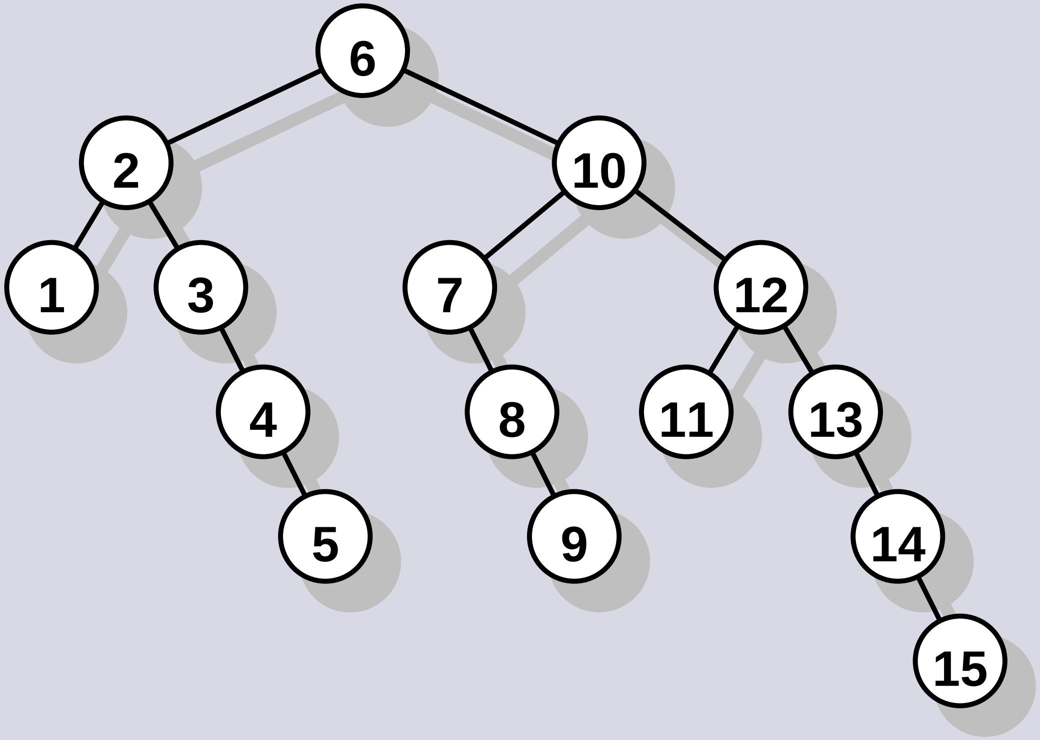
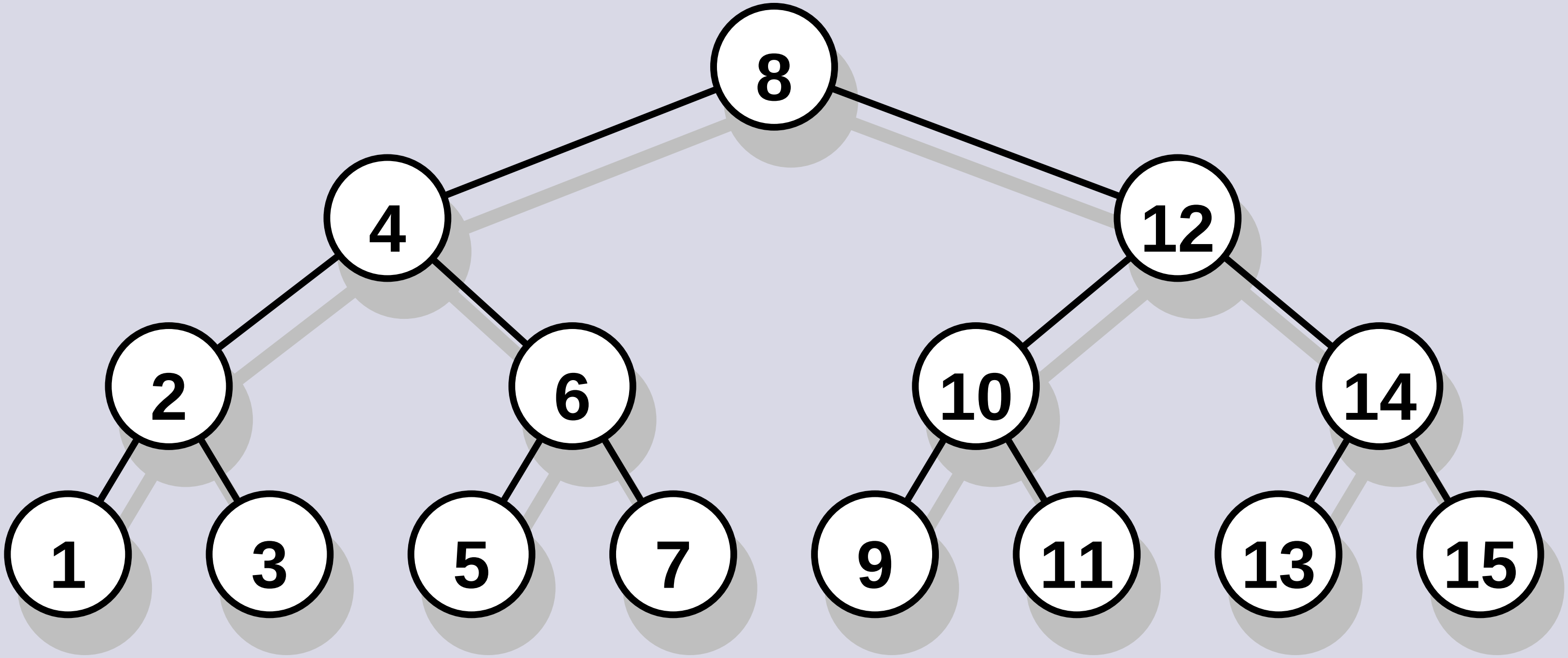
# Skewed Binary Search Trees



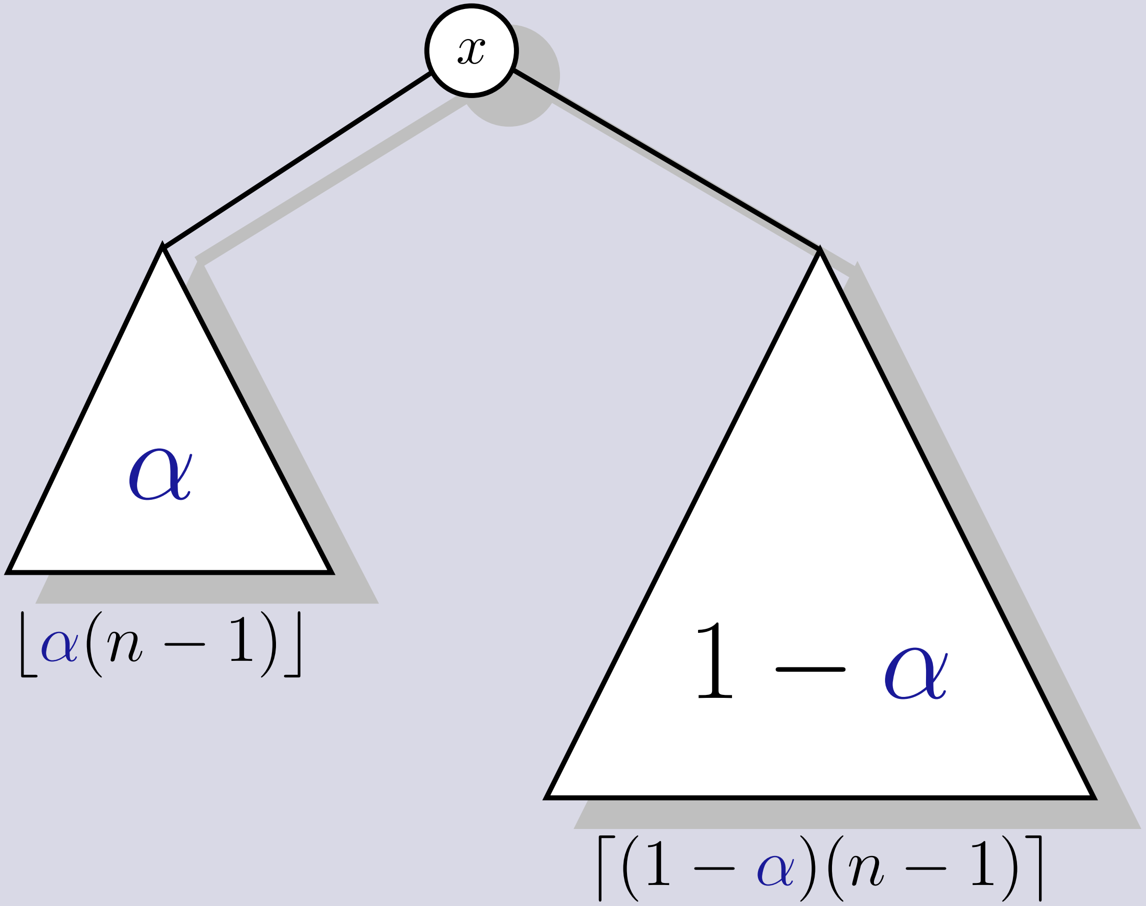

**Gerth Stølting Brodal**
University of Aarhus

Joint work with Gabriel Moruz presented at ESA'06

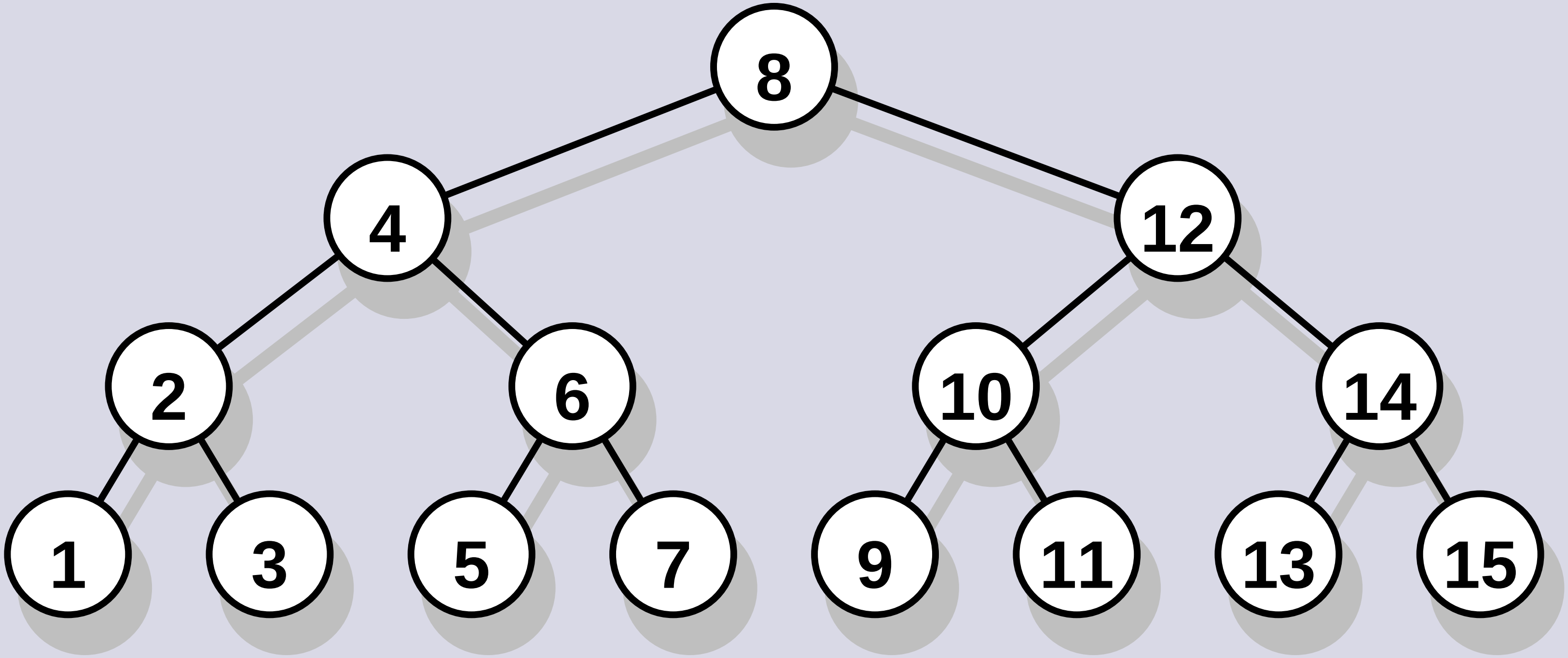
# Perfectly Balanced Search Trees



# Skewed Binary Search Trees

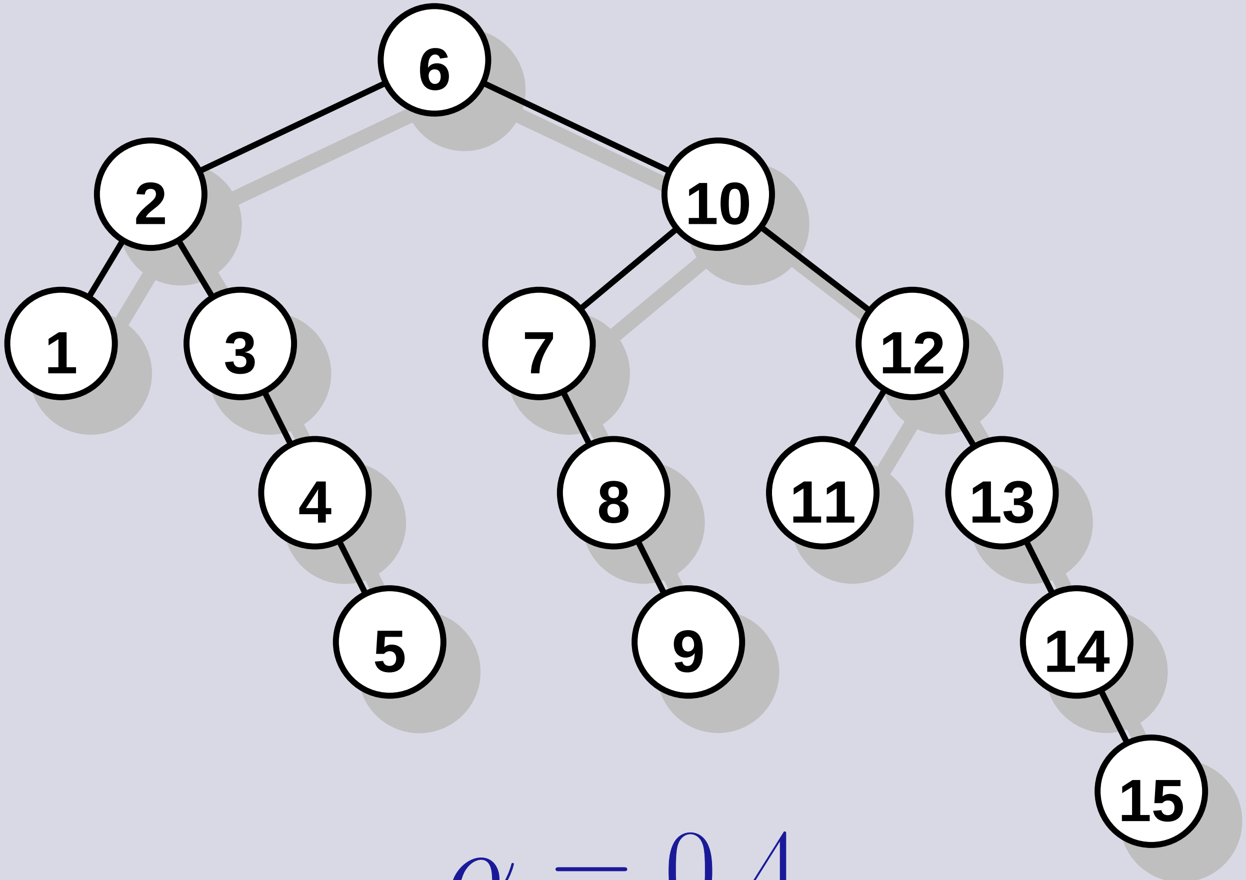


# Skewed Binary Search Trees



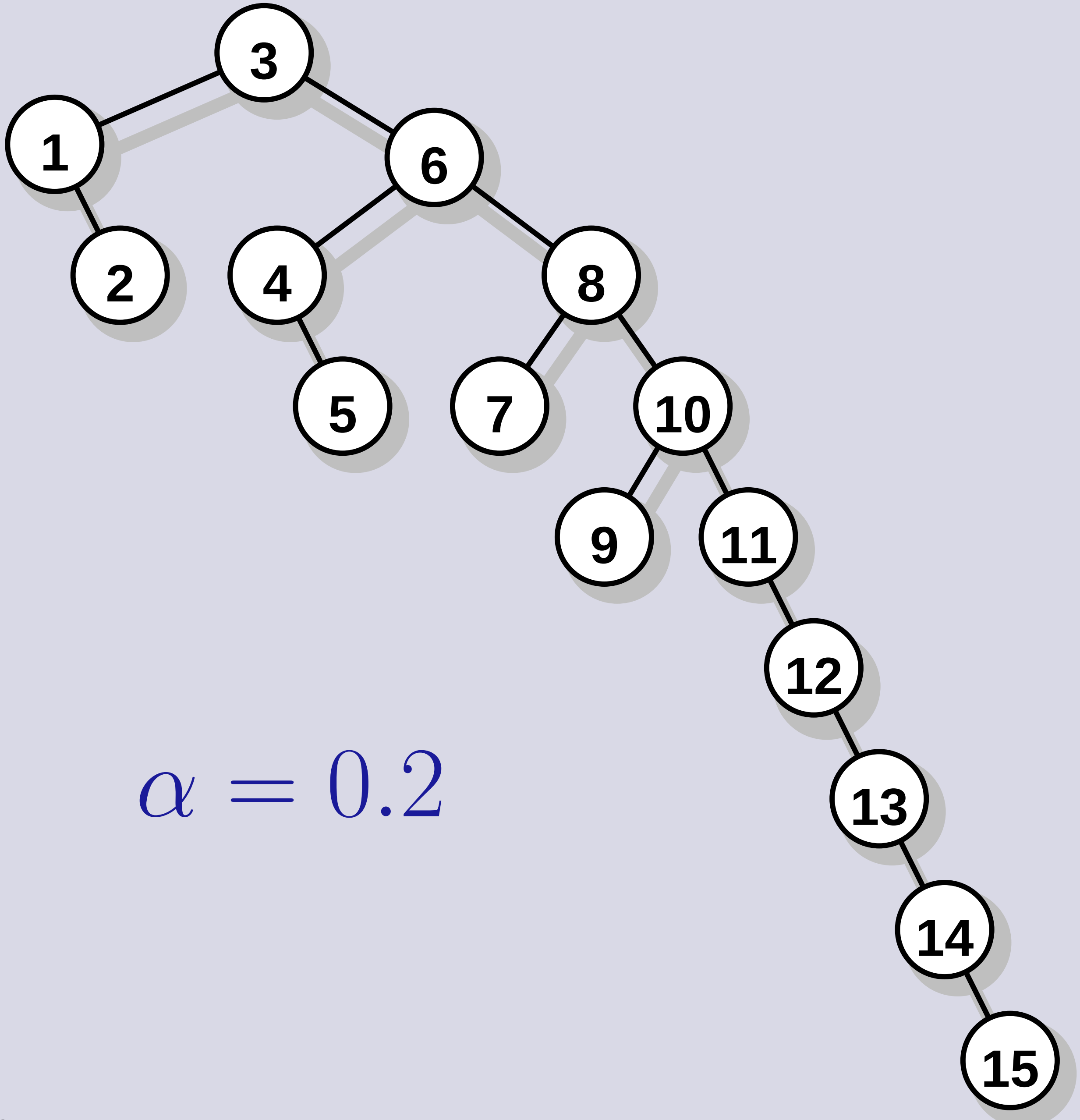

$$\alpha = 0.5$$

# Skewed Binary Search Trees



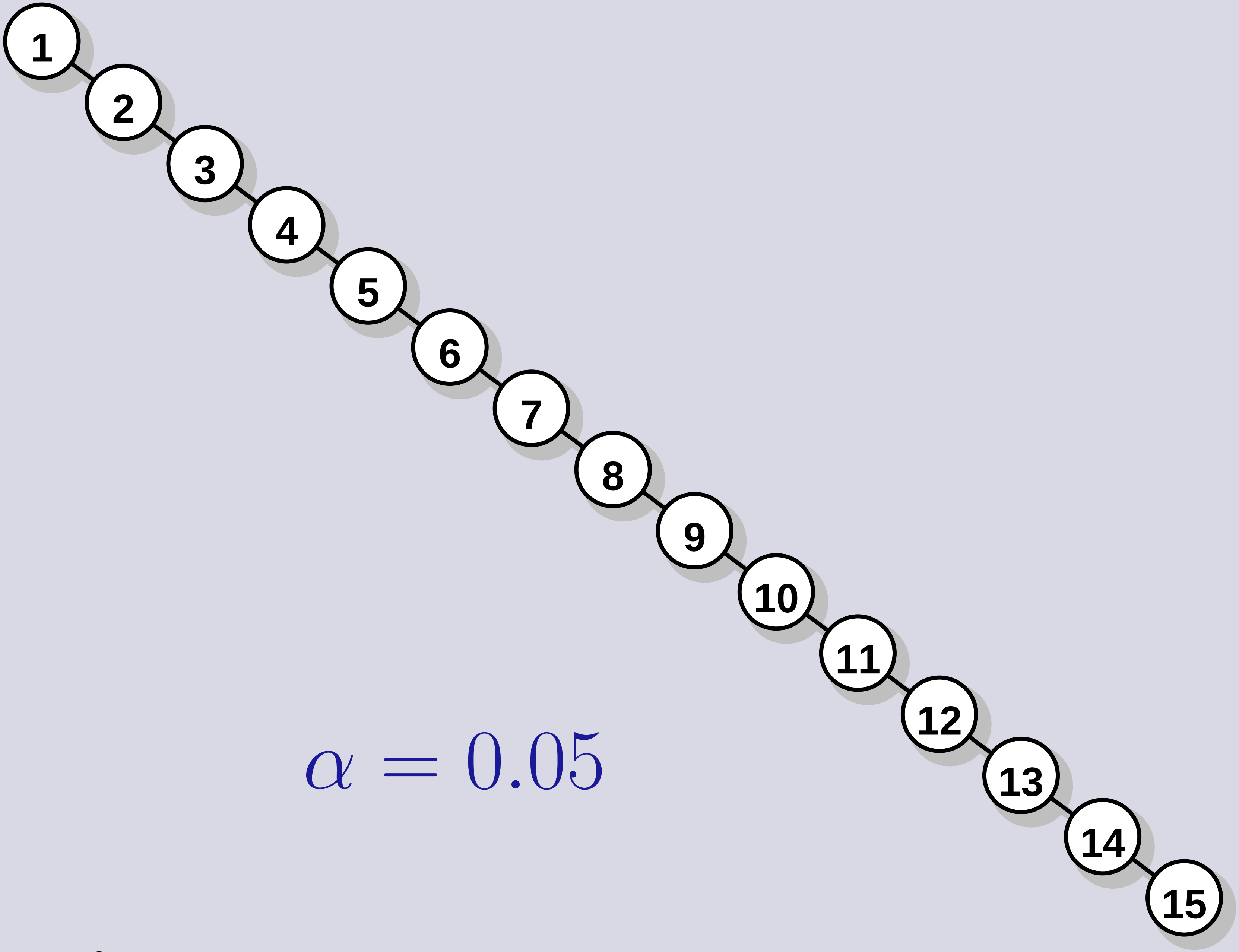

$$\alpha = 0.4$$

# Skewed Binary Search Trees




$$\alpha = 0.2$$

# Skewed Binary Search Trees



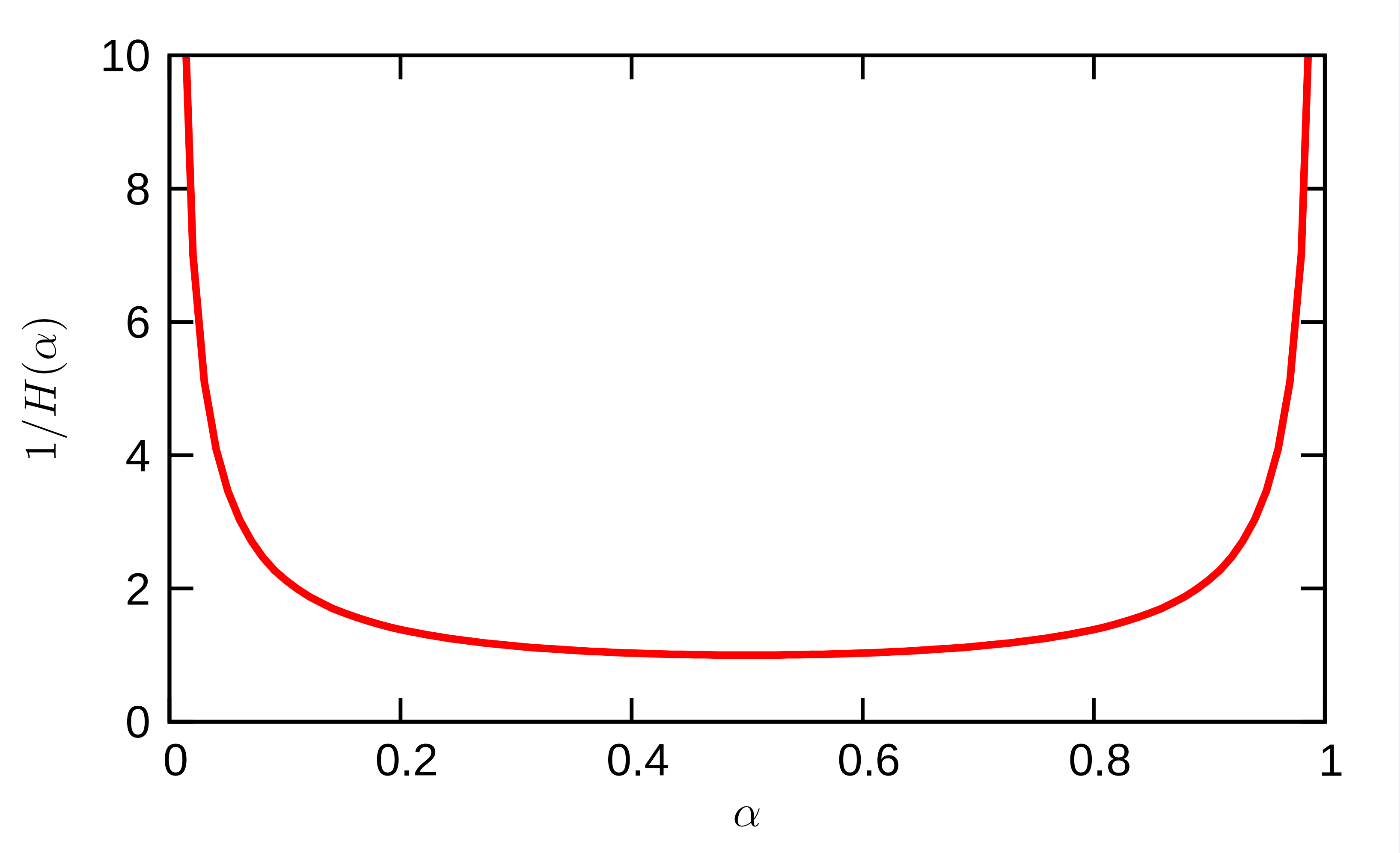

$$\alpha = 0.05$$

# Skewed Binary Search Trees
## — Average Node Depth

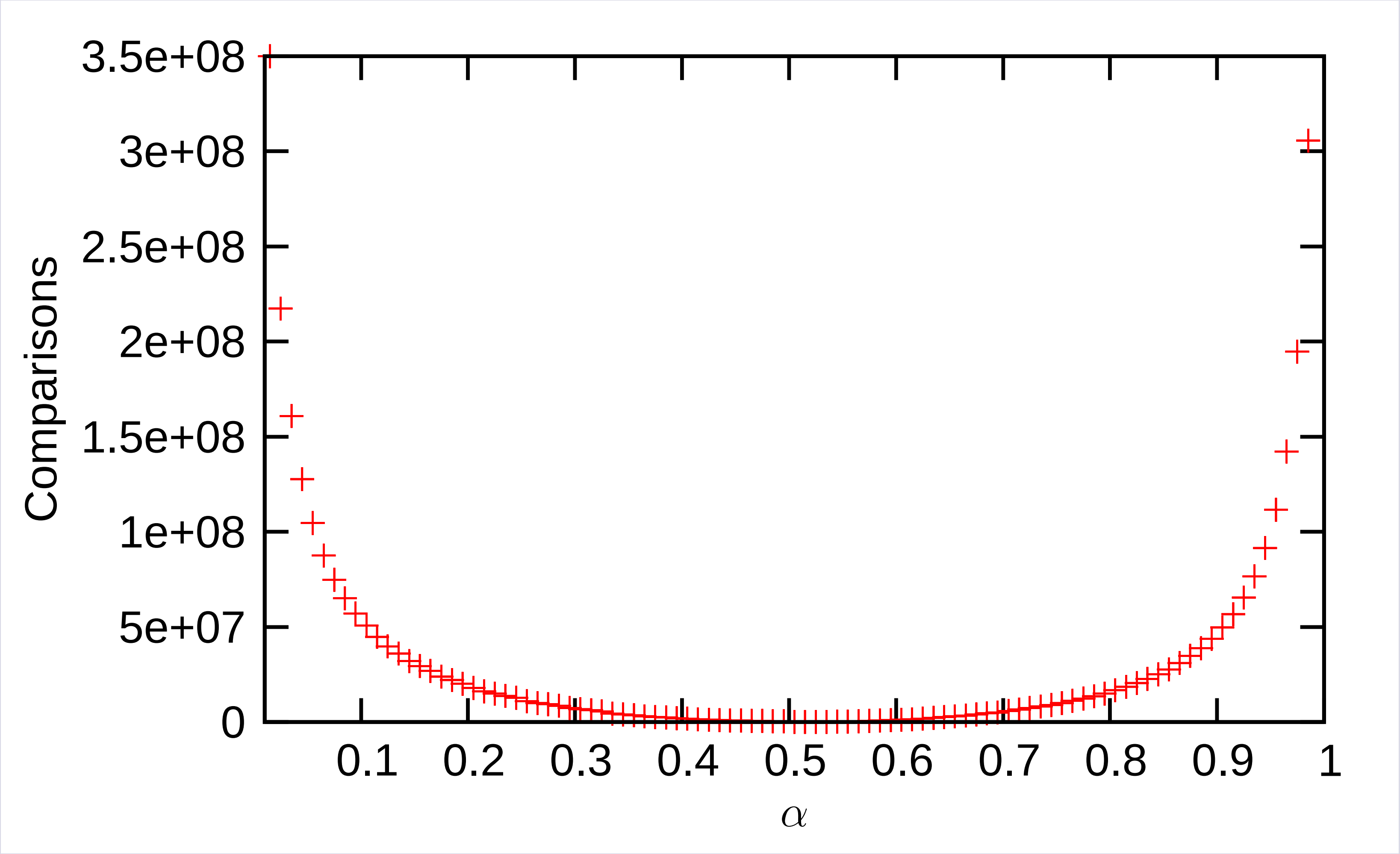
$$\leq \underbrace{\frac{1}{-\alpha \log_2 \alpha - (1-\alpha)\log_2(1-\alpha)}}_{H(\alpha)} \cdot \frac{n+1}{n} \cdot \log_2(n+1) - 2$$

Nievergelt and E. M. Reingold, 1972

# $1/H(\alpha)$

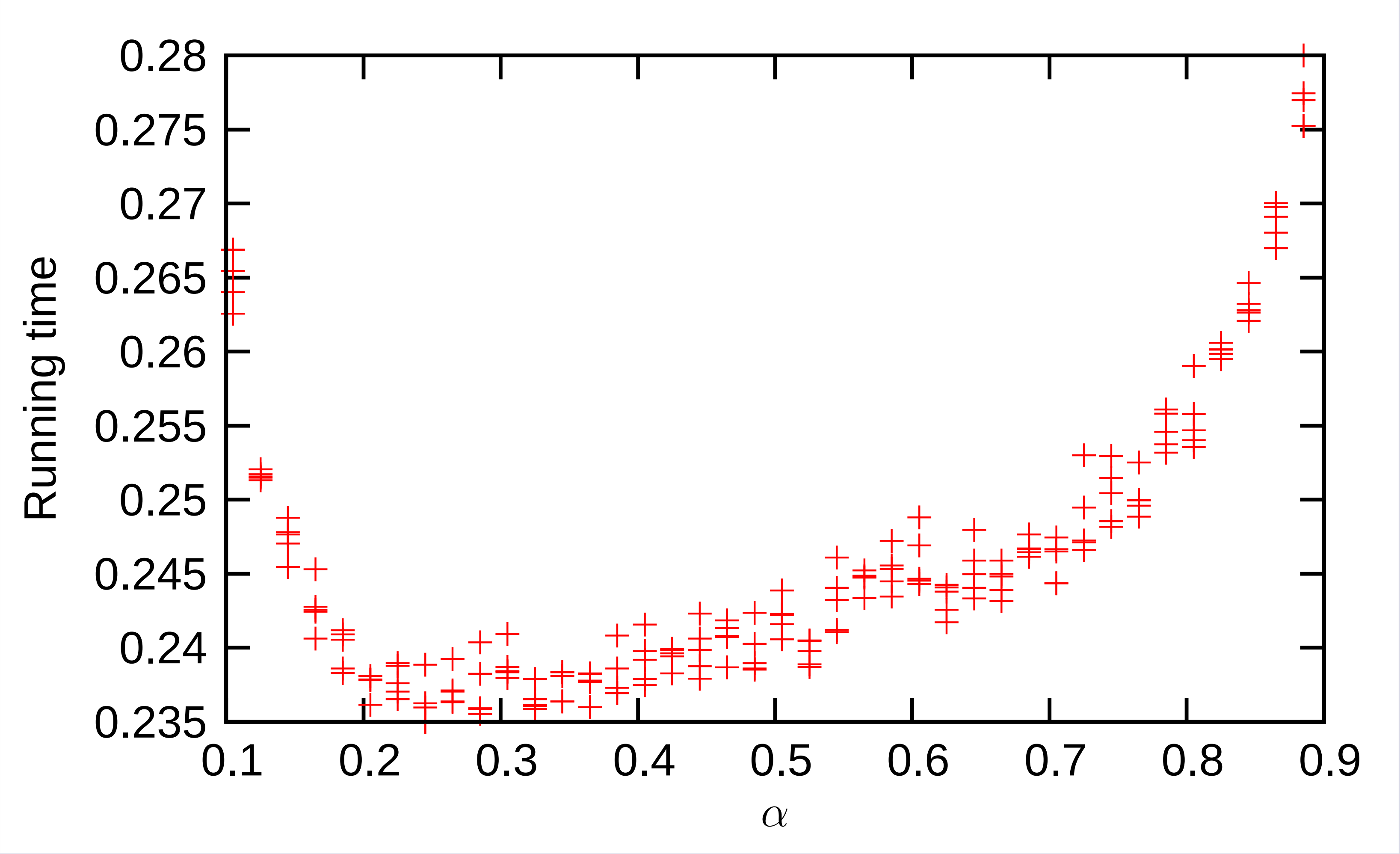


# Comparisons




$n = 50.000$

# Running Time




Best running time achieved for $\alpha \approx 0.3$ !?

# Conclusion

Skewed binary search trees

can beat

Perfectly balanced binary search trees !

# Why ?

# Why ?

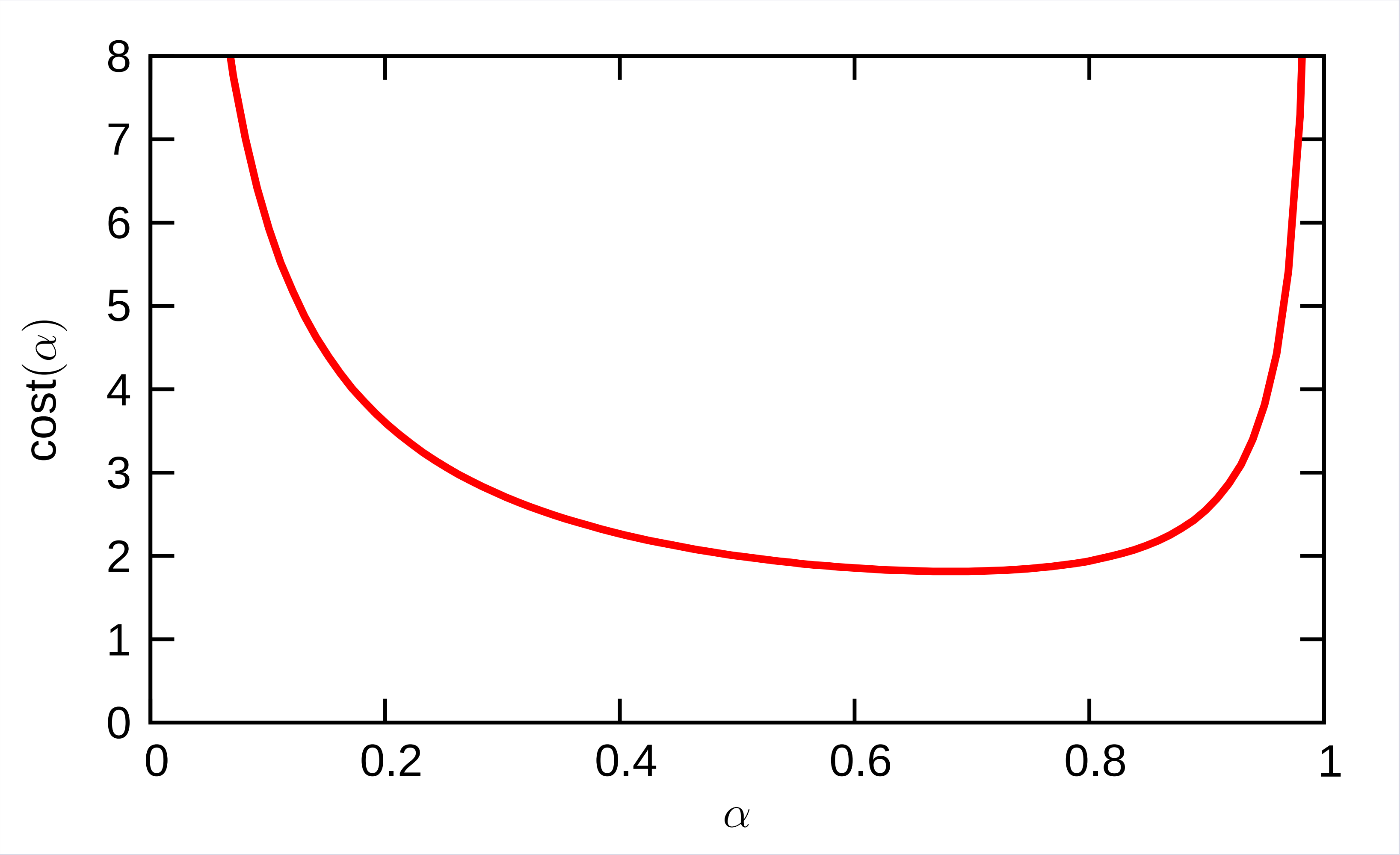
The costs going left and right are different !

## Possible reasons

- Number of instructions
- Branch mispredictions
- Cache faults (what is a good memory layout?)
- ...

# Expected Cost

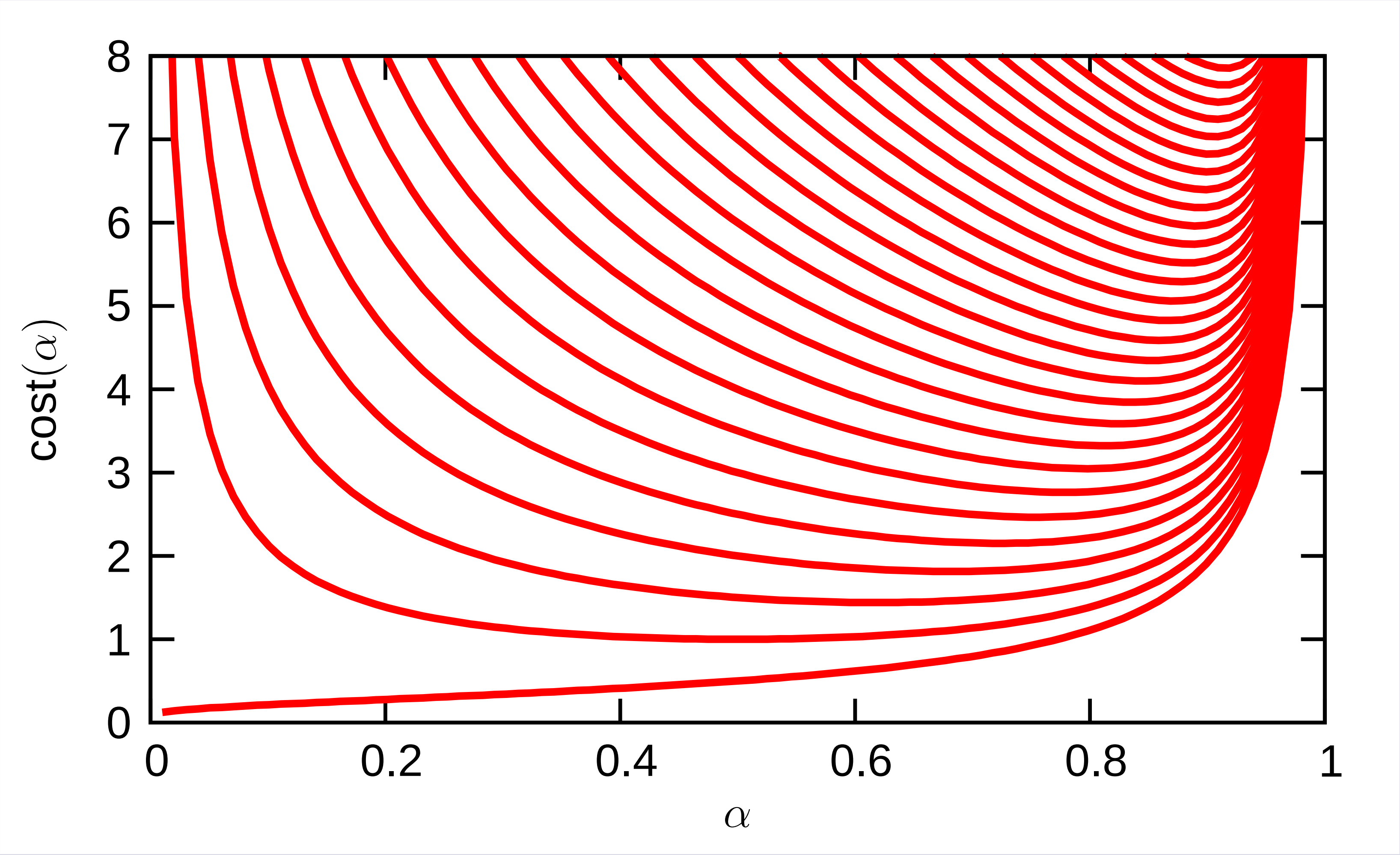
$$\text{cost}(\alpha) = (\alpha \cdot \{\text{left cost}\} + (1-\alpha) \cdot \{\text{right cost}\})/H(\alpha)$$




left cost = 1 and right cost = 3

# Expected Cost

$$\text{cost}(\alpha) = (\alpha \cdot \{\text{left cost}\} + (1-\alpha) \cdot \{\text{right cost}\})/H(\alpha)$$




left cost = 1 and right cost = 0 .. 28

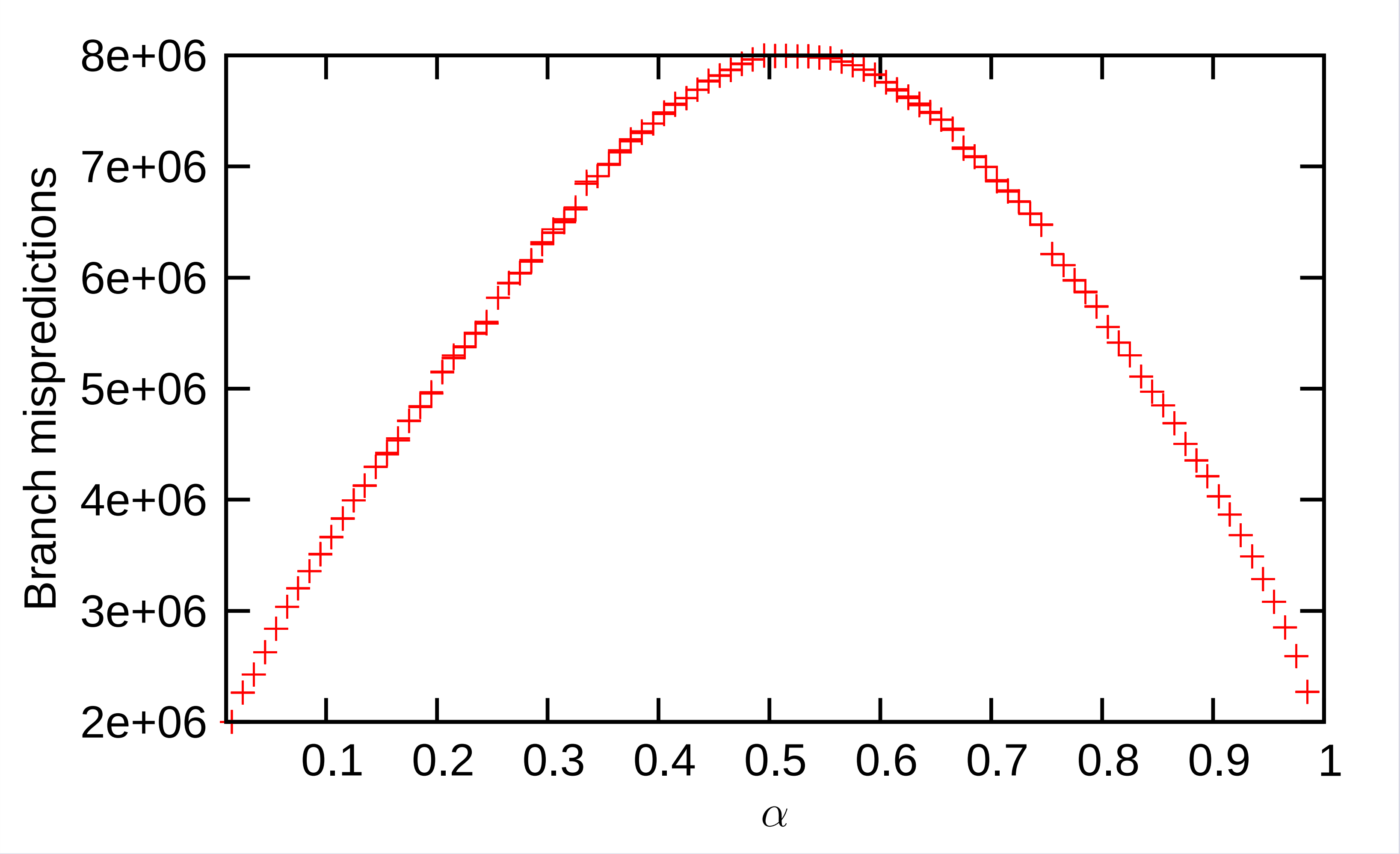
# Experimental setup

- AMD Athlon XP 2400+
- 2.0 GHz
- 256 KB L2 cache
- 64 KB L1 data cache
- 64 KB L1 instruction cache
- 1GB RAM
- Linux 2.6.8.1
- GCC 3.3.2
- Tree nodes = 12 bytes
- No unsuccesful searches

# Search Code

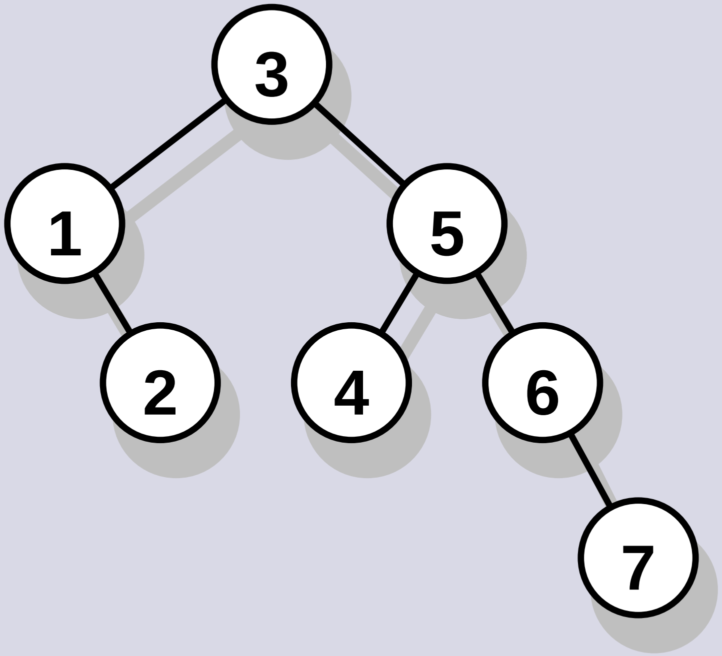
```
while(root!=NULLV)
{
  if(key==t[root].key)
    return root;
  if(key>t[root].key)
    root=t[root].right;
  else
    root=t[root].left;
}
```

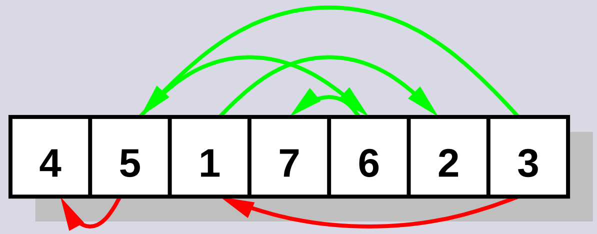
# Branch Mispredictions



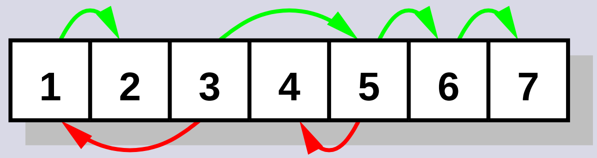

$n = 50.000$

# Simple Layouts

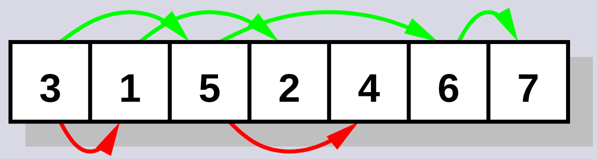


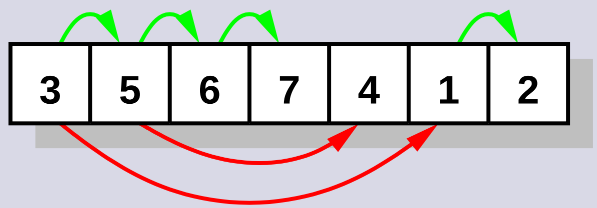

**Random** – $O(\frac{\log n}{H(\alpha)})$ I/Os




**Inorder** – $O(\frac{\log n}{H(\alpha)} - \log B)$ I/Os

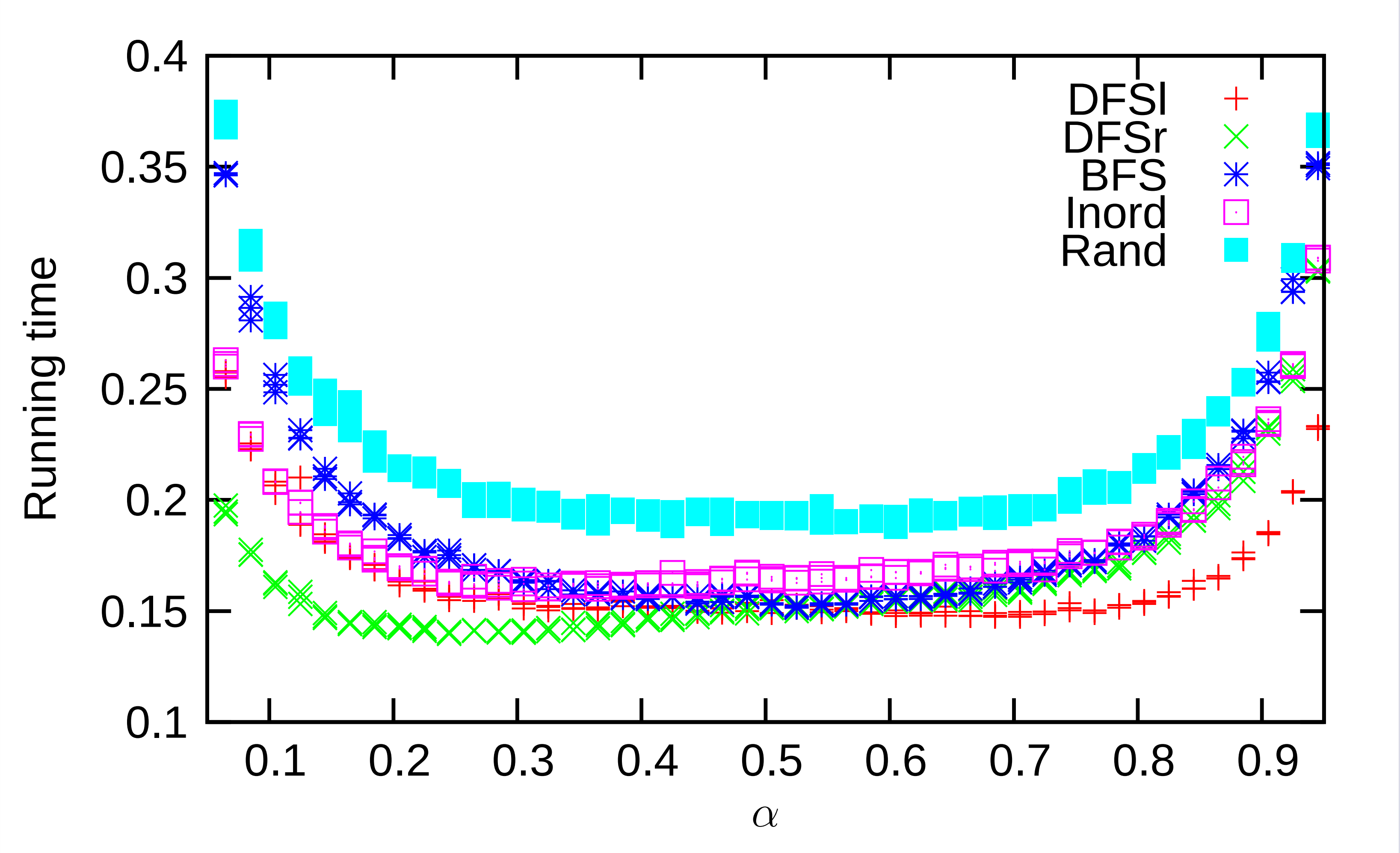



**BFS** – $O(\frac{\log n}{H(\alpha)} - \log B)$ I/Os




**DFSr** – $O(\frac{\alpha+(1-\alpha)/B}{H(\alpha)} \cdot \log n)$ I/Os.

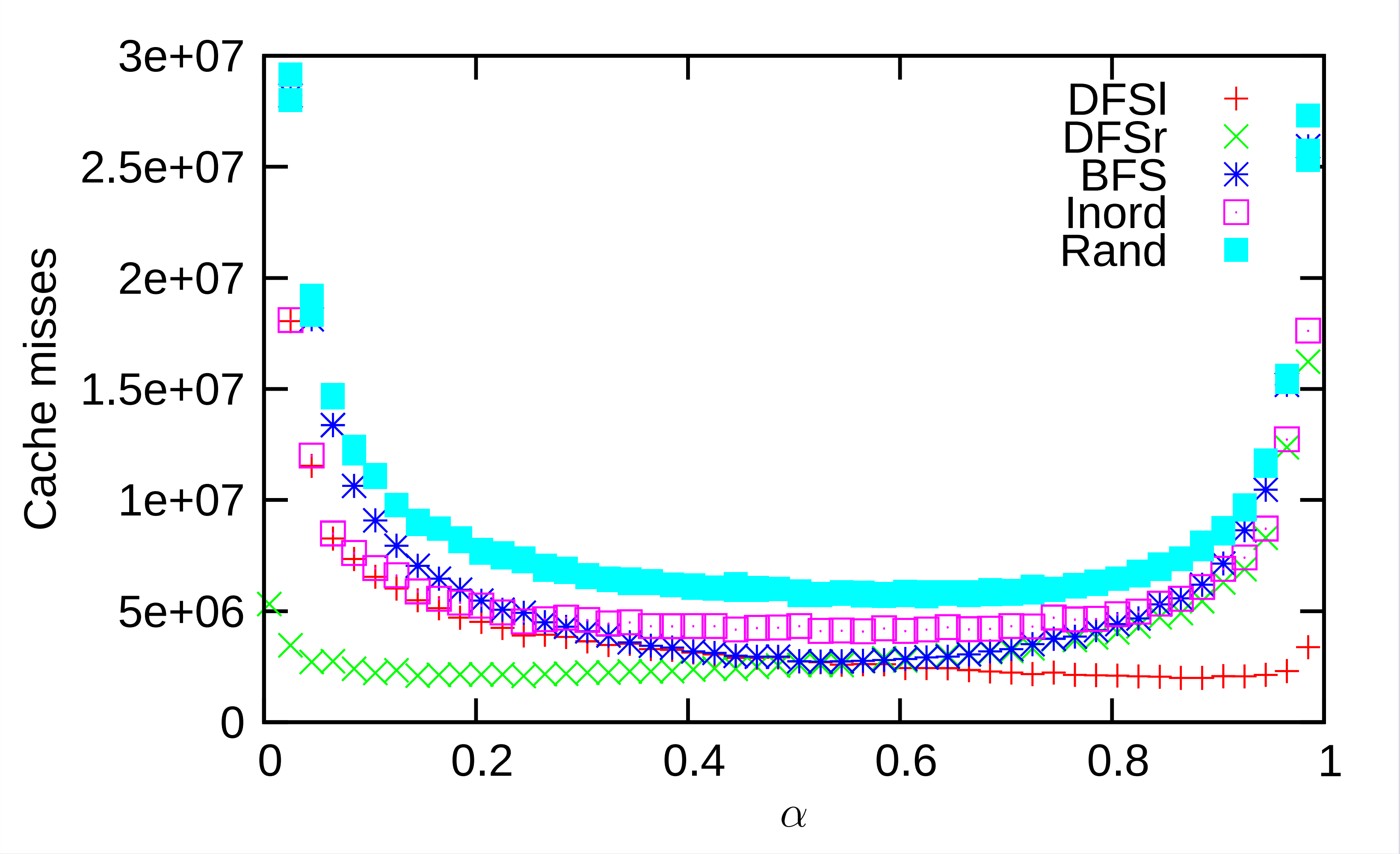
# Running Time for Simple Layouts




DFS < Inorder < BFS < Random

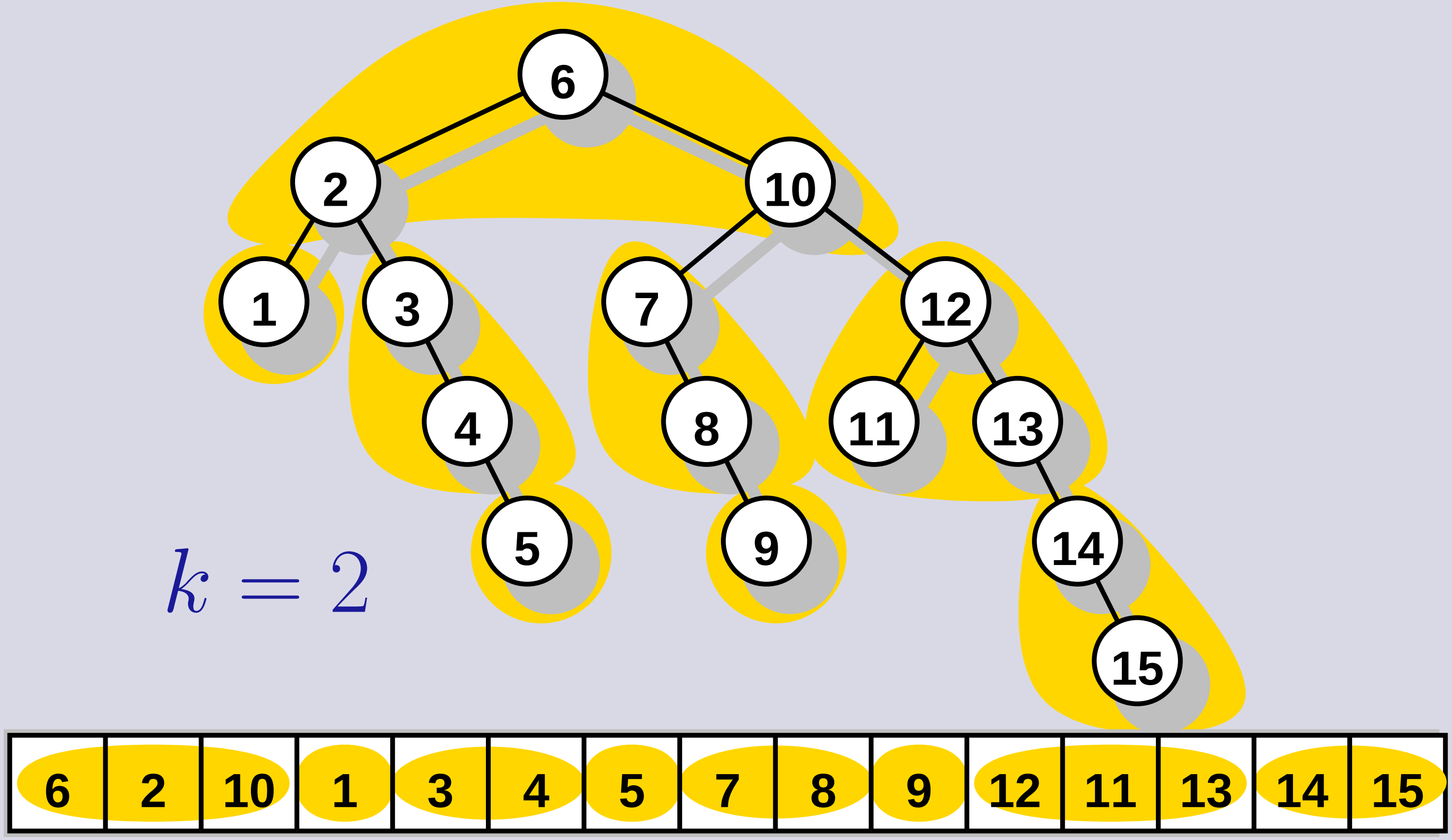
DFS achieves the best performance for $\alpha \approx 0.2$ !

# Cache Faults for Simple Layouts



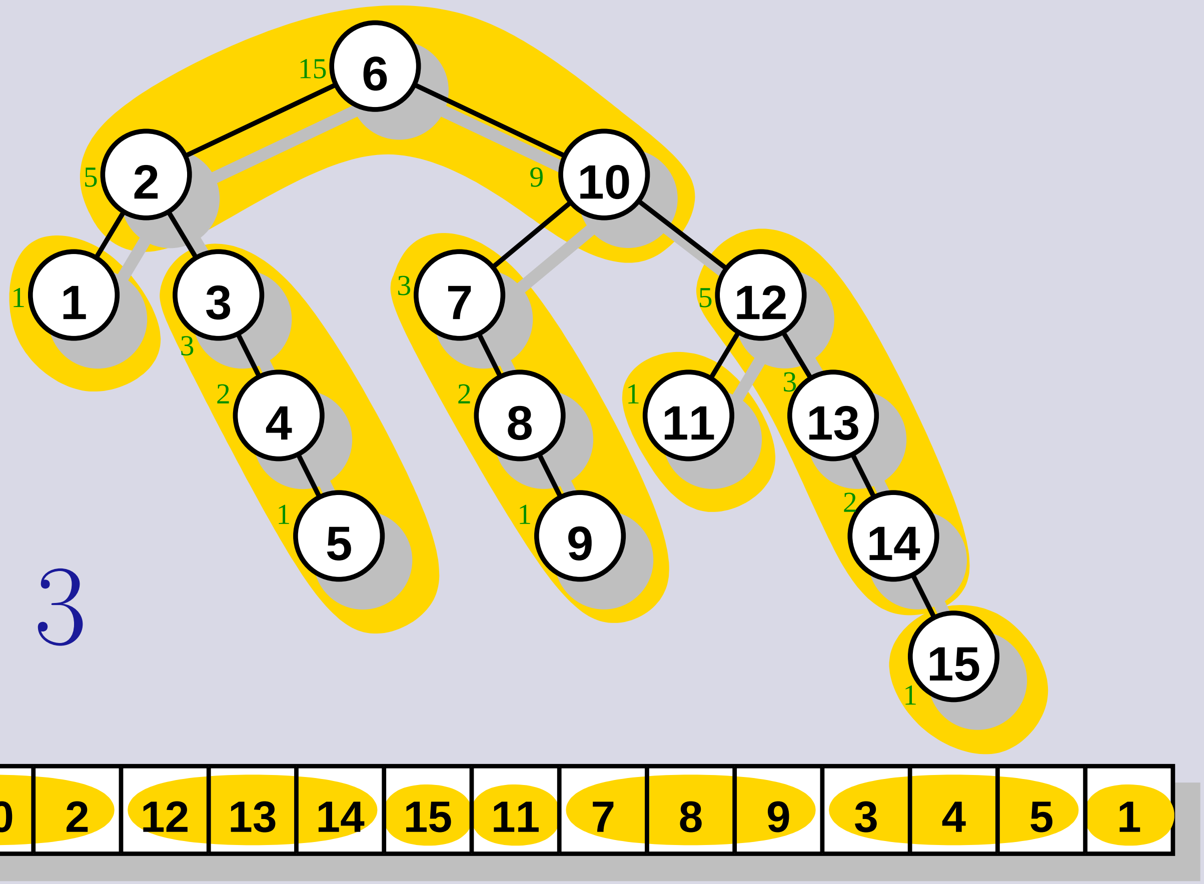

DFS $\approx$ expected left cost = 1 and right cost = $1/B$.

# Blocked Layouts — $k$-level blocking




- layout the nodes of the first $k$ levels
- recurse on subtrees
- a search uses $O(\log_B n / H(\alpha))$ I/Os

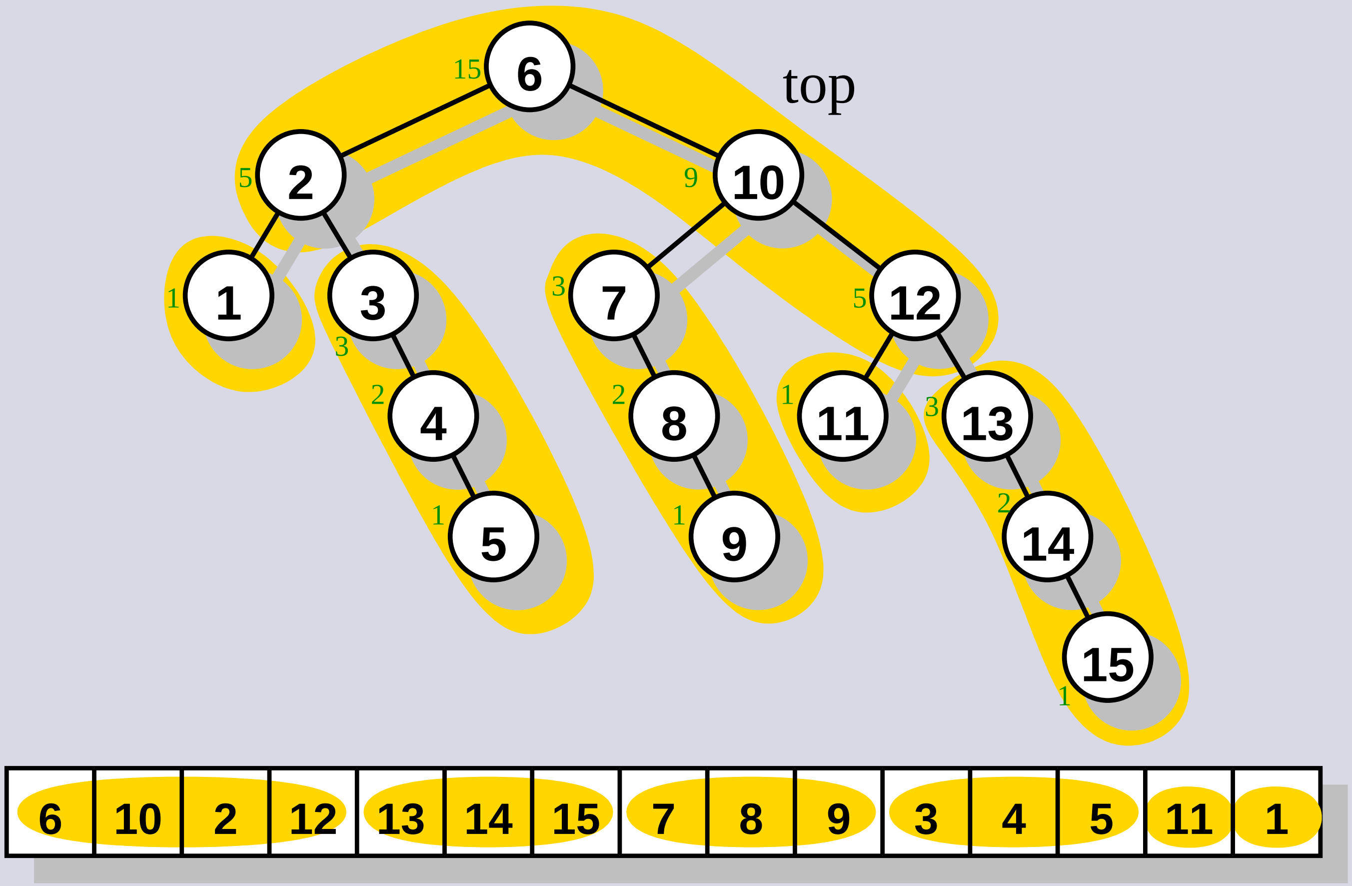
# Blocked Layouts — pqDFS$k$




$k = 3$

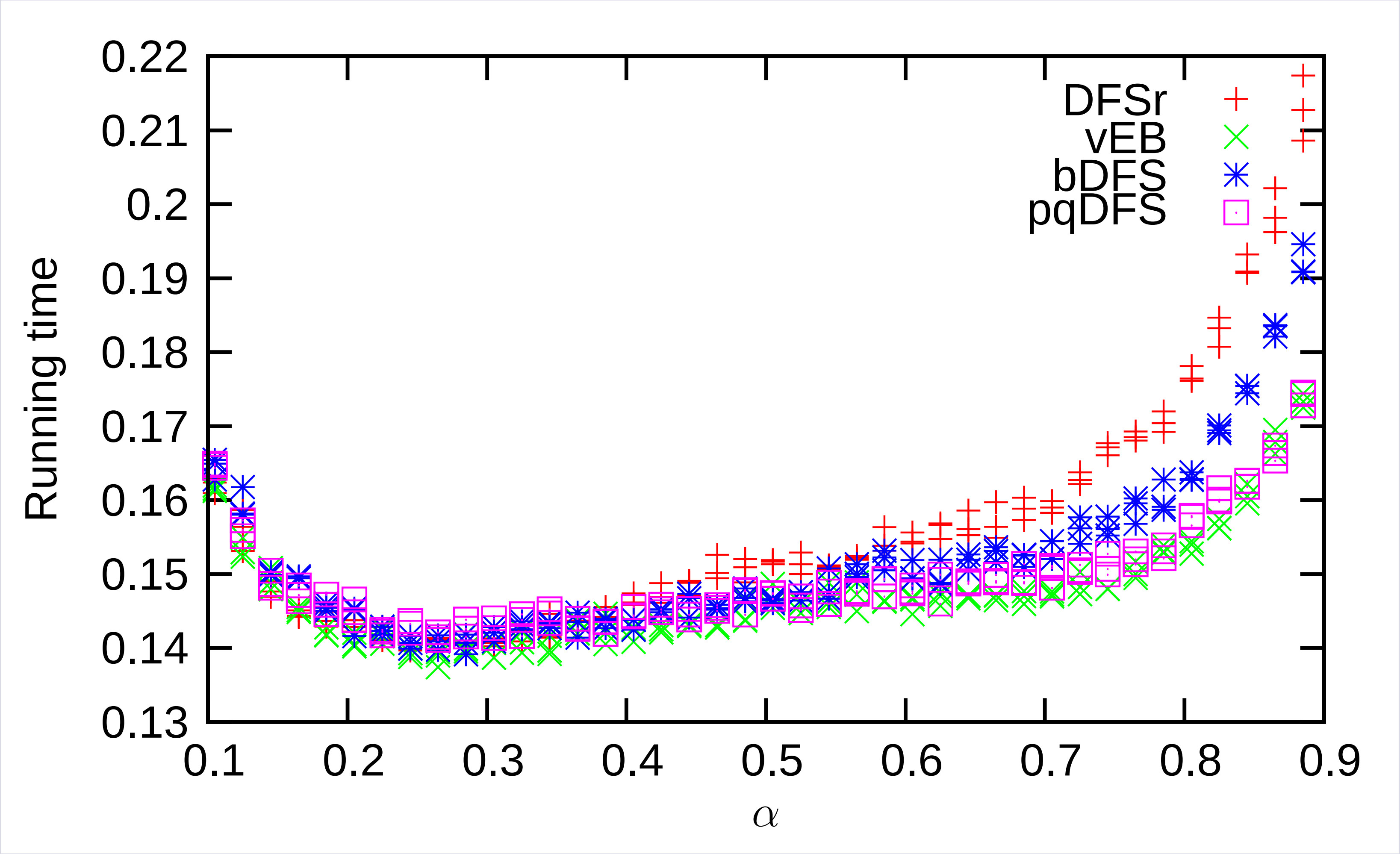
- layout the $k$ heavest nodes in order of decreasing size
- recurse on subtrees in order of decreasing size
- a search uses $O(\log_{B\alpha+1} n)$ I/Os

# Blocked Layouts — veb



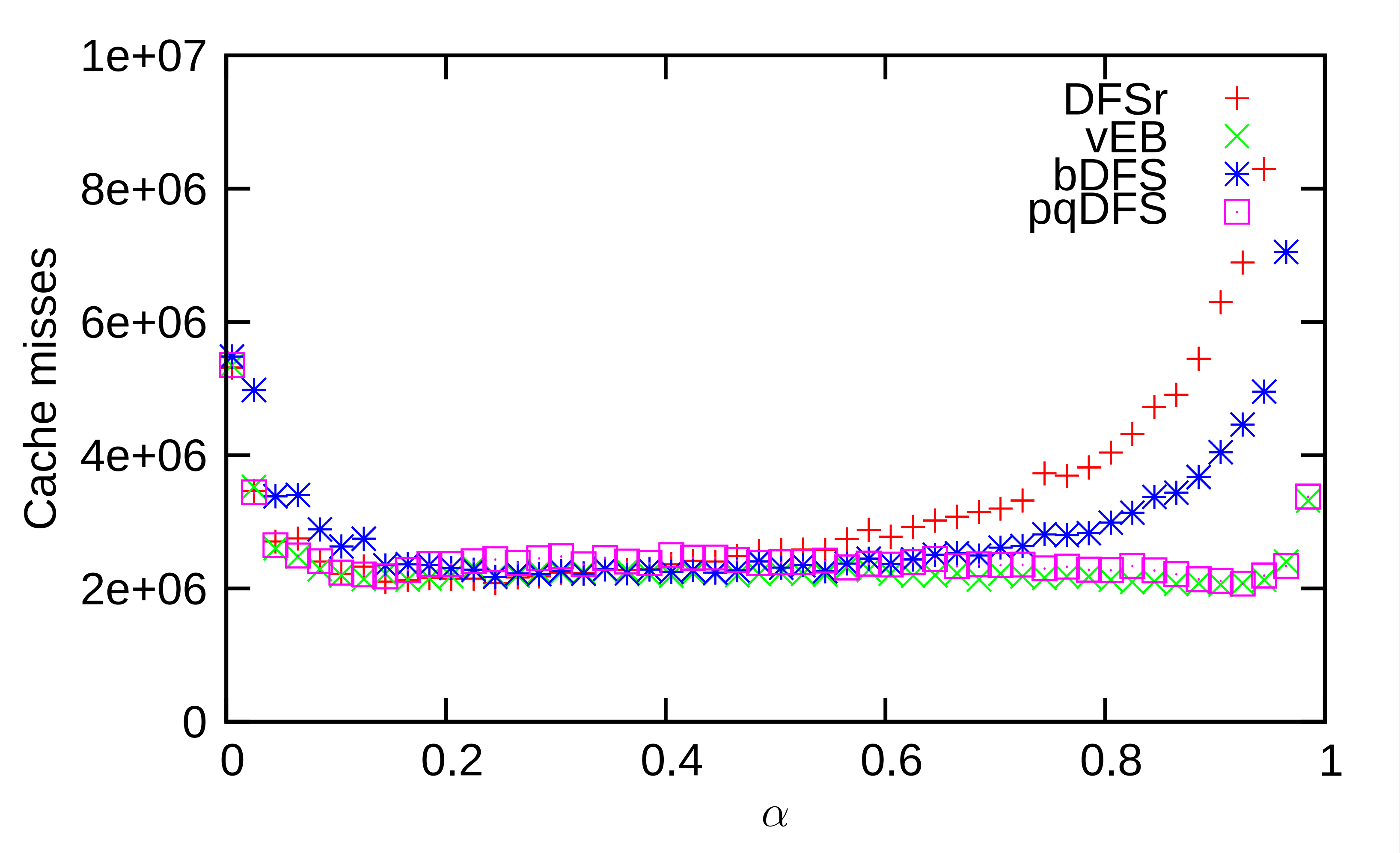

- top = $\lceil \sqrt{n} \rceil$ heavest nodes
- recurse on top and bottom trees in order of decreasing size
- a search uses $O(\log_{B\alpha+1} n)$ I/Os

# Running Time for Blocked Layouts



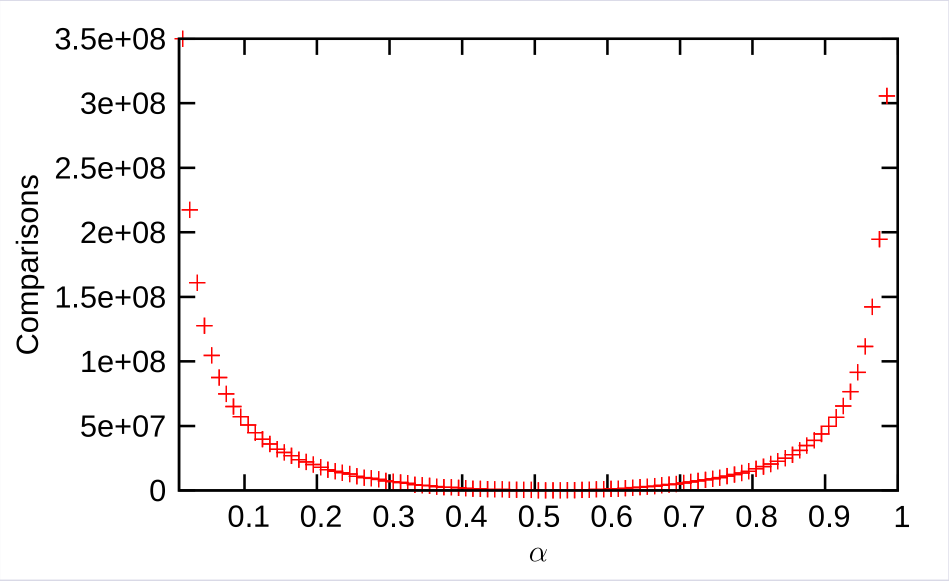

vEB achieves the fastest running time for $\alpha \approx .25$

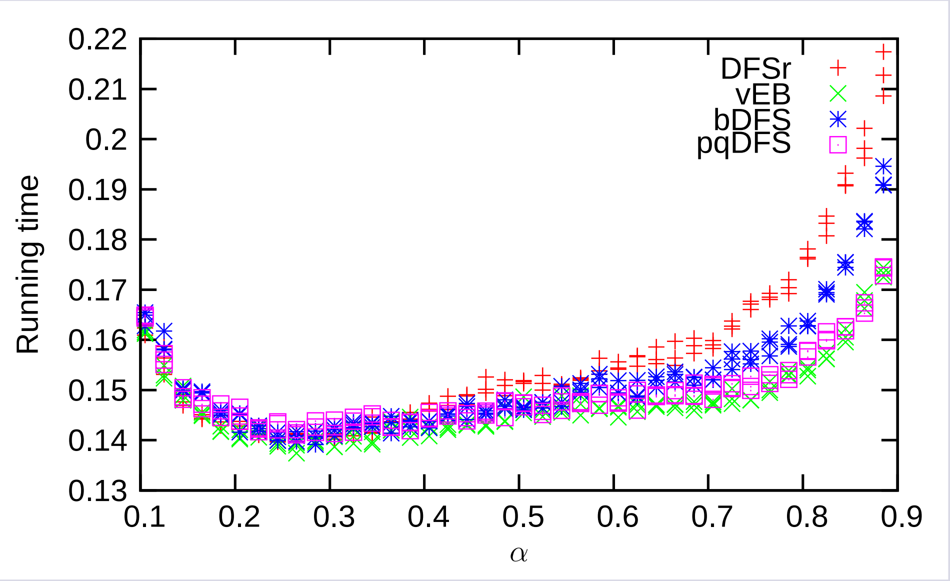
# Cache Faults for Blocked Layouts

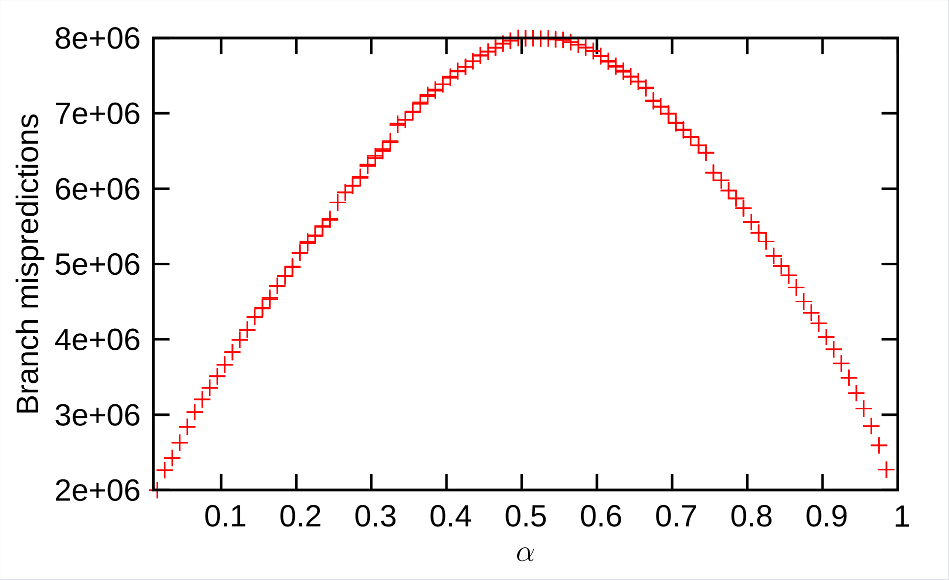


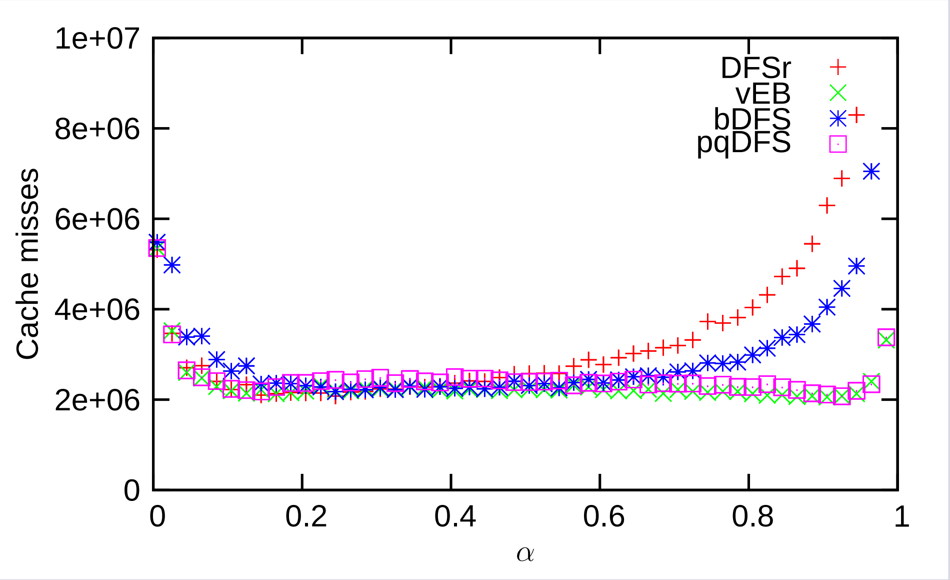

vEB achieves the smallest number of cache faults

# Experimental Summary



# Conclusion

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