Assignment 7
Hand in date: May 26, 2017

Exercise 1. Hand in your solution to Exercise 4.10 from the notes on categorical logic.

Exercise 2. Hand in your solution to Exercise 4.14 from the notes on categorical logic.

Exercise 3. Hand in your solution to Exercise 5.3 from the notes on categorical logic.

Exercise 4. Let \( \left( X_i \right\}_{i=0}^{\infty} \) be a complete ordered family of equivalences, \( \left\{ x_i \right\}_{i=0}^{\infty} \) and \( \left\{ y_i \right\}_{i=0}^{\infty} \) two Cauchy sequences in \( X \), and \( n \in \mathbb{N} \).

Show that if \( x_i = y_i \) for all \( i \in \mathbb{N} \) then

\[
\lim_{i \to \infty} x_i = \lim_{i \to \infty} y_i.
\]

Exercise 5. Let \( X \) and \( Y \) be two complete ordered families of equivalences and let \( X \) be inhabited. Let \( f : Y \times X \to X \) be a non-expansive function such that for all \( y \in Y \) the function \( f(y, -) : X \to X \) is contractive.

Show that there exists a unique non-expansive function \( g : Y \to X \) such that \( f(y, g(y)) = g(y) \).