Combinatorial Search. First problem set.

The first question (out of six) on the exam is “P, NP and NPC”. Let three random students present this question, based on the first two lectures.

Then, consider the following informally defined problems.

1. MULTIPLICATION: Given two integers, compute their product.
2. COMPOSITENESS: Given a natural number, decide if it is a non-trivial product of smaller numbers.
3. FACTORING: Given a natural number, compute its factorization into prime numbers.
4. LP: Given a linear program, compute an optimal solution. If no such solution exists, report this.
5. MILP: Given a mixed integer linear program, compute an optimal soloution. If no such solution exists, report this.
6. 01-INTEGER LINEAR INEQUALITIES: Given a system of linear inequalities, find a solution where all variables are either 0 or 1. If no such solution exists, report this.
7. SUBSET SUM: Given a finite set of numbers, partition the set into two subsets so that the sum of the numbers in one subset is the same as the sum of the numbers in the other subset. If no such partition exists, report that this is the case.
8. BIN PACKING: Given a finite set of numbers and an integer n, partition the numbers in n bins, so that the sum of the numbers in the bin with the largest such sum is minimized.
9. GRAPH COLOURING: Given an undirected graph, find a colouring of its vertices using the fewest possible number of colours, so that no two vertices of the same colour are connected by an edge.
10. EUCLIDEAN TSP: Given a set of points in the plane with integer coordinates, find the shortest traveling salesman tour (relative to the usual Euclidean distance).
11. BOOLEAN FORMULA EVALUATION: Given a Boolean (AND, OR, NOT) formula without variables (but with constants TRUE and FALSE) such as ”TRUE AND FALSE”, what is the boolean value of the formula?
12. BOOLEAN EQUATION: Given a single Boolean equation with several variables (such as ”X OR Y = X AND NOT(Y)”) find a solution, or report that no solution exists.
13. H: Given a Turing machine, does it eventually halt on the empty input tape?
14. HB: Given a Turing machine an a natural number t, does the machine eventually halt on the empty input tape in t or fewer steps?

Formally define associated formal languages for each of the problees. For which problems do you consider the defined language as completely capturing the difficulty of the associated computational problem? For which problems not quite so? Can you find alternative languagues for the latter that you are more happy about? For which problems do the choice of representation of the associated objects make a difference? Which of the associated languauges can you put in **P**? How? Which of the associated languages can you put in **NP**? How? Which of the problems do you *know* is not in **P**? Which of the problems do you *know* is not in **NP**? Which reductions can you find between the problems?