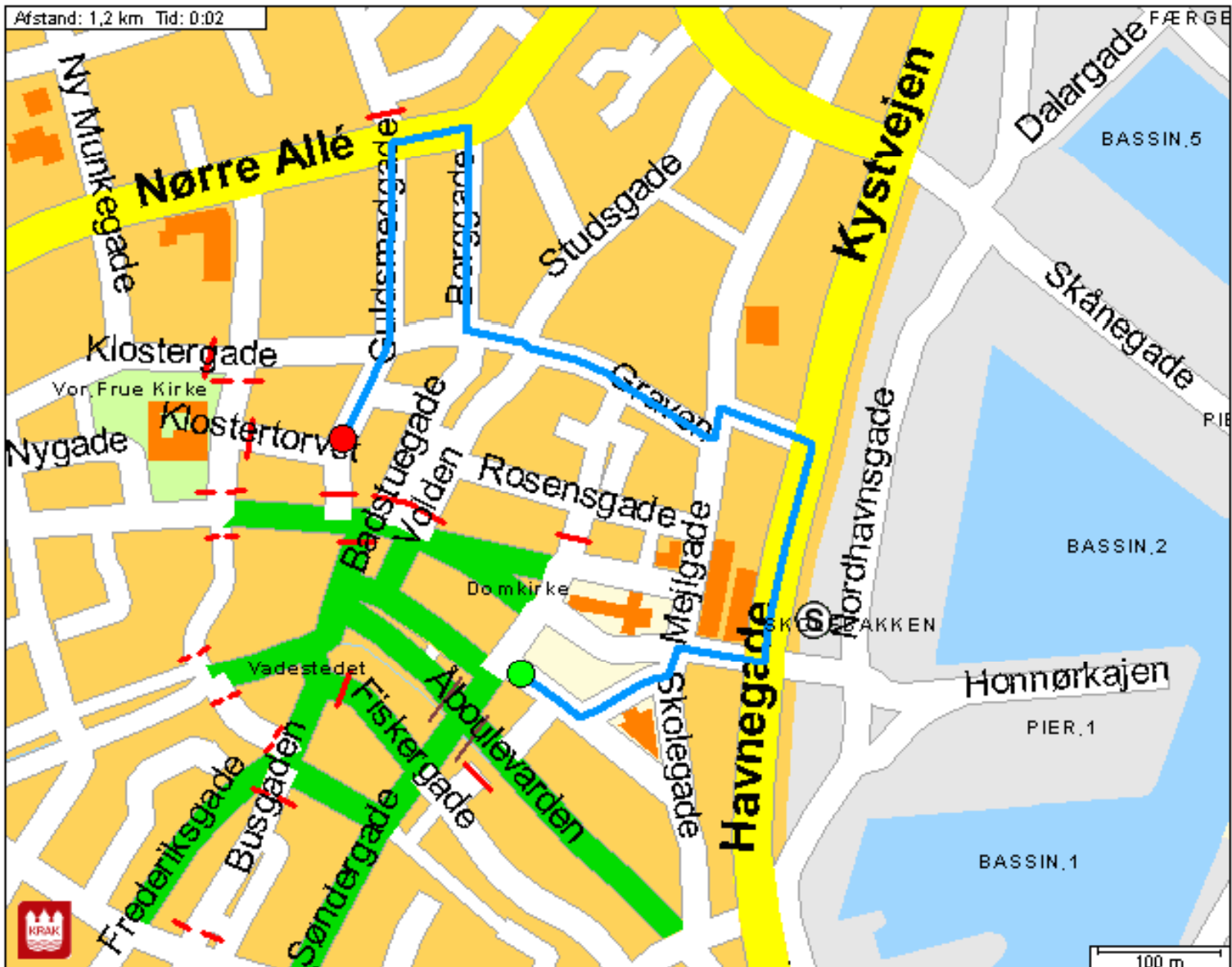


Rute på kort

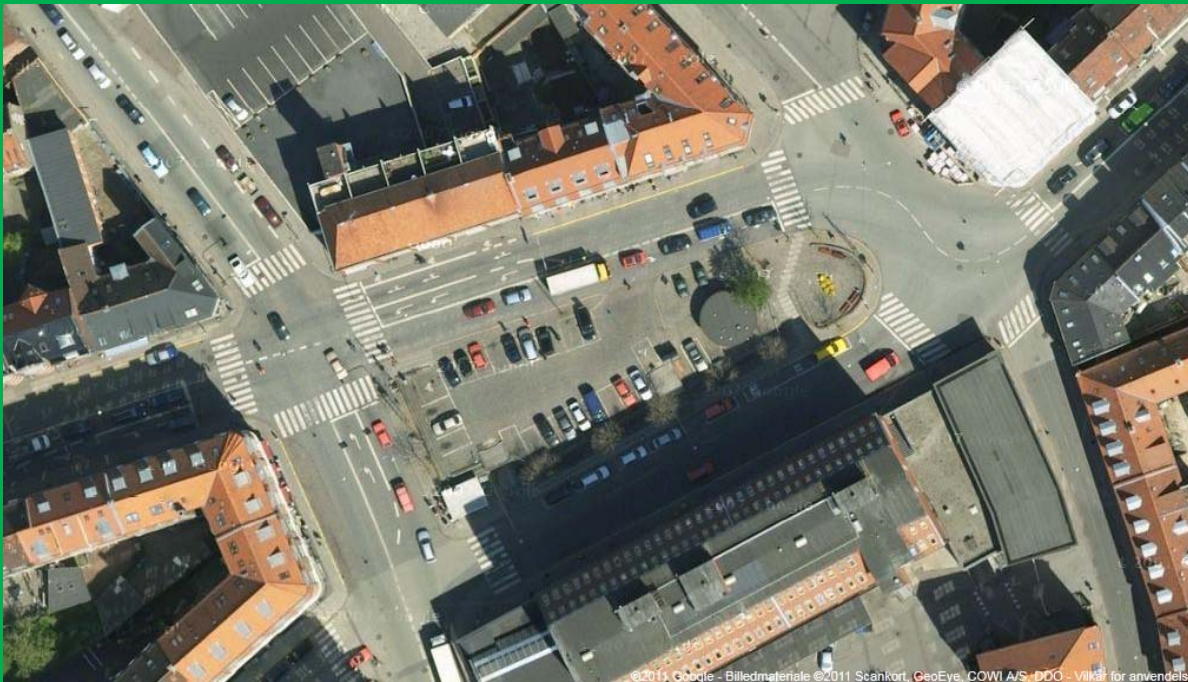
Fra Kannikegade 1 , 8000 Århus C

Til Guldsmedgade 1 , 8000 Århus C

Via



Hvor mange knuder skal man bruge for at repræsentere et vejkryds?



- a) 1
- b) 2
- c) 4
- d) 5
- e) 8
- f) 9
- g) 12
- h) Ved ikke

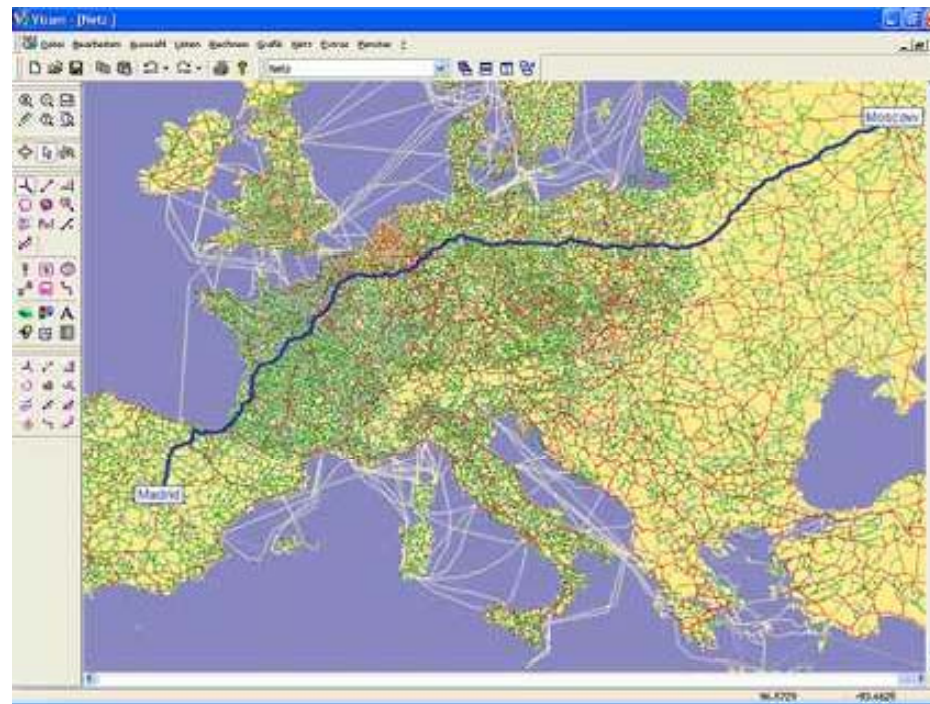
Hvor mange knuder skal man bruge for at repræsentere et vejkryds?

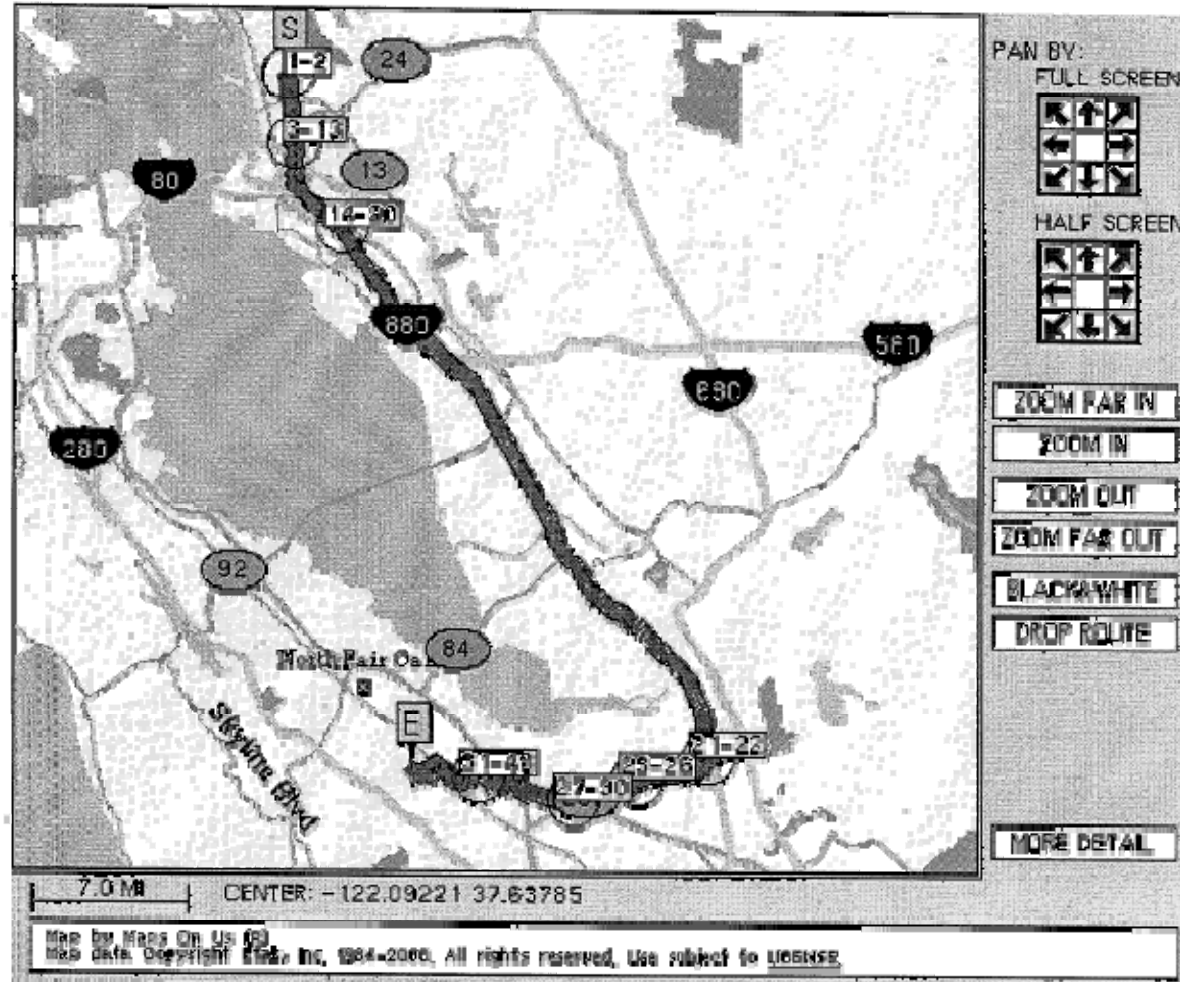


- a) 1
- b) 2
- c) 4
- d) 5
- e) 8
- f) 9
- g) 12
- h) Ved ikke

Kort over Vest-Europa

- 18.029.721 knuder
- 42.199.587 orienterede kanter





“However, because of the size of the routing data, we have to use heuristics when planning routes. As a result, sometimes a Favor Highways route will be slightly faster than the Fastest route.”

— MapsOnUs

Dine valg

Fra: Skagen st
 Til: Rødby Færge Vælg anden Fra/Til
 Udrejse: 27.04.07
 Kl.: 10:00 (Afgang)

Oversigt

Tidligere forbindelser

	Station/Stop	Dato	Kl.	Varighed	Skift	Transportmidler
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 08:56 Ank. 17:35	8:39	2	L yn EC
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 09:56 Ank. 17:35	7:39	3	L yn IC Re
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 09:56 Ank. 18:30	8:34	2	L yn Re
<input type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 11:54 Ank. 19:35	7:41	3	L yn IC EC
<input checked="" type="checkbox"/>	Skagen st Rødby Færge	27.04.07 27.04.07	Afg. 13:54 Ank. 22:31	8:37	3	L yn Re

Senere forbindelser

Vis valgte Vis alle

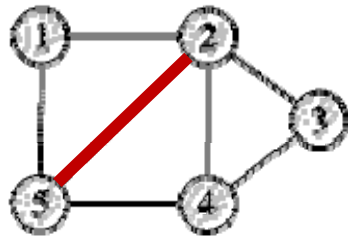
Din rejseplan

Station/Stop	Dato	Kl.	Spor	Transportmidler	Bemærkninger
Skagen st Frederikshavn st	27.04.07 27.04.07	Afg. 13:54 Ank. 14:31		PP 79	Privatbane Retning: Frederikshavn st
Frederikshavn st Frederikshavn Busterminal	27.04.07 27.04.07			Til fods Se kort	0 min.
Frederikshavn Busterminal Aalborg Busterminal	27.04.07 27.04.07	Afg. 14:35 Ank. 15:48		X-B 973X	X-BUS Retning: Aalborg Busterminal
Aalborg Busterminal Aalborg st	27.04.07 27.04.07			Til fods Se kort	5 min.
Aalborg st Høje Taastrup st	27.04.07 27.04.07	Afg. 15:59 Ank. 20:14	3 2	ICL 54	IC Lyntog Retning: København H Spornummeret er kun vejledende.
Høje Taastrup st Rødby Færge	27.04.07 27.04.07	Afg. 20:23 Ank. 22:31		RE 82273	Regionaltog Retning: Rødby Færge

Varighed: 8:37; kører 27. apr, 11. maj

Bemærkning: En station/stop er passeret flere gange, hvilket kan have betydning for prisudregningen af billetten.

Graf repræsentationer: Incidenslister og incidensmatricer

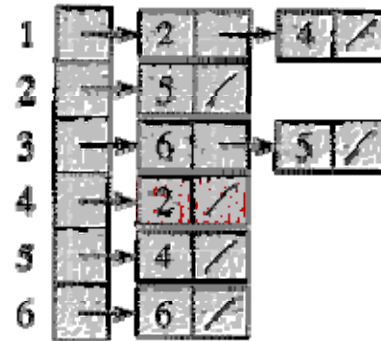
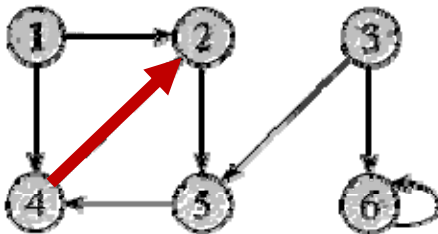


	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Uorienterede grafer

Plads $O(n+m)$

Plads $O(n^2)$



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

Orienterede grafer

Bredde først søgning (BFS)

BFS(G, s)

```

1  for each vertex  $u \in G.V - \{s\}$ 
2     $u.color = WHITE$ 
3     $u.d = \infty$ 
4     $u.\pi = NIL$ 
5   $s.color = GRAY$ 
6   $s.d = 0$ 
7   $s.\pi = NIL$ 
8   $Q = \emptyset$ 
9  ENQUEUE( $Q, s$ )
10 while  $Q \neq \emptyset$ 
11    $u = DEQUEUE(Q)$ 
12   for each  $v \in G.Adj[u]$ 
13     if  $v.color == WHITE$ 
14        $v.color = GRAY$ 
15        $v.d = u.d + 1$ 
16        $v.\pi = u$ 
17       ENQUEUE( $Q, v$ )
18    $u.color = BLACK$ 

```

$u.color$:

WHITE = knuderne endnu ikke besøgt

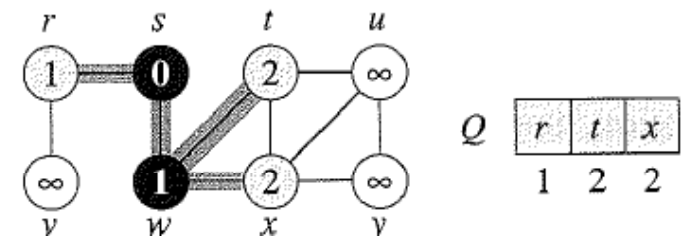
GRAY = knuderne i køen Q

BLACK = knuderne besøgt

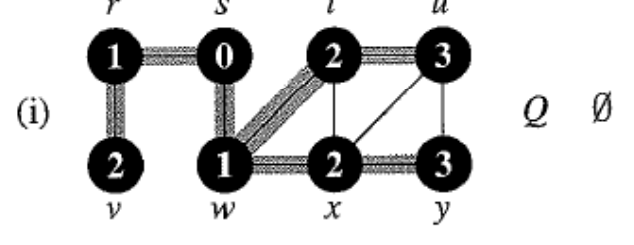
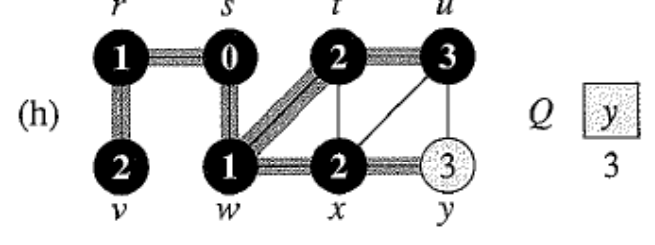
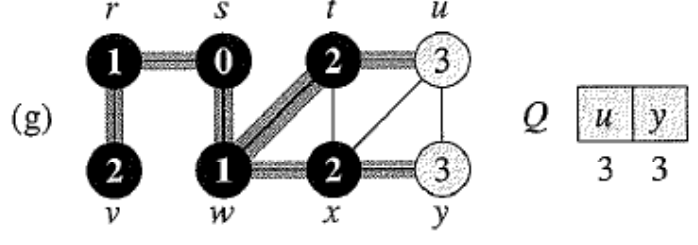
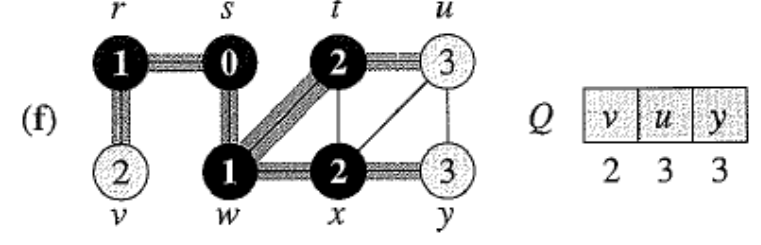
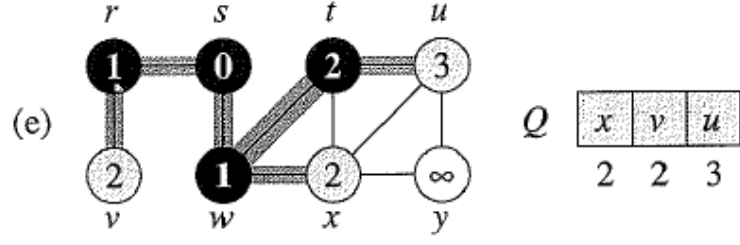
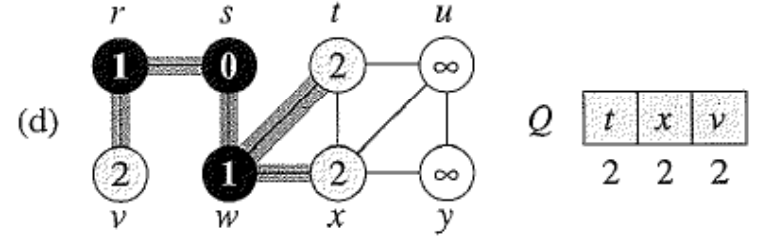
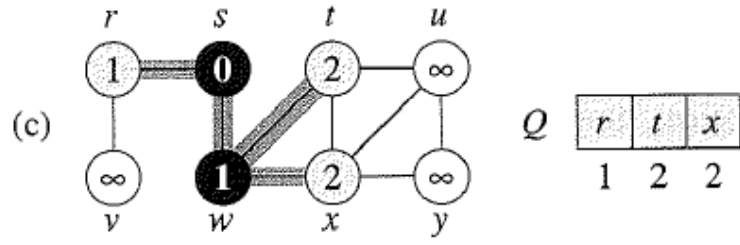
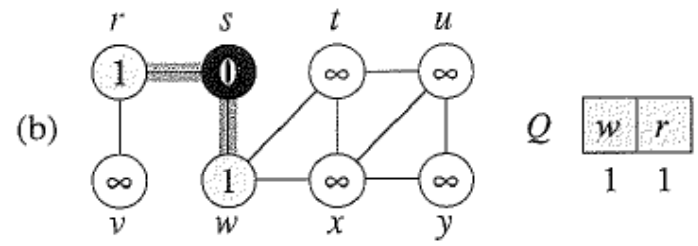
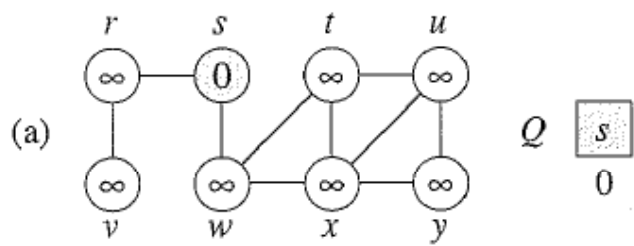
$u.d =$ afstand til s

$u.\pi =$ faderen til u i BFS træet

$Q =$ kø af grå knuder (som er forbundet til sorte knuder)

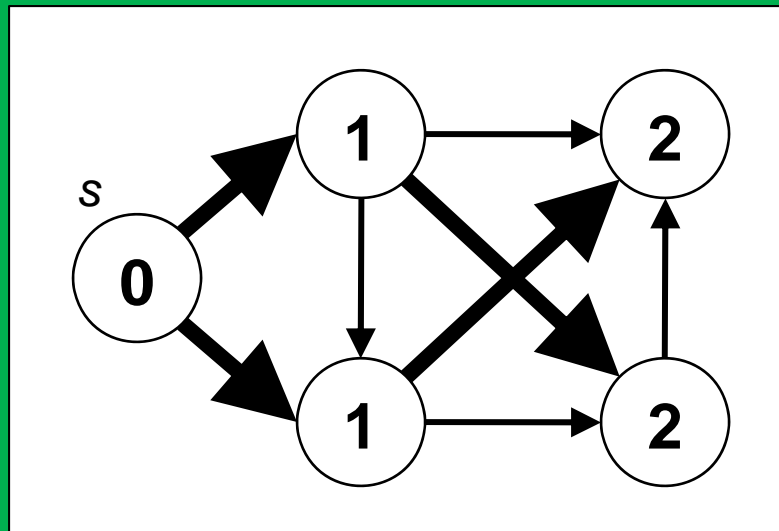


Tid $O(n+m)$



Er nedenstående et BFS træ?

- a) Ja
- b) Nej
- c) Ved ikke



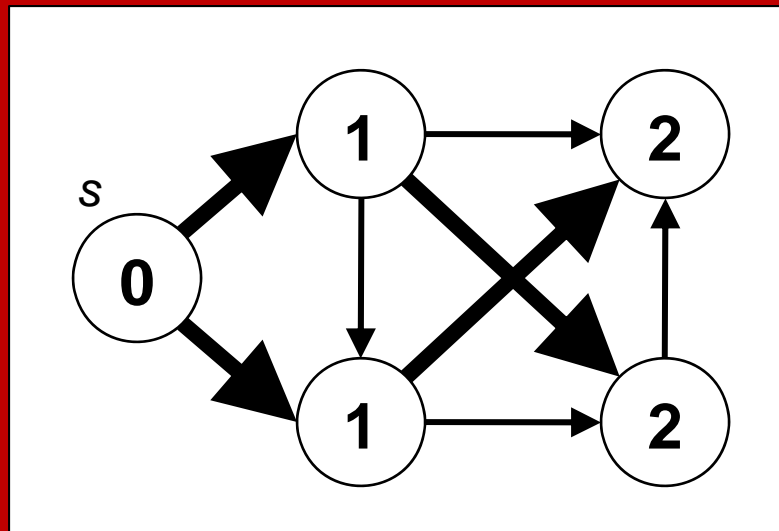
Er nedenstående et BFS træ?

a) Ja



b) Nej

c) Ved ikke



BFS : Udskrivning af sti fra s til v

PRINT-PATH(G, s, v)

```
1  if  $v == s$ 
2      print  $s$ 
3  elseif  $v.\pi == \text{NIL}$ 
4      print “no path from”  $s$  “to”  $v$  “exists”
5  else PRINT-PATH( $G, s, v.\pi$ )
6      print  $v$ 
```

Dybde Først Søgning (DFS)

DFS(G)

```
1 for each vertex  $u \in G.V$ 
2    $u.color = WHITE$ 
3    $u.\pi = NIL$ 
4    $time = 0$ 
5 for each vertex  $u \in G.V$ 
6   if  $u.color == WHITE$ 
7     DFS-VISIT( $G, u$ )
```

$u.color$

WHITE = knuderne endnu ikke besøgt
GRAY = knuder på rekursionsstakken
BLACK = knuderne besøgt

$u.\pi$ = faderen til u i DFS træet

$u.d$ = "discover time" for u
 $u.f$ = "finishing time" for u

DFS-VISIT(G, u)

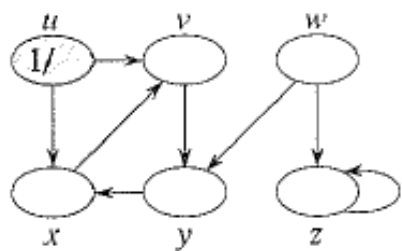
```
1  $time = time + 1$ 
2  $u.d = time$ 
3  $u.color = GRAY$ 
4 for each  $v \in G.Adj[u]$ 
5   if  $v.color == WHITE$ 
6      $v.\pi = u$ 
7     DFS-VISIT( $G, v$ )
8  $u.color = BLACK$ 
9  $time = time + 1$ 
10  $u.f = time$ 
```

// white vertex u has just been discovered

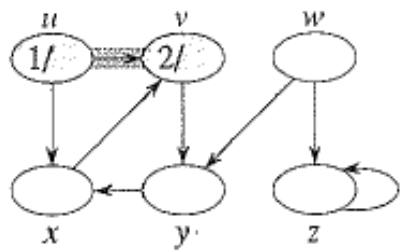
// explore edge (u, v)

// blacken u ; it is finished

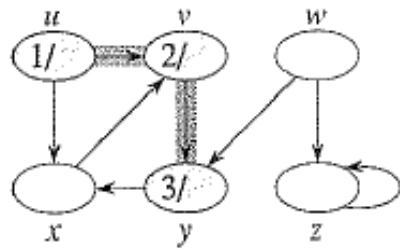
Tid $O(n+m)$



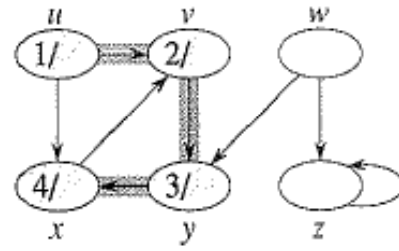
(a)



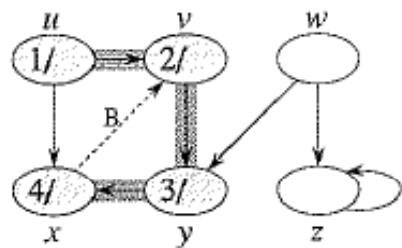
(b)



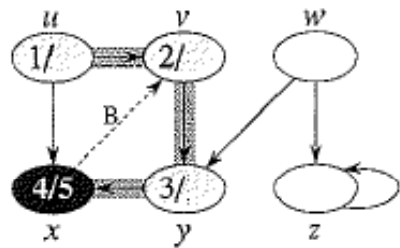
(c)



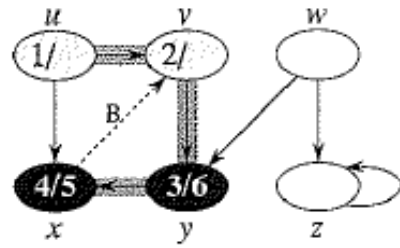
(d)



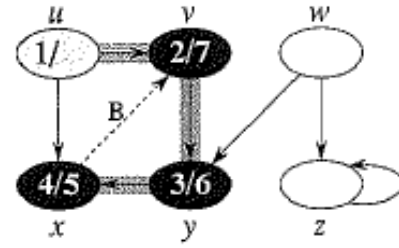
(e)



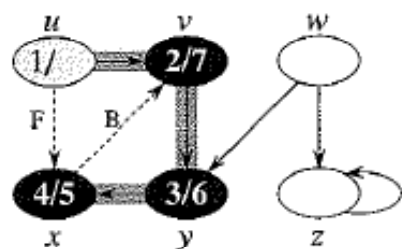
(f)



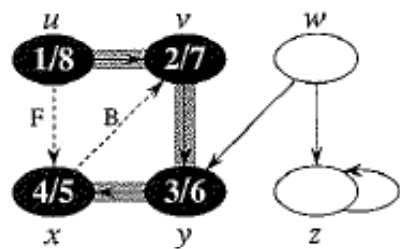
(g)



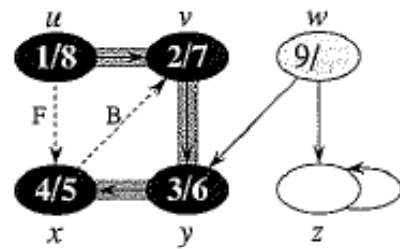
(h)



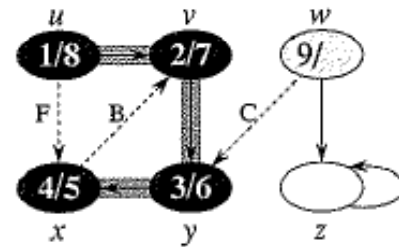
(i)



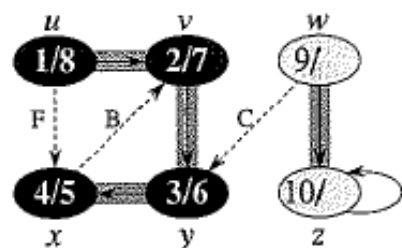
(j)



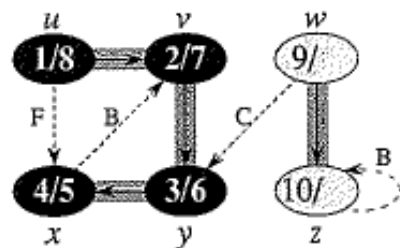
(k)



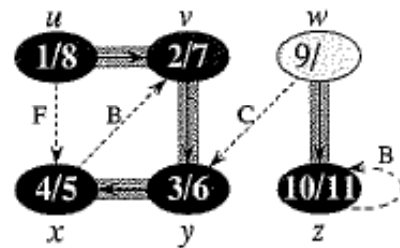
(l)



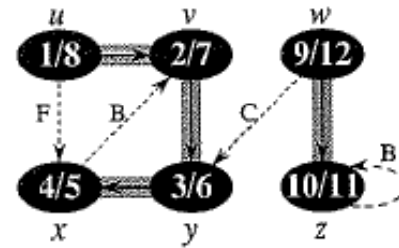
(m)



(n)



(o)



(p)

Kan en knude have 13/17 som
DFS discover/finishing tider ?

- a) Ja
- b) Nej
- c) Ved ikke

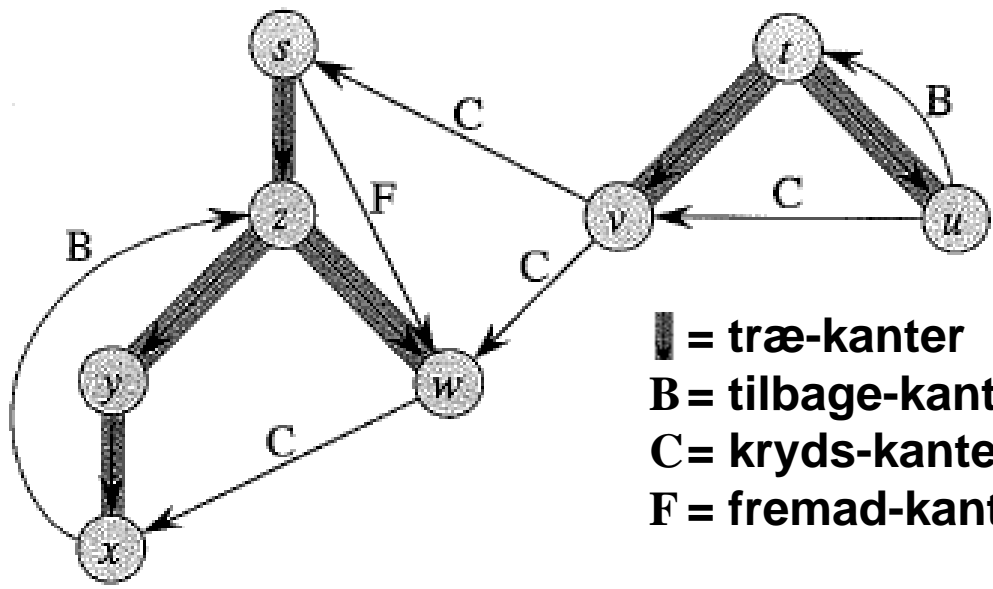
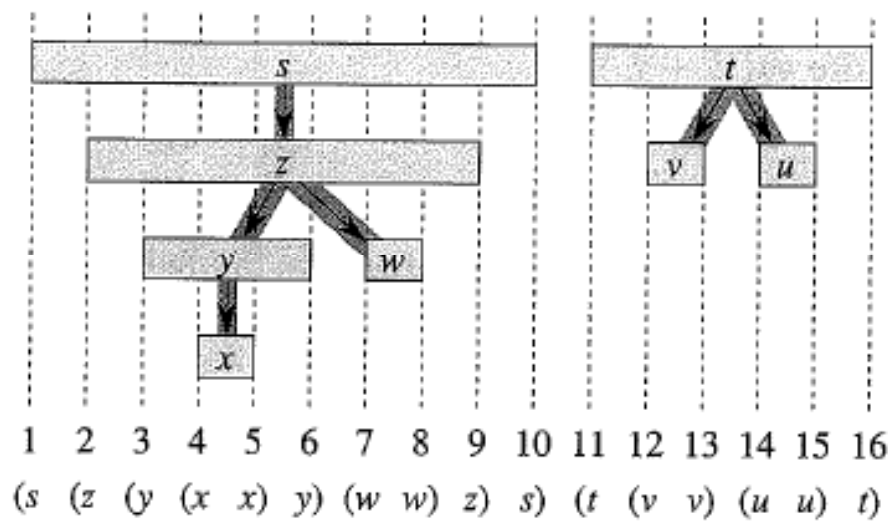
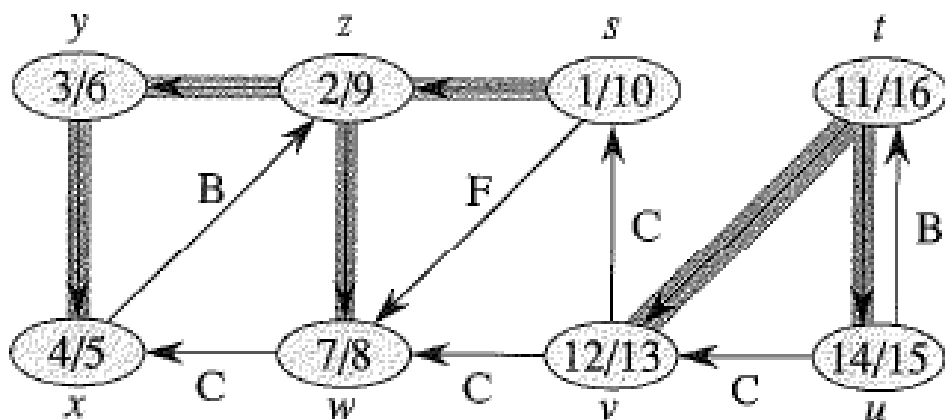
Kan en knude have 13/17 som
DFS discover/finishing tider ?

a) Ja



b) Nej

c) Ved ikke



█ = træ-kanter
B = tilbage-kanter
C = kryds-kanter
F = fremad-kanter

BFS og DFS anvendelser

BFS Finde afstande til startknuden
(afstand = antal kanter)

DFS Topologisk sortering,
stærke sammenhængskomponenter
(næste forelæsning)