

Algoritmer og Datastrukturer 1

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Merge-Sort [CLRS, kapitel 2.3]

Heaps [CLRS, kapitel 6]



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Merge-Sort

(Eksempel på Del-og-kombiner)

MERGE-SORT(A, p, r)

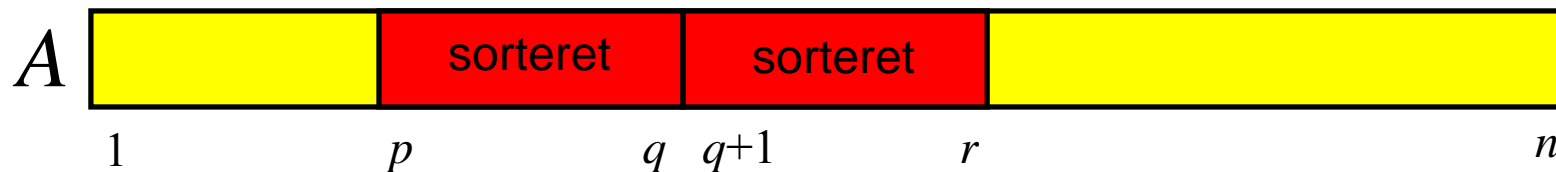
1 **if** $p < r$

2 $q = \lfloor (p + r) / 2 \rfloor$

3 MERGE-SORT(A, p, q)

4 MERGE-SORT($A, q + 1, r$)

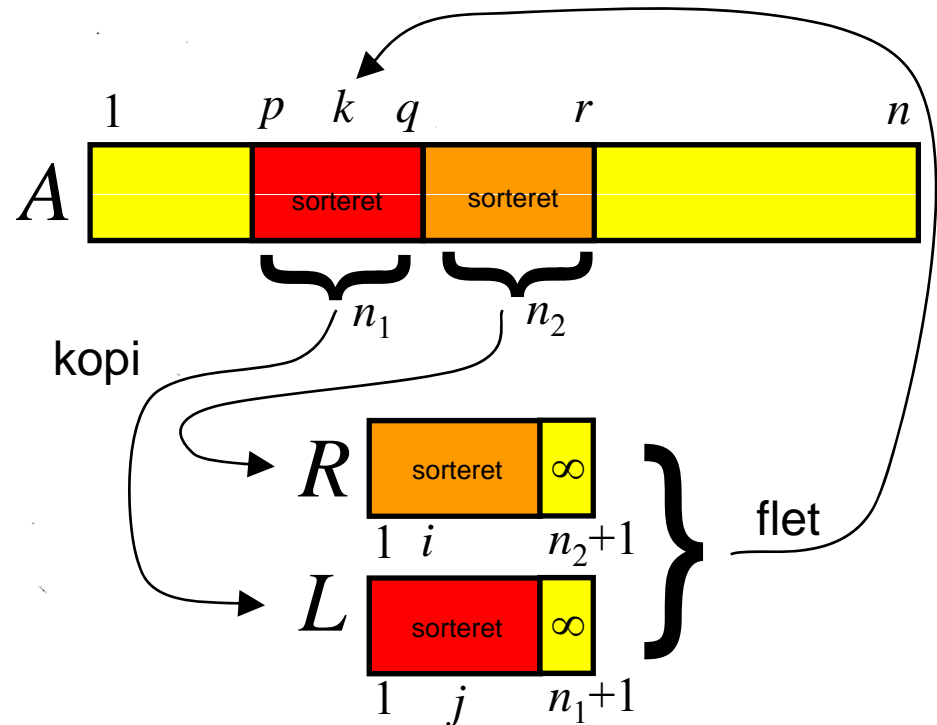
5 MERGE(A, p, q, r)



I starten kaldes MERGE-SORT($A, 1, n$)

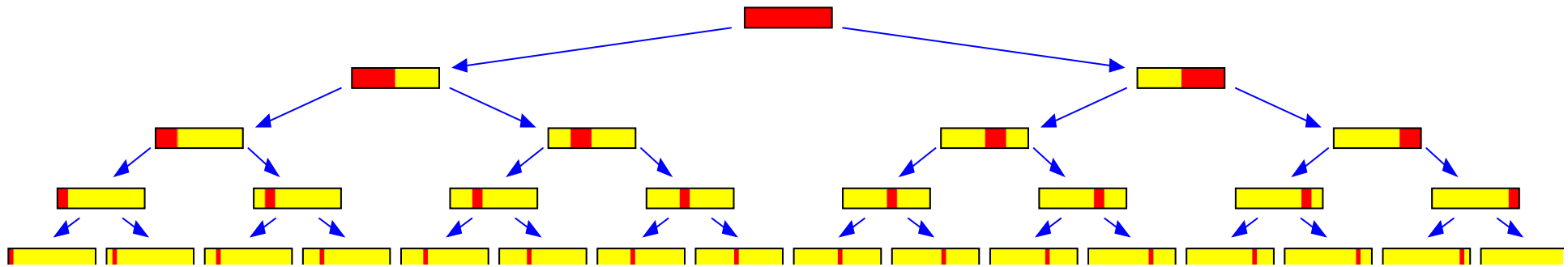
MERGE(A, p, q, r)

```
1   $n_1 = q - p + 1$ 
2   $n_2 = r - q$ 
3  let  $L[1..n_1 + 1]$  and  $R[1..n_2 + 1]$  be new arrays
4  for  $i = 1$  to  $n_1$ 
5       $L[i] = A[p + i - 1]$ 
6  for  $j = 1$  to  $n_2$ 
7       $R[j] = A[q + j]$ 
8   $L[n_1 + 1] = \infty$ 
9   $R[n_2 + 1] = \infty$ 
10  $i = 1$ 
11  $j = 1$ 
12 for  $k = p$  to  $r$ 
13     if  $L[i] \leq R[j]$ 
14          $A[k] = L[i]$ 
15          $i = i + 1$ 
16     else  $A[k] = R[j]$ 
17          $j = j + 1$ 
```



Merge-Sort : Analyse

Rekursionstræet



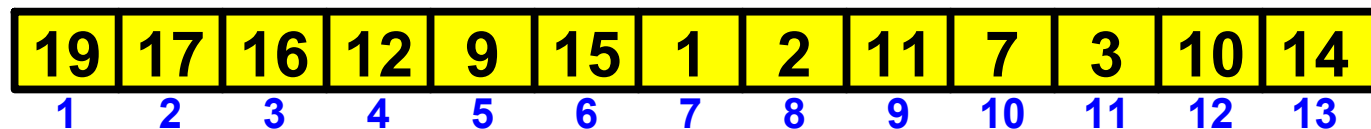
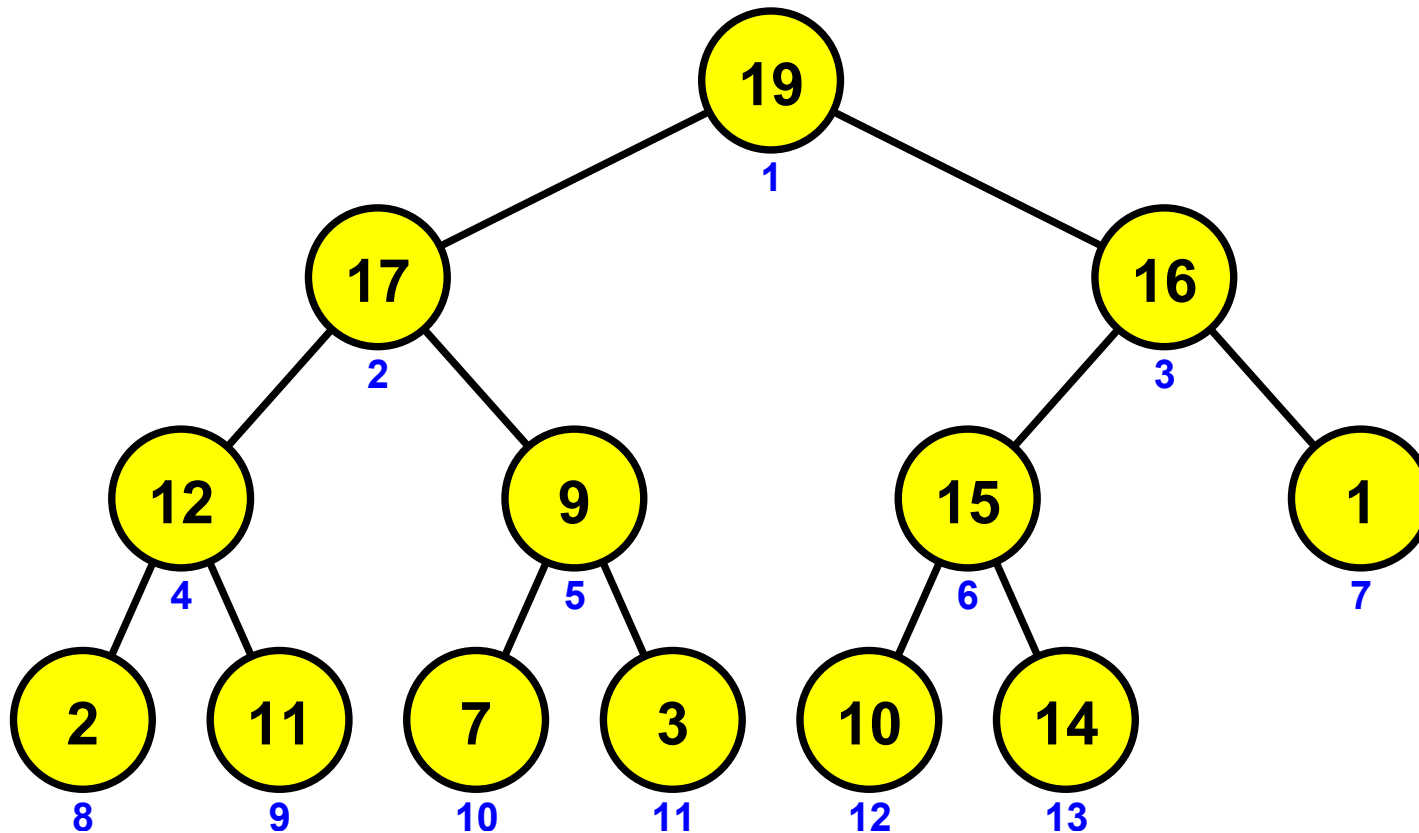
Observation

Samlet arbejde per lag er $O(n)$

Arbejde

$$O(n \cdot \# \text{ lag}) = O(n \cdot \log_2 n)$$

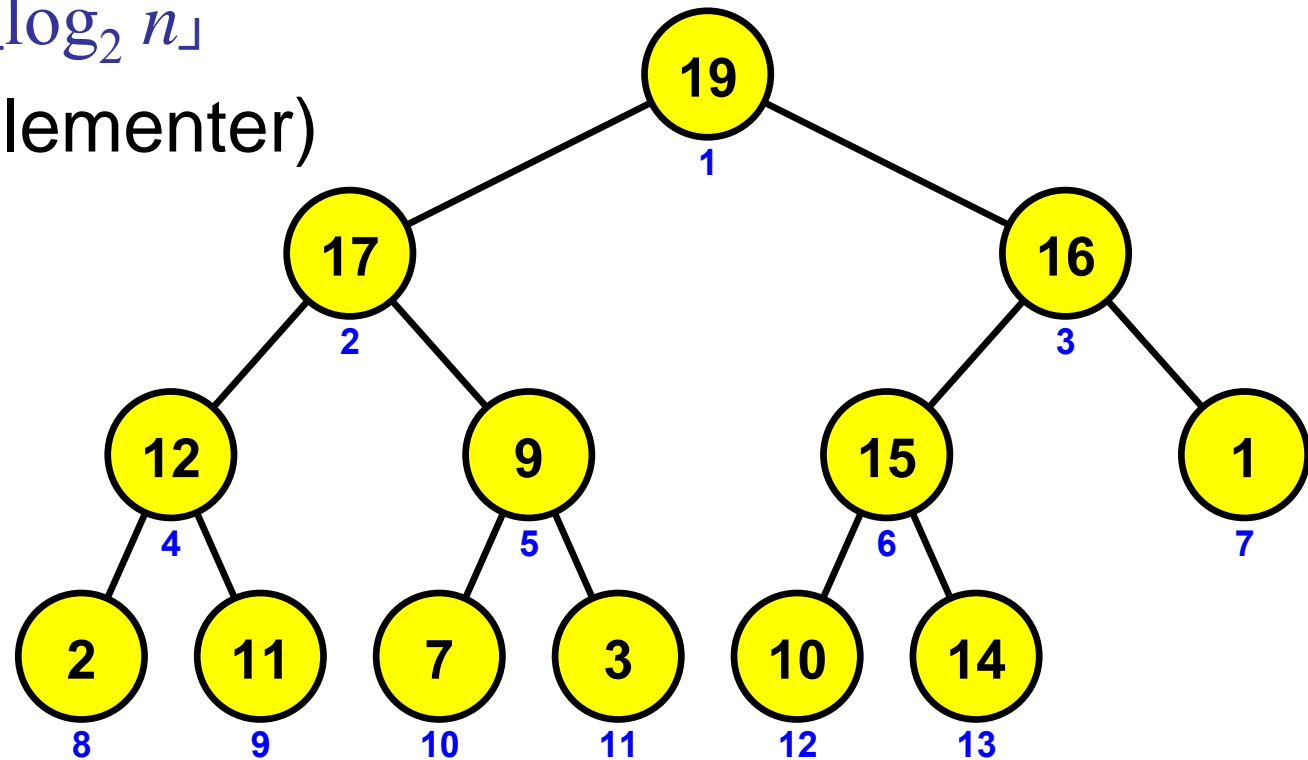
Binær (Max-)Heap



Williams, 1964

Max-heap : Egenskaber

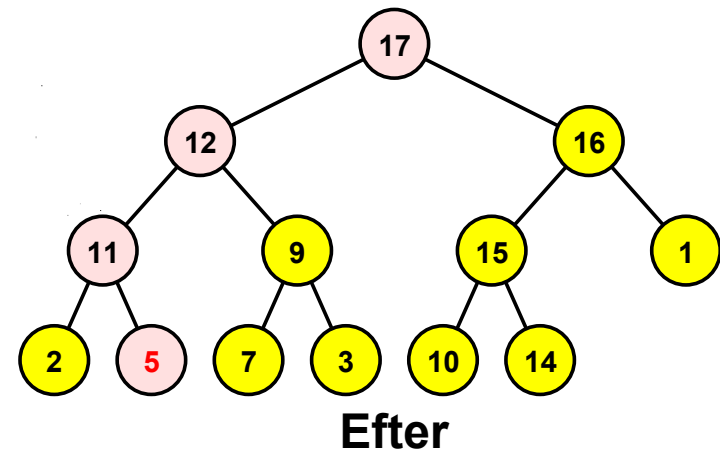
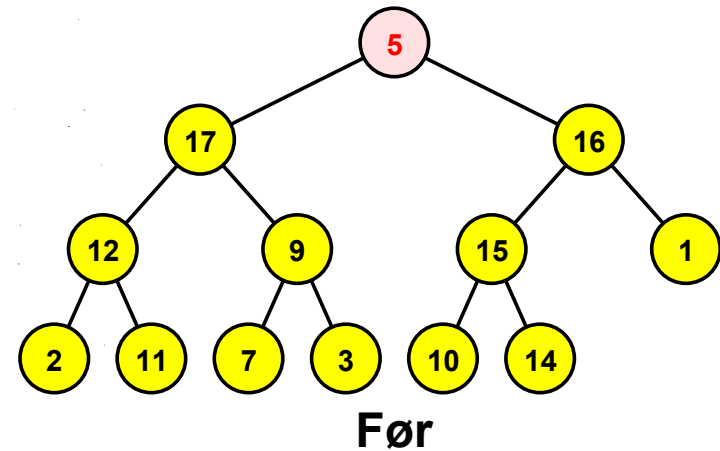
- Roden : knude 1
- Børn til knude i : $2i$ og $2i+1$
- Faren til knude i : $\lfloor i / 2 \rfloor$
- Dybde : $1 + \lfloor \log_2 n \rfloor$
(n = antal elementer)



Max-Heapify

MAX-HEAPIFY(A, i)

- 1 $l = \text{LEFT}(i)$
- 2 $r = \text{RIGHT}(i)$
- 3 **if** $l \leq A.\text{heap-size}$ and $A[l] > A[i]$
- 4 $\text{largest} = l$
- 5 **else** $\text{largest} = i$
- 6 **if** $r \leq A.\text{heap-size}$ and $A[r] > A[\text{largest}]$
- 7 $\text{largest} = r$
- 8 **if** $\text{largest} \neq i$
- 9 exchange $A[i]$ with $A[\text{largest}]$
- 10 MAX-HEAPIFY($A, \text{largest}$)



Tid $O(\log n)$

Heap-Sort

BUILD-MAX-HEAP(A)

Floyd, 1964

```
1   $A.heap-size = A.length$ 
2  for  $i = \lfloor A.length/2 \rfloor$  downto 1
3      MAX-HEAPIFY( $A, i$ )
```

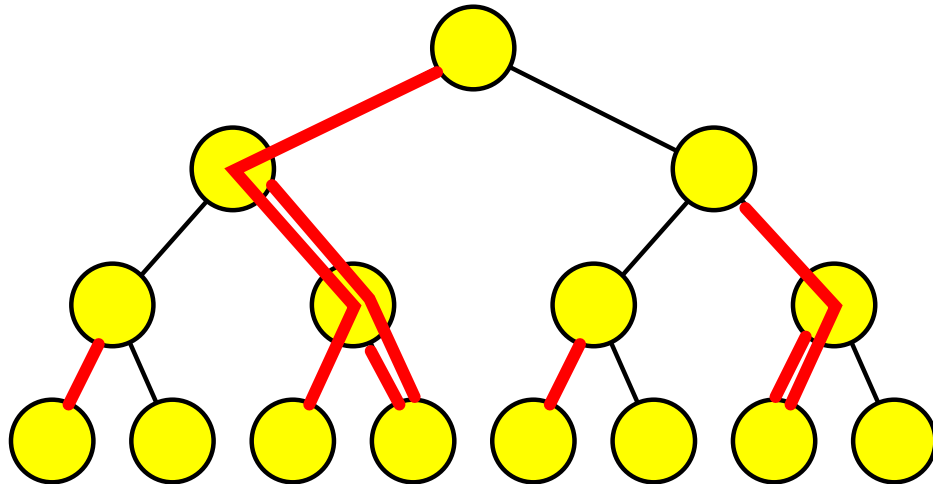
HEAPSORT(A)

Williams, 1964

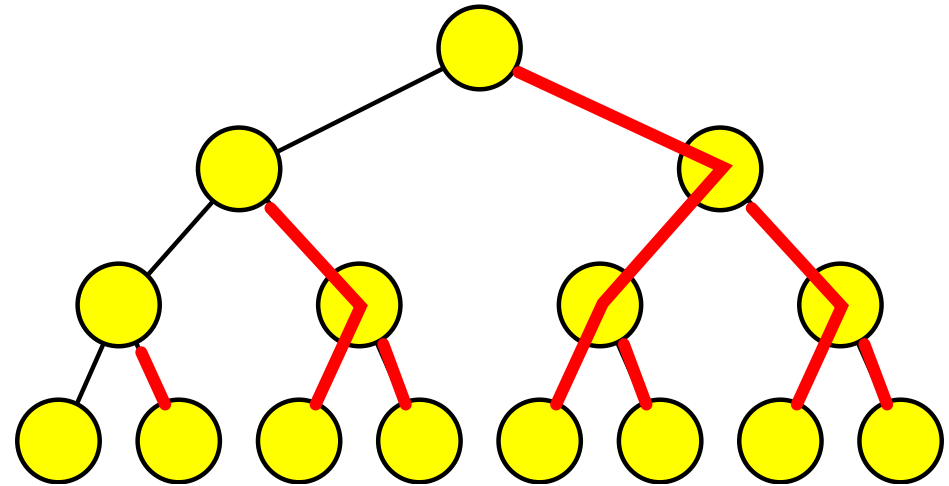
```
1  BUILD-MAX-HEAP( $A$ )
2  for  $i = A.length$  downto 2
3      exchange  $A[1]$  with  $A[i]$ 
4       $A.heap-size = A.heap-size - 1$ 
5      MAX-HEAPIFY( $A, 1$ )
```

Tid $O(n \cdot \log n)$

Build-Max-Heap



Max-Heapify stierne (eksempel)



Ikke-overlappende stier med samme #kanter (højre, venstre, venstre...)

Tid for Build-Max-Heap
= \sum tid for Max-Heapify
= **# røde kanter**

\leq **# røde kanter**
= n - dybde
= $O(n)$

Tid $O(n)$

Sorterings-algoritmer

Algoritme	Worst-Case Tid
Heap-Sort	$O(n \cdot \log n)$
Merge-Sort	
Insertion-Sort	$O(n^2)$

Max-Heap operationer

HEAP-MAXIMUM(A)

1 **return** $A[1]$

MAX-HEAP-INSERT(A, key)

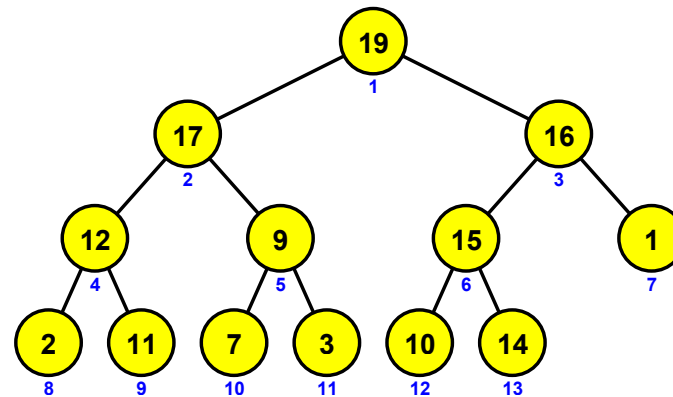
1 $A.heap-size = A.heap-size + 1$
2 $A[A.heap-size] = -\infty$
3 HEAP-INCREASE-KEY($A, A.heap-size, key$)

HEAP-EXTRACT-MAX(A)

1 **if** $A.heap-size < 1$
2 **error** “heap underflow”
3 $max = A[1]$
4 $A[1] = A[A.heap-size]$
5 $A.heap-size = A.heap-size - 1$
6 MAX-HEAPIFY($A, 1$)
7 **return** max

HEAP-INCREASE-KEY(A, i, key)

1 **if** $key < A[i]$
2 **error** “new key is smaller than current key”
3 $A[i] = key$
4 **while** $i > 1$ and $A[PARENT(i)] < A[i]$
5 exchange $A[i]$ with $A[PARENT(i)]$
6 $i = PARENT(i)$



Max-Heap operation

Operation	Worst-Case Tid
Max-Heap-Insert	$O(\log n)$
Heap-Extract-Max	
Max-Increase-Key	
Heap-Maximum	$O(1)$

n = aktuelle antal elementer i heapen

Prioritetskø

En **prioritetskø** er en abstrakt datastruktur der gemmer en mængde af **elementer** med tilknyttet **nøgle** og understøtter operationerne:

- **Insert**(S, x)
- **Maximum**(S)
- **Extract-Max**(S)

Maximum er med hensyn til de tilknyttede nøgler.

En mulig **implementation** af en prioritetskø er en **heap**.